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INTUITIONISTIC FUZZY o-SUBALGEBRAS OF BCK-ALGEBRAS WITH CONTIDITON (S)

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Abstract. In this paper, some properties of intuitionistic fuzzy o-subalgebras of BCK-algebra with condition (S) are investigated.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [12]. Since then these ideas have been applied to other algebraic structures such as semigroups, groups and rings, etc. In 1991, Xi [11] applied the concept of fuzzy sets to BCK-algebras which are introduced by Y. Imai and K. Iséki in 1966 [6]. K. Iséki [5] introduced the notion of BCK-algebra with condition (S) and several researchers considered the fuzzification of it. Recently, Y. B. Jun et al. [7] introduced the notion of fuzzy $\circ$-subalgebras in BCK-algebras with condition (S). In 1986, K. T. Atanassov [1] introduced the notion of intuitionistic fuzzy set which is a generalization of the notion of fuzzy set. S. M. Hong et al. [4], using the Atanassov's idea, introduced the concept of intuitionistic fuzzy subalgebras in BCK-algebras and S. M. Hong and H. G. Kim [3] studied the Cartesian product of fuzzy $\circ$-subalgebras in BCK-algebras with condition (S). In this paper, we introduce the notion of intuitionistic fuzzy o-subalgebra in BCK-algebras with condition (S) and investigated some of their properties.

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2. Preliminaries

DEFINITION 2.1. An algebra $(X, *, 0)$ of type (2,0) is called a BCK-algebra if for all $x, y, z \in X$ the following conditions hold:

(a) $((x * y) * (x * z)) * (z * y) = 0$
(b) $(x * (x * y)) * y = 0$
(c) $x * x = 0$
(d) $0 * x = 0$
(e) $x * y = 0$ and $y * x = 0$ imply $x = y$.

For any BCK-algebra X, the relation $\leq$ defined by $x \leq y$ if and only if $x * y = 0$ is a partial order on X.

DEFINITION 2.2. A BCK-algebra X is said to be *with condition (S)* if for all $x, y \in X$, the set $\{z \in X | z * x \leq y\}$ has a greatest element, written $x \circ y$.

In any BCK-algebra X with condition (S), the following holds : for all $x, y \in X$

(1) $x \leq x \circ y$, $y \leq x \circ y$,
(2) $x \circ 0 = 0 \circ x = x$,
(3) $x \circ y = y \circ x$.

DEFINITION 2.3. [3] Let X be a BCK-algebra with condition (S) and let S be a nonempty subset of X. Then S is called a *$\circ$-subalgebra* of X if, for any $x, y \in S$, $x \circ y \in S$.

DEFINITION 2.4. [3] A map $f : X \to Y$ of BCK-algebras with condition (S) is called a *$*$-homomorphism* (resp. *$\circ$-homomorphism*) if $f(x * y) = f(x) * f(y)$ (resp. $f(x \circ y) = f(x) \circ f(y)$) for all $x, y \in X$. If f is both a $*$-homomorphism and a $\circ$-homomorphism of X, we say that f is a *homomorphism*.

We now review some fuzzy logic concepts. Let X be a set. By a *fuzzy set* μ in X we mean a function $\mu : X \to [0, 1]$, and the complement of μ, denoted by $\overline{\mu}$, is the fuzzy set in X given by $\overline{\mu}(x) = 1 - \mu(x)$ for all $x \in X$.

DEFINITION 2.5. [11] Let X be a BCK-algebra. A fuzzy subset μ of X is called a *fuzzy $*$-subalgebra* of X if for all $x, y \in X$, $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$.

DEFINITION 2.6. [7] Let X be a BCK-algebra with condition (S). A fuzzy subset μ of X is called a *fuzzy $\circ$-subalgebra* of X if for all $x, y \in X$, $\mu(x \circ y) \geq \min\{\mu(x), \mu(y)\}$.

THEOREM 2.7. [3] *A fuzzy subset μ of a BCK-algebra X with conditin (S) is a fuzzy $\circ$-subalgebra of X if and only if, for every $t \in [0,1]$, μ_t is either $\emptyset$ or a $\circ$-subalgebra of X.*

DEFINITION 2.8. [11] Let μ be a fuzzy subset of a set S. For $t \in [0,1]$, the set

$$\mu_t = \{x \in S | \mu_t(x) \geq t\}$$

is called a *level subset* of μ.

DEFINITION 2.9. [3] Let X be a BCK-algebra with condition (S) and let μ be a fuzzy o-subalgebra of X. Then o-subalgebra μ_t, $t \in [0,1]$ are called *level $\circ$-subalgebras* of μ.

