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Case Studies of Developing Creativity through Integrating Algorithmic Teaching into Mathematical Activities

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In this increasingly technological world, the creativity development has been highlighted much in many countries. In this paper, two mathematical activities with Chinese characteristics are presented to illustrate how to integrate algorithmic teaching into mathematical activities to develop students' creativity. Case studies show that the learning of algorithm can be transferred into creative learning when students construct their own algorithms in Logo environment rather than being indoctrinated the existing algorithms. Creativity development in different stages of mathematical activities and creativity development in programming are also discussed.

Keywords: creativity development, algorithmic teaching, mathematical activity, Logo
ZDM Classification: D30, U50
MSC2000 Classification: 97D30, 97C30

INTRODUCTION

Innovation is the soul of a nation, also the great impetus for a thriving and prosperous country[1]. In this increasingly technological world, the creativity development has been highlighted much in many countries (Shin 2000). In china, the cultivation of innovative spirit and practical ability was documented as one of the most important curricular goals in *the guidelines for curriculum reform in the primary and secondary school* in 2001. As its subjective status and features, mathematics has the closer relation with the topic of creativity. Some studies show that providing problems related to games, puzzles, or sports, studies and projects can promote students' interests and develop mathematical creativity

[1] Jiang, Z. M.: The talk with the commissioners of science and technology field in political negotiation committee. *People Daily* May 3, 1998.

(Linda 2005), but there are not many studies about how to integrate mathematical teaching of some certain fields into mathematical activities. With the background of the mathematics curriculum reform for primary and secondary school in China, the author will explore the creativity development through integrating algorithmic teaching into mathematical activities.

The algorithm refers to the step-by-step systematic procedure used to accomplish an operation (Arthur 1978). Being considered as a kind of necessary mathematical quality for modern people, the algorithmic idea is gradually emphasized in mathematical educational circles. In 2003, *the Standards of Mathematics Curriculum for the Senior High School* were promulgated in China and the algorithm was listed as one of the new contents. This is the first time that the term of algorithm was been represented in the mathematical textbooks for school. So the study about algorithmic teaching and the study about transferring the learning of algorithm into creative learning undoubtedly have practical implications for mathematical education especially in China.

At present, as the process of problem solving and mathematical application in the realistic life are highlighted in modern mathematics teaching reform, the learning and understanding of the algorithmic process is especially emphasized. It's advocated in almost every country that the traditional way of teaching algorithm should be changed by the way of teaching students to design their own algorithms and to solve realistic problems through using the algorithmic idea, what's more, students should decide their own approaches and steps (Xu 2001).

In this paper, two mathematical activities with Chinese characteristics are presented to illustrate how to integrate algorithmic teaching into mathematical activities, in which students are assigned concrete tasks which can be divided into some sub-tasks with the guidance of the teacher. Case studies show that in the task-driven teaching, the teacher guides students to accomplish the sub-tasks to experience the *initiative* of algorithms through their constructive learning in the Logo environment rather than indoctrinating the existing algorithms, thus develop students' creativity and transfer the learning of algorithm into a kind of creative learning. Creativity development in different stages of mathematical activities and creativity development in programming are also discussed.

CASE STUDY AND THE MATHEMATICAL ACTIVITY BASED ON GAME "JIGSAW PUZZLE"

The Jigsaw Puzzle is a great creation of Chinese ancient people, consisting of seven regularly shaped pieces of cardboard, plastic, or wood that can form a picture when fitted together, also named Oriental Magic Cardboard or Picture Puzzle. As simple it is, it can form hundreds and thousands of pictures, such as animals, architectures, alphabets and so on, from which enjoyment can be gotten and intellectual development can be obtained.

