

# Optimization of Intentional Mistuning for Bladed Disk : Damping and Coupling Effect

## 블레이드 디스크의 Intentional Mistuning 최적화 : 감쇠와 커플링효과<sup>#</sup>

ByeongKeun Choi and WonChul Kim  
최 병 근<sup>†</sup> · 김 원 철<sup>\*</sup>

(Received December 27, 2004 : Accepted March 10, 2005)

**Key Words** : Bladed Disk(블레이드 디스크), Intentional Mistuning(의도적 비동조), Optimization(최적화), Genetic Algorithm(유전알고리즘)

### ABSTRACT

In turbomachinery rotor, there are small differences in the structural and/or geometrical properties of individual blades, which are referred to as blade mistuning. Mistuning effects of the forced response of bladed disks can be extremely large as often reported in many studies. In this paper, the pattern optimization of intentional mistuning for bladed disks considering with damping and coupling effect is the focus of the present investigation. More specifically, the class of intentionally mistuned disks considered here is limited, for cost reasons, to arrangements of two types of blades (A and B, say) and Genetic Algorithm and steepest descent method are used to optimize the arrangement of these blades around the disk to reduce the forced response of blade with different damping and coupling stiffness. Examples of application involving both simple bladed disk models and a 17-blade industrial rotor clearly demonstrate the significant benefits of using this class of intentionally mistuned disks.

### 요 약

터보기계에서 mistuning은 구조적, 기하학적인 측면에서의 blade와 blade 사이의 미소한 특성차이를 의미하며, blade의 제작과정이나 운전 중 발생하는 마모의 차이에 의해 발생한다고 알려져 있다. Blade사이에서 발생하는 이러한 미소한 차이가 강제 진동 시 아주 큰 국부진동을 야기 시킬 수 있다는 사실이 여러 논문들에 의해 확인되었다. 최근에는 조화패턴의 intentional mistuning배열을 사용하여 제작 및 사용 중에 발생하는 unintentional mistuning에 의한 blade의 강제진동 응답을 줄일 수 있다는 연구가 발표되었다. 따라서 본 논문에서는 두 가지 형태의 blade(A와 B)를 사용하고, blade감쇠와 coupling 효과를 고려하여 bladed disk의 강제진동응답을 줄일 수 있는 intentional mistuning의 최적배열패턴을 인공지능 알고리즘의 하나인 유전알고리즘과 steepest descent법을 이용하여 구하고자 한다. 그리고 단순 bladed disk와 17-blade로 된 산업체 로터의 수치예제를 통하여 intentional mistuning된 bladed disk의 이점을 증명하려고 한다.

<sup>†</sup> 책임저자 : 정희원, 경상대학교 기계항공공학부, 해양산업연구소  
E-mail : bgchoi@gsnu.ac.kr,

Tel : (055) 640-3059, Fax : (055) 640-3188

<sup>\*</sup> 정희원, 경상대학교 기계항공공학부, 해양산업연구소

<sup>#</sup> 이 논문은 2004추계학술대회에서 우수발표논문으로 추천되었음.

## 1. Introduction

In a dynamic analysis of a turbomachinery rotor, one traditionally has assumed that the blades are identical. But, in practice there are small

differences in the structural and/or geometrical properties of individual blades, which are referred to as blade mistuning. Much of the vast literature on this topic<sup>(1-5)</sup> has assumed these differences to be small and to arise either during the manufacturing process and/or as a consequence of in service wear. The motivation for considering such small variations is that their effects of the forced response of bladed disks can be extremely large as often reported in the above studies. Interestingly, the large sensitivity of the tuned system to these small variations has been linked<sup>(3)</sup> to its high level of symmetry.

In this light, it would appear beneficial to design bladed disk not to be tuned, namely to exhibit intentional mistuning, to reduce the sensitivity of the forced response to unintentional mistuning. Certainly, the consideration of intentional mistuning is not new<sup>(1,2)</sup> however, in the context of forced response, some papers<sup>(6,7)</sup> have only recently investigated the use of harmonic patterns of mistuning. Recently, the authors have investigated and identified the effect of intentional mistuning which can significantly reduce the magnification of the forced response due to unintentional random mistuning.<sup>(8)</sup>

In this paper, the pattern optimization of intentional mistuning for bladed disks considering with damping and coupling effect is the focus of the present investigation using the two sets of blades A and B. Genetic Algorithm and steepest descent method are used to obtain the pattern(s) that yields small/the smallest value of the largest amplitude of response to a given excitation in the absence of unintentional mistuning using simple model (one-degree-of-freedom per blade) of bladed disks and a 17-blade industrial rotor.