DEFINITION 2.10. [1] Let X be a nonempty fixed set. An *intuitionistic fuzzy set* (IFS for short) A is an object having the form

$$A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$$

where the function $\mu_A : X \to [0,1]$ and $\gamma_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for all $x \in X$.

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \gamma_A)$ for the IFS $A = \{(x, \mu_A(x), \gamma_A(x)) | x \in X\}$.

DEFINITION 2.11. [4] An IFS $A = (\mu_A, \gamma_A)$ in a BCK-algebra X is called an *intuitionistic fuzzy subalgebra* of X if for all $x, y \in X$,

(I1) $\mu_A(x * y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(I2) $\gamma_A(x * y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$.

PROPOSITION 2.12. [4] *Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy subalgebra of BCK-algebra X. Then $\mu_A(0) \geq \mu_A(x)$ and $\gamma_A(0) \leq \gamma_A(x)$ for all $x \in X$*

3. Intuitionistic fuzzy $\circ$-subalgebras

In what follows, let X denote a BCK-algebra with condition (S) unless otherwise specified.

DEFINITION 3.1. An IFS $A = (\mu_A, \gamma_A)$ of X is called an *intuitionistic fuzzy $\circ$-subalgebra* of X if for all $x, y \in X$,

(IF1) $\mu_A(x \circ y) \geq \min\{\mu_A(x), \mu_A(y)\}$,
(IF2) $\gamma_A(x \circ y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$.

EXAMPLE 3.2. Let $X = \{0, 1, 2, 3\}$ in which $*$ is defined by :

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	2	0	0
3	3	3	3	0

Then $(X; *, 0)$ is a BCK-algebra with condition (S) and we can find the following $\circ$-table

$\circ$	0	1	2	3
0	0	1	2	3
1	1	1	2	3
2	2	2	2	3
3	3	3	3	3

Let $s, t \in [0, 1]$ be such that $s + t \leq 1$. Define an IFS $A = (\mu_A, \gamma_A)$ in X as follows :

$$\mu_A(0) = 1, \mu_A(1) = \mu_A(2) = s, \mu_A(3) = 0,$$

$$\gamma_A(0) = 0, \gamma_A(1) = \gamma_A(2) = t, \gamma_A(3) = 1.$$

By routine calculation we know that $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy $\circ$-subalgebra of X.

LEMMA 3.3. *An IFS $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy $\circ$-subalgebra of X if and only if the fuzzy sets μ_A and $\overline{\gamma}_A$ are fuzzy $\circ$-subalgebras of X.*

Proof. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy $\circ$-subalgebra of X. Then μ_A is a fuzzy $\circ$-subalgebra of X. Now, for every $x, y \in X$, we have

$$\begin{aligned}\overline{\gamma}_A(x \circ y) &= 1 - \gamma_A(x \circ y) \\ &\geq 1 - \max\{\gamma_A(x), \gamma_A(y)\} \\ &= \min\{1 - \gamma_A(x), 1 - \gamma_A(y)\} \\ &= \min\{\overline{\gamma}_A(x), \overline{\gamma}_A(y)\}\end{aligned}$$

Hence $\overline{\gamma}_A(x)$ is a fuzzy $\circ$-subalgebra of X.

Conversely, assume that both μ_A and $\overline{\gamma}_A$ are fuzzy $\circ$-subalgebras of X. For every $x, y \in X$, we have $\mu_A(x \circ y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and

$$\begin{aligned}1 - \gamma_A(x \circ y) &= \overline{\gamma}_A(x \circ y) \\ &\geq \min\{\overline{\gamma}_A(x), \overline{\gamma}_A(y)\} \\ &= \min\{1 - \gamma_A(x), 1 - \gamma_A(y)\} \\ &= 1 - \max\{\gamma_A(x), \gamma_A(y)\}.\end{aligned}$$

It follows that $\gamma_A(x \circ y) \leq \max\{\gamma_A(x), \gamma_A(y)\}$. Thus, $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy $\circ$-subalgebra of X. ☐

Theorem 3.4. *Let $A = (\mu_A, \gamma_A)$ be an IFS in X. Then it is an intuitionistic fuzzy $\circ$-subalgebra of X if and only if $\sharp A := (\mu_A, \overline{\mu}_A)$ and $\Diamond A := (\overline{\gamma}_A, \gamma_A)$ are intuitionistic fuzzy $\circ$-subalgebras of X.*

Proof. If $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy $\circ$-subalgebra of X, then $\mu_A = \overline{\overline{\mu}}_A$ and γ_A are fuzzy $\circ$-subalgebras from Lemma 3.3.