Task

To form a cat-picture through fitting the shaped pieces of the Jigsaw Puzzle in Logo

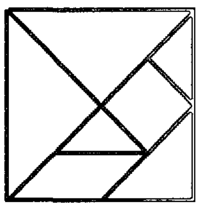


Figure 1. Jigsaw Puzzle

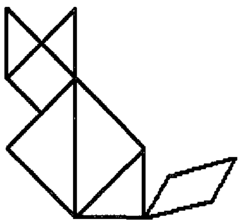


Figure 2. Cat

Purpose

To apply the algorithmic ideas to solve mathematical problems and to develop creative thinking through experiencing the process of transferring the operation of problem solving into programs.

Stages

Stage 1 Tasks analysis

1) Analysis of the elements of Jigsaw Puzzle firstly; The seven regularly shaped pieces of the Jigsaw Puzzle include 5 isosceles right triangles, 1 parallelogram and 1 square, which can form a bigger square (see Figure 1). To understand the inherent relation of the sides and angles of these shaped pieces is the key of fitting, which can be obtained through the geometrical computing: Suppose that the length of the side of Jigsaw Puzzle (namely the bigger square) is *a*, then the 5 isosceles right triangles with sides of length of

$$\frac{\sqrt{2}}{2}a,\ \frac{\sqrt{2}}{2}a,\ a;\quad \frac{\sqrt{2}}{2}a,\ \frac{\sqrt{2}}{2}a,\ a;\quad \frac{1}{2}a,\ \frac{1}{2}a,\ \frac{\sqrt{2}}{2}a;\quad \frac{\sqrt{2}}{4}a,\ \frac{\sqrt{2}}{4}a,\ \frac{1}{2}a;\quad \frac{\sqrt{2}}{4}a,\ \frac{\sqrt{2}}{4}a,\ \frac{1}{2}a,$$

respectively; the parallelogram with the adjacent sides of length of

$$\frac{\sqrt{2}}{4}a \text{ and } \frac{1}{2}a,$$

one of the internal angle of 45°; the square with side of length of

$$\frac{\sqrt{2}}{4}a \text{ (in this paper, } \sqrt{2} \approx 1.414\text{)}.$$

2) The sub-tasks are to program for the isosceles right triangle, the parallelogram and the square.

Stage 2 Sub-tasks accomplishment

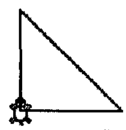
According to the characteristics of these regular geometrical figures, students can program easily in Logo which has some unique characteristics (we will discuss it in the next subsection).

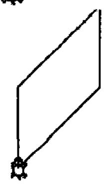
The programs for the isosceles right triangle, the parallelogram and the square are as follows.

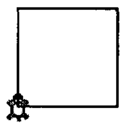
```
TO TRIANGLE: X
    FD: X RT 135 FD: X*1.414 RT 135 FD: X RT 90
END

TO PARALLELOGRAM: X
   REPEAT 2 [FD: X RT 45 FD: X*1.414 RT 135]
END

TO SQUARE: X
   REPEAT 4 [FD: X RT 90]
END
```







Stage 3 Task performance

From Figure 2, we can find the elements of the cat-picture that the two smaller congruent isosceles right triangles are something similar to the ears, the square to the face the two bigger congruent isosceles right triangles to the chest and the other isosceles right triangle to the crouching legs, while the parallelogram to the tail.

So the operation can begin with the procedure of TRIANGLE to draw the two smaller congruent isosceles right triangle with the right side of length of

$$\frac{1}{2}a,$$

through translation and rotation of the turtle to the right location, continue the similar operation of procedure of the square, then the two bigger congruent isosceles right triangles, the other isosceles right triangle and the parallelogram lastly.

A new algorithm can be created through this process, which can be translated into the following CAT program.

```
TO CAT: B
    CS HT
    MAKE "B1: B/1.414
    MAKE "B2: B/2
    MAKE "B3: B1/2
    RT 45 TRIANGLE: B3 RT 180 TRIANGLE: B3
    LT 90 SQUARE: B3 FD :B3 LT 90
    BK: B1 TRIANGLE: B1 FD: B1 RT 90 F: B1
    RT 90 TRIANGLE: B1 LT 45 FD: B2 RT 90
    TRIANGLE: B2 RT 120 PARALLELOGRAM: B3
END
```

Stage 4 Enrichment exploration

The CAT algorithm is just only one of the many ways to accomplish the operation. Can you create other algorithms to form the same cat, such as beginning with the parallelogram, or the square?