## 2. Optimization Approach

In view of the complexity and cost of intentional mistuning, one should not look simply at set patterns, for example the harmonic patterns<sup>(7)</sup> but

rather one should optimize the pattern to reduce as much as possible the amplification of the forced response to a given excitation or set thereof. Accordingly, it was suggested by authors that the use of intentional mistuning is probably not a standard design tool but would be very valuable if: (a) it yields a large decrease in sensitivity to unintentional mistuning, and (b) it involves a minimum number of types of blades, ideally 2.<sup>(8)</sup>

Therefore, in this paper, the disk will first be assumed to support only two different types of blades (blades A and B, say) and their arrangement that yields the smallest amplification of the forced response will be sought with different coupling stiffness and damping. The two sets of blades A and B were selected to have natural frequencies 5% lower and 5% higher, respectively, than the tuned ones (type C) in this paper.

The process described above has some rather dramatic computational implications. Indeed, for each mistuning pattern considered, it requires the determination of the largest amplitude of blade response that can be observed on a disk exhibiting both intentional and unintentional mistuning. At this point in time, however, reliable estimates of this largest amplitude can only be obtained by Monte Carlo simulations that are so computationally expensive that they can only be performed after a problem has been detected and on very simple dynamic models of the bladed disk. Accordingly, a straightforward application of the proposed optimization strategy could only be done for very simple bladed disk models. It was thus proposed to proceed slightly differently by: (1) performing the optimization in the absence of unintentional mistuning, and (2) obtaining a qualitative/quantitative estimate of the sensitivity of a given intentionally mistuned disk to additional unintentional random mistuning.<sup>(8)</sup> In this manner, the optimization effort as step (1) requires only one forced response evaluation per intentionally mistuned disk considered, as opposed to an entire

population.

Mistuning also produces a very nonlinear effect on the forced response. That is, by switching the order of the blades around the disk, dramatic differences can be obtained in the variability of the blade-to-blade amplitudes of vibration as exemplified in particular by the harmonic mistuning analysis of Mignolet et al.<sup>(1)</sup> It might thus be suspected that there exists a series of local optima in the complex, high dimensional space over which the optimization must take place. In this light, the present optimization effort has relied on the use of Genetic Algorithm (GA)<sup>(9)</sup>.

### 3. Simple Genetic Algorithm

GA are particularly well suited for the present effort because the design variables only admit discrete values (i.e. a specific blade is only of type A or B), see References (9,10) for further details. The simple genetic algorithm (SGA) used here relies on a population of  $n_{pop}$  bladed disks each of which is a random arrangement of N genes (the type A or B of the different blades). Accordingly, each bladed disk can be characterized by a sequence of N A and B letters, for example AABBBBAA..., which evolves from one

generation to the next according to the rules of selection, crossover, and mutation until all the chromosomes yield essentially similar values of the fitness or objective function (the maximum amplitude of blade response).

The fitness proportionate selection, the single point crossover technique and an exponentially decreasing mutation function was used in the present investigation. Also, the one elite reservation strategy was used in this paper according to which the best disk is retained unchanged from one generation to the next.

### 4. Simple Bladed Disk Model

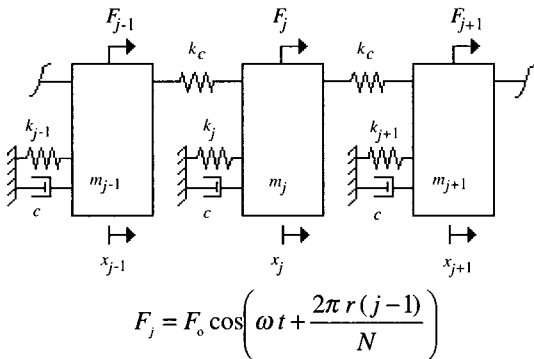
To identify the effect of coupling stiffness and damping to optimum pattern of intentional mistuning, the SDOF per blade model shown in Fig. 1 was first considered with four values of the coupling stiffness ( $k_c = 5000, 8606, 20000$  and  $45430$  N/m) and three values of damping coefficient ( $c = 0.143, 0.4, 0.9$  N·s/m) with only 12 blades was considered in this paper. Specifically, each of the  $N$  blades is represented as a single mass ( $m$ ) which is connected to the ground (i.e. the disk) and the aerodynamic and structural coupling between blades are modeled by springs ( $k_c$ ) and