Conversely, if $\sharp A = (\mu_A, \overline{\mu}_A)$ and $\Diamond A = (\overline{\gamma}_A, \gamma_A)$ are intuitionistic fuzzy $\circ$-subalgebras of X, then the fuzzy sets μ_A and $\overline{\gamma}_A$ are fuzzy $\circ$-subalgebras of X. Thus, $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy $\circ$-subalgebra of X. ☐

Definition 3.5. Let $A = (\mu_A, \gamma_A)$ be an IFS in X and let $t \in [0, 1]$. Then the set $U(\mu_A; t) = \{x \in X \mid \mu_A(x) \geq t\}$ (resp. $L(\gamma_A; t) = \{x \in X \mid \gamma_A(x) \leq t\}$) is called *upper t-level cut* (resp. *lower t-level cut*) of A.

Theorem 3.6. *If an IFS $A = (\mu_A, \gamma_A)$ in X is an intuitionistic fuzzy $\circ$-subalgebra of X, the upper t-level cut and lower t-level cut of A are $\circ$-subalgebras of X for every $t \in [0, 1]$ such that $t \in Im(\mu_A) \cap Im(\gamma_A)$,* *which are called an upper level subalgebra and a lower level subalgebra respectively.*

Proof. If $x, y \in U(\mu_A; t)$, then $\mu_A(x) \geq t$ and $\mu_A(y) \geq t$. Hence we have $\mu_A(x \circ y) \geq \min\{\mu_A(x), \mu_A(y)\} \geq t$. It follows that $x \circ y \in U(\mu_A; t)$. Thus, $U(\mu_A; t)$ is a $\circ$-subalgebra of X. Now let $x, y \in L(\gamma_A; t)$. Then $\gamma_A(x \circ y) \leq \max\{\gamma_A(x), \gamma_A(y)\} \leq t$ and hence $x \circ y \in L(\gamma_A; t)$. Thus, $L(\gamma_A; t)$ is a $\circ$-subalgebra of X. □

THEOREM 3.7. *Let $A = (\mu_A, \gamma_A)$ be an IFS in X such that the nonempty sets $U(\mu_A; t)$ and $L(\gamma_A; t)$ are $\circ$-subalgebras of X for every $t \in [0, 1]$. Then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy $\circ$-subalgebra of X.*

Proof. We need to prove that $A = (\mu_A, \gamma_A)$ satisfies the conditions (IF1) and (IF2). First, if the condition (IF1) does not hold, then there exist $x_0, y_0 \in X$ such that $\mu_A(x_0 \circ y_0) < \min\{\mu_A(x_0), \mu_A(y_0)\}$. Let

$$t_0 = \frac{1}{2}[\mu_A(x_0 \circ y_0) + \min\{\mu_A(x_0), \mu_A(y_0)\}].$$

Then $\mu_A(x_0 \circ y_0) < t_0 < \min\{\mu_A(x_0), \mu_A(y_0)\}$ and hence, $x_0 \circ y_0 \notin U(\mu_A; t_0)$, but $x_0, y_0 \in U(\mu_A; t_0)$. This is a contradiction.

Second, if the condition (IF2) does not hold, then

$$\gamma_A(x_0 \circ y_0) > \max\{\gamma_A(x_0), \gamma_A(y_0)\},$$

for some $x_0, y_0 \in X$. Let

$$s_0 = \frac{1}{2}[\gamma_A(x_0 \circ y_0) + \max\{\gamma_A(x_0), \gamma_A(y_0)\}].$$

Then $\max\{\gamma_A(x_0), \gamma_A(y_0)\} < s_0 < \gamma_A(x_0 \circ y_0)$. It follows that $x_0, y_0 \in L(\gamma_A; s_0)$ and $x_0 \circ y_0 \notin L(\gamma_A; s_0)$, which is a contradiction. This completes the proof. □

THEOREM 3.8. *Any $\circ$-subalgebra of X can be realized as both an upper level subalgebra and a lower level subalgebra of some intuitionistic fuzzy $\circ$-subalgebra of X.*

Proof. Let S ba a $\circ$-subalgebra of X and let μ_A and γ_A be fuzzy sets of X defined by

$$\mu_A(x) = \begin{cases} \alpha, & \text{if } x \in S, \\ 0, & \text{otherwise}, \end{cases}$$

and

$$\gamma_A(x) = \begin{cases} \beta, & \text{if } x \in S, \\ 1, & \text{otherwise,} \end{cases}$$

for all $x \in X$, where α and β are fixed numbers in $(0, 1)$ such that $\alpha + \beta < 1$. Let $x, y \in X$. If $x, y \in S$, then $x \circ y \in S$. Thus $\mu_A(x \circ y) = \min\{\mu_A(x), \mu_A(y)\}$ and $\gamma_A(x \circ y) = \max\{\gamma_A(x), \gamma_A(y)\}$. If at least one of x and y does not belong to S, then at least one of $\mu_A(x)$ and $\mu_A(y)$ is equal to 0, and at least one of $\gamma_A(x)$ and $\gamma_A(y)$ is equal to 1. It follows that $\mu_A(x \circ y) \geq 0 = \min\{\mu_A(x), \mu_A(y)\}$ and $\gamma_A(x \circ y) \leq 1 = \max\{\gamma_A(x), \gamma_A(y)\}$. Thus $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy $\circ$-subalgebra of X. Clearly, we have $U(\mu_A; \alpha) = S = L(\gamma_A; \beta)$. This completes the proof. □