Many animal-pictures can be formed through fitting the shaped pieces of the Jigsaw Puzzle together; can you create some algorithms for them, for example, a penguin (see Figure 3)? (The following is one of the programs for a penguin.)

```
TO PENGUIN: B
    CS HT
    MAKE "B1: B / 1.414
    MAKE "B2: B / 2
    MAKE "B3: B1 / 2
    RT 135 SJX: B3 RT 135 SJX: B1
    RT 90 FD: B1 RT 90 SJX: B1
    FD: B1/7 LT 90 SJX: B2
    FD: B2 LT 45 BK: B3 SBX: B3
    RT 45 FD: B2 LT 45 ZFX: B3
    RT 90 FD: B3 LT 90 SJX: B3
END
```

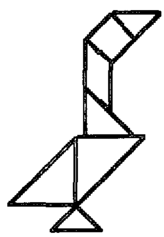


Figure 3. Penguin

CASE STUDY AND THE MATHEMATICAL ACTIVITY BASED ON DRAWING "NATIONAL FLAG OF CHINA"

The National Flag of China is the red five-star-shaped flag, consisting of five yellow pentagrams, the bigger one surrounded by the four similar smaller ones (see Figure 4), with profound symbolic meaning, very familiar to students, easily to arouse their interests and motivation.

Task

To draw a National Flag of China

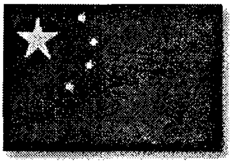


Figure 4. National Flag of China

Purpose

To develop the ability of solving realistic problems and to develop algorithmic ideas through programming in Logo.

Stages

Stage 1 Tasks analysis

1) To draw the pentagrams and to color they are the sub-tasks;
2) Searching for related resources to further confirm the distances between the pentagrams and their right locations in the flag.

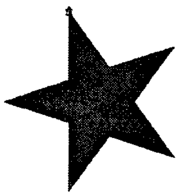
Stage 2 Sub-tasks accomplishment

The angle is the key of solving the algorithmic problem of pentagram. Through connecting the diagonal of the pentagon, we can conclude that the internal angles of the intersection of two adjacent sides are 36° and 252°. A study (Peng 2005) shows that due to the specialty of the angles, students are easily confused by the relation between the angles.

About the color, to add one more parameter to the program for drawing a pentagram is enough. If the beauty is considered, a precise thinking way and a open mind should be needed. The programs for the sub-tasks are as follows.

```
TO STAR1: I
    REPEAT 5 [FD: I RT 144 FD: I LT 72]
END
```

```
TO STAR2: I: C
    SETPC: C REPEAT 5[FD: I RT 144 FD: I LT 72]
    RT 72 RU FD 2 PD FILL
    PU BK 2 LT 72 FD: I
END
```



Stage 3 Task performance

Through review the collected information about the design of National Flag of China, we can find it's such rather professional work that much elaborate work should be deal with, such as the ratio of the height and the length of the flag is 3 to 2, the diameter of the circumscribed circle of the bigger pentagram and the smaller ones are the 3/10 and 1/10 of the height, respectively, and the smaller ones arch near the right of the bigger one, every pentagrams have the fixed and elaborating location and so on. For convenience, we attempt to draw a flag similar but not congruent to the National Flag of China, named red-flag.