**Table 1** The comparison of optimization result by Genetic Algorithm for each coupling stiffness level with the tune and other mistuning pattern

Coupling stiffness (N/m)	Tune system (all C)	Optimum solution by GA	Other pattern
5,000	1.1181e-3 m	2A2B : 1.0541e-3 m (5.7 %)	All A : 1.1787e-3 m, All B : 1.0684e-3 m AB : 1.1307e-3 m, 3A3B : 1.081e-3 m 4A2B : 1.2041e-3 m
8,606	1.10434e-3 m	2A2B : 1.0012e-3 m (9.34 %)	All B : 1.0561e-3 m, 2BA : 1.0249e-3 m 3A3B : 1.061e-3 m, CDCBAB : 1.11e-3 m
20,000	1.063e-3 m	2A2B : 8.388e-4 m (21.1 %)	All A : 1.1139e-3 m, All B : 1.0196e-3 m AB : 1.2047e-3 m, 3A3B : 1.0698e-3 m 4A2B : 8.4922e-4 m
45430	9.9167e-4 m	7A5B : 7.9776e-4 m (19.56 %)	All A : 1.0289e-3 m, All B : 9.5615e-4 m 2A2B : 8.5173e-4 m, A2B : 8.42271e-4 m 5A7B : 9.1139e-4 m

dashpots ( $c$ ). In the sequel, it is assumed that the coefficient vanishes in this paper. The values of mass ( $m=0.0114$  kg), and stiffness ( $k_t=430,300$  N/m) have already been used in a previous investigations to model a high-pressure turbine stage is used in this paper.

The computations proceeded as follows. The bladed disk model and engine order of the excitation were first selected. The SGA described above was then used to obtain the intentionally mistuned disk formed of blades A and B such that the maximum of its response over the entire frequency range was the smallest possible. First, the coupling stiffness effect to optimum pattern of intentional mistuning is identified. The value of the damping coefficient  $c$  was set to 0.143 N·s/m (approximately, 0.1 % of the critical value) and the two sets of blades A and B were selected to have natural frequencies 5 % lower and 5 % higher, respectively, than the tuned ones (type C). In analysis, the stiffness of A and B type blade is selected to have 10 % lower and 10 % higher than the tuned ones. The optimum pattern of intentional mistuning is searched with  $k_c=5000, 8606, \text{ and } 20,000$  N/m (weakly coupling system) and 45,430 N/m (an average to strong blade-to-blade coupling level) by Genetic algorithm.



$$F_j = F_0 \cos\left(\omega t + \frac{2\pi r(j-1)}{N}\right)$$

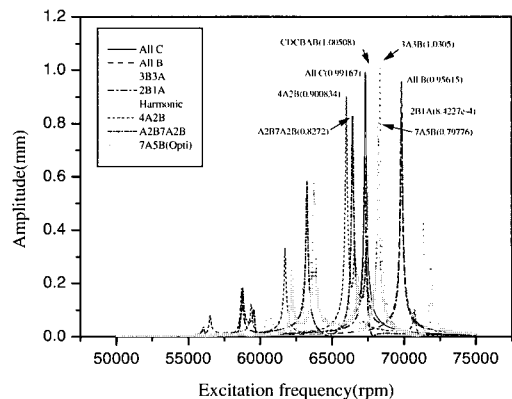
$m_i = 0.0114$  kg,  $k_t = 430,300$  N/m,  $c = 0.143$  Ns/m,  
 $N = 12$  blades,  $F_0 = 1$  N,  $r = 4$ ,  $k_j$ : normal distribution of mean  $k_t$  and standard deviation  $\sigma$

**Fig. 1** Single-degree-of-freedom per blade disk model

Table 1 show the comparison of optimization result by Genetic Algorithm for each coupling stiffness level with the tune system (all C) and other possible mistuning pattern in 4<sup>th</sup> engine order case.

In Table 1, the optimum pattern that yields small/the smallest value of the largest amplitude of response to a given excitation by GA was changed according to the coupling stiffness level. The genetic optimization algorithm yielded the configuration 7A5B the highest responding blade of which experiences an amplitude of vibration of 0.79 times (reduced amplitude about 19.56 %) the tuned value in strong coupling level while 2A2B is searched as optimum pattern in weakly coupling level. The largest amplitude on the disk can be reduced by up to 21.1 % by using a disk with an A/B blade pattern as opposed to a tuned one.

Fig. 2 and 3 shows the forced response of optimum, tuned and other A/B patterns with  $k_c=8606$  (weakly coupling system) and 45,430 N/m (strong coupling system). The notation 3A3B refers to disks formed of 2 groupings AAABBB or (AAABBB)2 and similarly 2B1A represents (BBA)4. It is identified that 2A2B(weakly coupling system) and 7A5B(strong coupling system) are better then tuned and harmonic patterns in an amplitude of vibration.



**Fig. 2** Comparison with forced response of optimum, tuned and other A/B pattern ( $k_c = 45,430$  N/m)

Now, the damping coefficient effect to optimum pattern of intentional mistuning is identified. The value of the coupling stiffness  $k_c$  was set to 45,430 N/m and the two sets of blades A and B were also used. The optimum pattern of intentional mistuning is searched with  $c=0.143, 0.4$  and  $0.9$  N·s/m by Genetic Algorithm. Table 2 show the comparison of optimization result by Genetic Algorithm for each damping coefficient with the tune system (all C) and other possible mistuning pattern in 4<sup>th</sup> engine order case.

The optimum pattern by GA was changed according to the damping coefficient in Table 2. The GA yielded the configuration 7A5B in damping coefficient  $c=0.143$  and  $0.4$  while 2A2B is founded as optimum pattern in damping

coefficient  $c=0.9$  N·s/m. But in case of  $c=0.9$  N·s/m, the largest amplitude of forced response for optimum pattern by GA, 2A2B have a little bit smaller than that of 7A5B pattern.

## 5. Application to an Industrial Rotor

A 17 bladed centrifugal compressor was selected to exemplify the optimization strategy described above to an industrial rotor. The compressor considered is an Auxiliary Power Unit (APU) load compressor with upstream inlet guide vanes (IGV's). The blisk rotor is known to be subjected to a strong upstream disturbance from the IGV's that excites the 5<sup>th</sup> characteristic blade mode. Accordingly, it is with this excitation that the optimization was conducted

The finite element model shown in Fig. 4 was built using cyclic symmetry and consisted of 17,474 eight-noded solid elements per sector. Specifically, the airfoil and disk portion of the model required 24,900 and 41,844 degrees-of-freedom, respectively.

The objective of the present application was to evaluate the benefits of intentional mistuning in reducing the sensitivity to random mistuning of the response of the rotor to the IGV engine order excitation ( $=14$ ). The external loading was represented as a single unit load located along the leading edge of the airfoil and normal to the

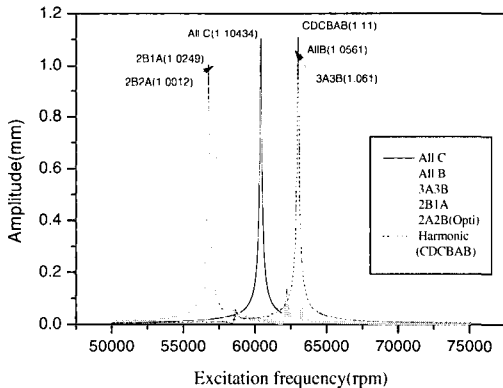


Fig. 3 Comparison with forced response of optimum, tuned and other A/B pattern ( $k_c=8,606$  N/m)

Table 2 The comparison of optimization result by Genetic Algorithm for each damping coefficient with the tune and other mistuning pattern

Damping coefficient (N·s/m)	Tune system (all C)	Optimum solution by GA	Other pattern
0.143	$9.9167e-4$ m	7A5B : $7.9776e-4$ m (19.56 %)	All A : $1.0289e-3$ m, All B : $9.5615e-4$ m 2A2B : $8.5173e-4$ m, A2B : $8.4227e-4$ m 5A7B : $9.1139e-4$ m
0.4	$3.5461e-4$ m	7A5B : $2.9127e-4$ m (17.86 %)	All A : $3.6879e-4$ m, All B : $3.4187e-4$ m 2A2B : $3.1084e-4$ m, 1A2B : $3.1023e-4$ m 5A7B : $3.5063e-4$ m
0.9	$1.5761e-4$ m	2A2B : $1.432e-4$ m (9.14 %)	All A : $1.6396e-4$ m, All B : $1.5194e-4$ m A2B : $1.45567e-4$ m, 5A7B : $1.7515e-4$ m 7A5B : $1.4399e-4$ m

surface of the blade. This excitation is clearly a simplification of actual engine conditions in which the blades are subjected to unsteady pressure loads. Structural damping was estimated from test data and used in the forced response calculations. Finally, the viscous damping associated with the blade/air interaction was not considered.