We can take the side of the pentagram as the parameter, through changing the parameter to determine the right location of the turtle, and draw the four pentagrams one by one and the bigger pentagram lastly. The programs for the operations are in the following:

```
TO RED-FLAG: I
    SETPC 12 PD FILL
    STAR2: I 14
    FD 3*: I PD STAR2: I 14
    LT 90 FD 3*: I RT 90 FD 2*: I
    PD STAR2: I 14
    BK 12*: I
    PD STAR: I 14
    FD 5*: I LT 90 FD 3*: I RT 90
    PD STAR 2*: I 14
END
```

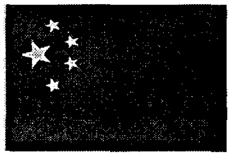


Figure 5 red-flag

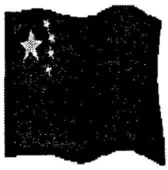


Figure 6 waving flag

Stage 4 Enrichment exploration

Try to imagine a National Flag of China waving in the wind (see Figure 6); can you create an algorithm for the waving flag? About the algorithm design for the animated flag, more creativity should be needed, other more related orders such as *while* should be introduced.

DISCUSSION

Creativity Development in Different Stages of the Mathematical Activities

Taking advantage of the mode of task-driven teaching, four stages with the different roles can be introduced in the mathematical activities, namely, the precondition, the base, the key and the enrichment, in which different levels of creativity can be developed. For example, in the second stage of sub-tasks accomplishment, generally, the exiting mathematical algorithms are expressed through the Logo language while the creative work is to determine parameters to expand the mathematical algorithms, thus it may help students gain a deeper insight of basic mathematical patterns and concepts, and enhance their problem solving skills. While in the last two stages, students should construct their own algorithms based on previous algorithms and the exploration of the connections between different mathematical procedures, thus it may enable students to attain an integrative and interdisciplinary understanding of knowledge and skills in diverse fields so as to become knowledge creators.

Creativity Development and Logo

Previous studies shows that Logo has some unique advantages (Clements & Battista 1989, 1992) in that it can link children's intuitive knowledge about moving and drawing to more explicit mathematical ideas, can encourage the manipulation of specific shapes in ways that help students to view them as mathematical creators of a class of shapes, can facilitate students' development of autonomy in learning (rather than seeking authority) and positive beliefs about the creation of mathematical ideas, can encourage wondering about and posing problems by providing an environment in which to test ideas and receive feedback about these ideas (Clements 1994). It is supported again by the evidence of the two cases illustrated here. Taking the Jigsaw Puzzle for example, it is not difficult for students to form a cat-picture by manual operation because they can attempt to fit freely, but not easy for them to work through computers in that accurate computing should be done at first. On the other hand, Logo drawing helps students create pictures that are more elaborate than those they can create by hand, which shows the power of deeper understanding mathematical algorithms of Logo.

Creativity Development and Programming

Due to the way of mechanical operation, the learning of algorithm was thought as a kind of uncreative learning in the psychology. Is it really so? Let's see the programs appeared in the above mathematical activities which almost involve one or more parameters. Firstly, through changing the parameter, the corresponding computing and drawing can be performed, thus it enables students to observe and find the mathematical rules; Secondly, through fitting the fundamental programs together, students can create much more complex programs to perform creative activities. Is not it creativity? Thirdly, after being indoctrinated an existing algorithm, students can explore other algorithms for the same problem, which is a more creative process. Taking the drawing "National Flag of China" for example, there are many other algorithms created by students themselves to draw a pentagram (Peng 2005). Thus, from the bottom of programming language, students can construct edifice of mathematical knowledge gradually and can use it more flexibly, during which students' creative ability can be developed.

CONCLUSION

Two examples of case studies and mathematical activities integrated with the algorithmic teaching are illustrated in this paper. These case studies and mathematical activities can be used in teaching and learning mathematics at various school levels, better for the lower secondary level. From the above discussion, we can conclude that through integrating algorithmic teaching into mathematical activities, it is helpful to promote students to understand the constructive process of algorithm. And the more meaningful thing is that students can create their own algorithms, ultimately nurturing them to become creative problem-solvers.

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