The effects of intentional mistuning on the maximum forced response of the rotor were considered. Following the previous discussion, this analysis was initially done without any unintentional mistuning and only patterns of two blades were considered. While blade "C" always corresponded to a tuned one, blade "A" was softer than tuned, i.e. with a Young's modulus reduced from its tuned counterpart by 2.5 %, 5 %, 7.5 %, and 10 %. These changes in blade stiffness can be construed as originating from a uniform distribution with standard deviations of 0.72 %, 1.44 %, 2.16 %, and 2.88%. The combination of subproblem approximation and steepest descent algorithms, as available in ANSYS, was used to obtain the optimum patterns of the A and C blades.

The optimized pattern corresponding to the 7.5 % decrease in stiffness, i.e.2CAC3A2C8A, was used to estimate the sensitivity of the response of such intentionally mistuned disks to random (unintentional) mistuning. The results, see Fig. 5, clearly show that this pattern has a much reduced

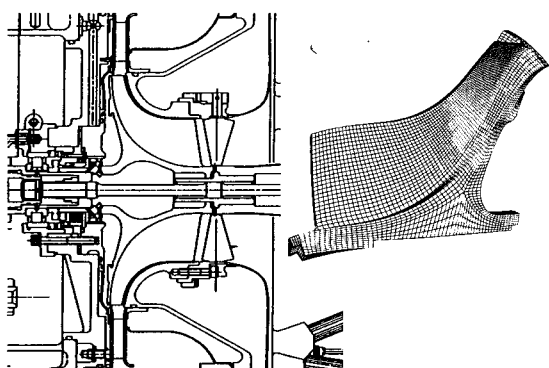


Fig. 4 Single sector structural model of the 17-blade centrifugal compressor rotor

sensitivity to unintentional mistuning as compared to the tuned (all C) disk for low values of the standard deviation of mistuning. When the level of unintentional mistuning becomes comparable to its intentional counterpart, the benefits of using different blades becomes substantially reduced but no worsening has been noted. Also, the standard deviation of unintentional mistuning at which the maximum magnification occurs tends to be pushed toward higher levels in Fig.5. It occurs at 0.5 %~1 % without intentional mistuning and at 2.5 %~3.5 % with intentional mistuning which creates an additional benefit when the variability of the frequencies of the manufactured blades is low. That is, the response penalty for achieving very

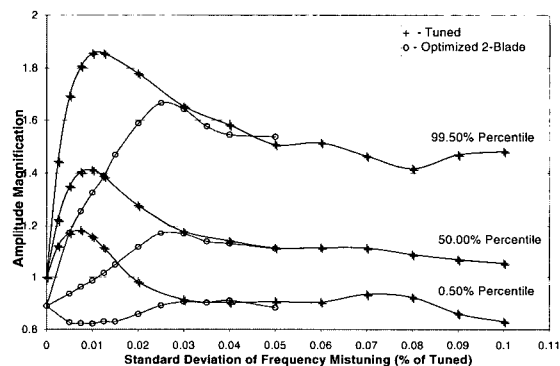


Fig. 5 Sensitivity of the tuned and optimized intentionally mistuned disks to unintentional random mistuning

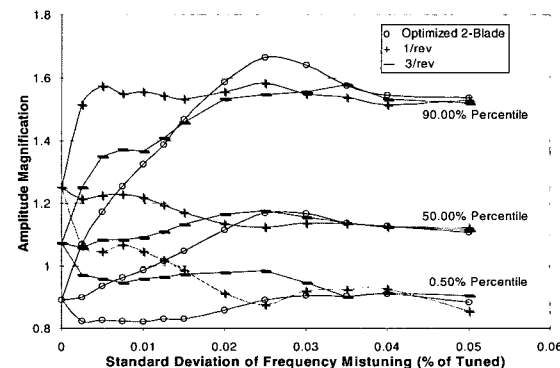


Fig. 6 Sensitivity of the optimized and some harmonically intentionally mistuned disks to unintentional random mistuning

tight manufacturing control of the geometric parameters of the airfoils has been substantially reduced.

As a final assessment of the benefits of the optimized pattern, a comparison of the above results with those produced with approximate 1 and 3 per rev arrangements of A and C blades (i.e. 9C8A and 3C3A3C3A3C2A, respectively) is shown in Fig. 6.

## 6. Summary

The investigation of this paper focused on the pattern optimization of intentional mistuning for bladed disks considering with damping and coupling effect using the two sets of blades. Two different optimization algorithms, one based on steepest descent and one genetic algorithm, have been tested and shown to perform well in finding the A/B pattern yielding the smallest maximum response in the absence of unintentional mistuning using simple model (one-degree-of-freedom per blade) of bladed disks and 17-blade industrial rotor.

Through the optimization, it is found that the optimized pattern considering with damping effect and coupling effect may or may not appear as variations of simple harmonic patterns, the patterns 7A5B, 2A2B on the 12-blade disk may appear as distorted 1, 2, 3, and 4 harmonics of mistuning but the distortion plays an important role in reducing the amplitude magnification. Further, the 2CAC3A2C8A arrangement obtained in connection with the 17-blade industrial rotor is not close to any specific harmonic pattern. It would appear that patterns of two blades close to simple harmonics are in general less robust to unintentional mistuning than those that are not.

According to the sensitivity of the response of intentionally mistuned disks to random (unintentional) mistuning, the standard deviation

of unintentional mistuning at which the maximum magnification occurs tends to be pushed toward higher levels. That is, the response penalty for achieving very tight manufacturing control of the geometric parameters of the airfoils has been substantially reduced

Also it is identified that the optimum pattern of intentional mistuning by GA was changed according to the coupling stiffness level and damping coefficient in Table 1 and 2. Therefore, the effect of coupling stiffness and damping coefficient should be considered to optimize the intentional mistuning pattern which can reduce the forced response in blade.

## Acknowledgements

This work was supported by the NURI and BK21 project.

## References

- (1) Kaza, K. R. V. and Kielb, R. E., 1984, "Flutter of Turbofan Rotors with Mistuned Blades", *AIAA Journal*, Vol.22, No.11, pp. 1618~1625.
- (2) Kaza, K. R. V. and Kielb, R. E., 1985, "Vibration and Flutter of Mistuned Bladed-Disk Assemblies", *Journal of Propulsion*, Vol.1, No.5, pp.336~344.
- (3) Wei, S. T. and Pierre, C., 1988, "Localization Phenomena in Mistuned Assemblies with Cyclic Symmetry - Part I: Free Vibrations", *Journal of Vibration, Acoustics, Stress, and Reliability in Design*, Vol.110, No.4, pp.429~438.
- (4) Mignolet, M. P., Hu, W. and Jadic I., 2000, "On the Forced Response of Harmonically and Partially Mistuned Bladed Disks, Part I: Harmonic Mistuning", *International Journal of Rotating Machinery*, Vol.6, No.1, pp.29~41.
- (5) Mignolet, M.P., Hu, W. and Jadic I., 2000, "On the Forced Response of Harmonically and

Partially Mistuned Bladed Disks. Part II: Partial Mistuning and Applications”, *International Journal of Rotating Machinery*, Vol. 6, No. 1, pp. 43~56.

(6) Castanier, M. P. and Pierre, C., 1997, “Predicting Localization via Lyapunov Exponent Statistics”, *Journal of Sound and Vibration*, Vol. 203, No. 1, pp. 151~157.

(7) Castanier, M. P. and Pierre, C., 1998, “Investigation of the Combined Effects of Intentional and Random Mistuning on the Forced Response of Bladed Disks”, Paper AIAA-98-3720.

(8) Choi, B. K., 2003, “Pattern Optimization of

Intentional Blade Mistuning for the Reduction of the Forced Response Using Genetic Algorithm”, *KSME International Journal*, Vol. 17, No. 7, pp. 966~977.

(9) Gen, M. and Cheng, R., 1997, *Genetic Algorithms & Engineering Design*. New York, Wiley-Interscience.

(10) Choi, B. K. and Yang, B. S., 2000, “Vibration Reduction of Rotor Systems Using Genetic Algorithm,” *Transactions of Korea Society for Noise and Vibration Engineering*, Vol. 10, No. 7, pp. 199~206.