Let f be a function from a set X to a set Y. If $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ are IFSs in X and Y respectively, then the preimage of B under f, denoted by $f^{-1}(B)$, is an IFS in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)),$$

and the image of A under f, denoted by $f(A)$, is an IFS of Y defined by

$$f(A) = (f_s(\mu_A), f_i(\gamma_A)),$$

where

$$f_s(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$f_i(\gamma_A)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \gamma_A(x), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

for each $y \in Y$ ([2]).

THEOREM 3.9. *Let $f : X \to Y$ be a $\circ$-homomorphism of BCK-algebras with condition (S). If $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy $\circ$-subalgebra of Y, then the preimage $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ of B under f is an intuitionistic fuzzy $\circ$-subalgebra of X.*

Proof. Suppose that $B = (\mu_B, \gamma_B)$ is an intuitionistic fuzzy $\circ$-subalgebra of Y. Let $x_1, x_2 \in X$. Then

$$\begin{aligned} f^{-1}(\mu_B)(x_1 \circ x_2) &= \mu_B(f(x_1 \circ x_2)) \\ &= \mu_B(f(x_1) \circ f(x_2)) \\ &\geq \min\{\mu_B(f(x_1)), \mu_B(f(x_2))\} \\ &= \min\{f^{-1}(\mu_B)(x_1), f^{-1}(\mu_B)(x_2)\} \end{aligned}$$

and

$$\begin{aligned} f^{-1}(\gamma_B)(x_1 \circ x_2) &= \gamma_B(f(x_1 \circ x_2)) \\ &= \gamma_B(f(x_1) \circ f(x_2)) \\ &\leq \max\{\gamma_B(f(x_1)), \gamma_B(f(x_2))\} \\ &= \max\{f^{-1}(\gamma_B)(x_1), f^{-1}(\gamma_B)(x_2)\}. \end{aligned}$$

Thus, $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B))$ is an intuitionistic fuzzy $\circ$-subalgebra of X. □

Theorem 3.10. *Let $f : X \to Y$ be an onto $\circ$-homomorphism of BCK-algebras with condition (S). If $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy $\circ$-subalgebra of X, then the image $f(A) = (f_s(\mu_A), f_i(\gamma_A))$ of A under f is an intuitionistic fuzzy $\circ$-subalgebra of Y.*

Proof. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy $\circ$-subalgebra of X and let $y_1, y_1 \in Y$. Observing that $\{x_1 \circ x_2 \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\} \subseteq \{x \in X \mid x \in f^{-1}(y_1 \circ y_1)\}$. We have

$$\begin{aligned} & f_s(\mu_A)(y_1 \circ y_2) \\ &= \sup\{\mu_A(x) \mid x \in f^{-1}(y_1 \circ y_2)\} \\ &\geq \sup\{\mu_A(x_1 \circ x_2) \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\} \\ &\geq \sup\{\min\{\mu_A(x_1), \mu_A(x_2)\} \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\} \\ &= \min\{\sup\{\mu_A(x_1) \mid x_1 \in f^{-1}(y_1)\}, \sup\{\mu_A(x_2) \mid x_2 \in f^{-1}(y_2)\}\} \\ &= \min\{f_s(\mu_A)(y_1), f_s(\mu_A)(y_2)\} \end{aligned}$$

and

$$\begin{aligned}
&f_i(\gamma_A)(y_1 \circ y_2)\\
&= \inf\{\gamma_A(x) \mid x \in f^{-1}(y_1 \circ y_2)\}\\
&\leq \inf\{\gamma_A(x_1 \circ x_2) \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\}\\
&\leq \inf\{\max\{\gamma_A(x_1), \gamma_A(x_2)\} \mid x_1 \in f^{-1}(y_1) \text{ and } x_2 \in f^{-1}(y_2)\}\\
&= \max\{\inf\{\mu_A(x_1) \mid x_1 \in f^{-1}(y_1)\}, \inf\{\mu_A(x_2) \mid x_2 \in f^{-1}(y_2)\}\}\\
&= \max\{f_s(\gamma_A)(y_1), f_s(\gamma_A)(y_2)\}.
\end{aligned}$$

Thus, $f(A) = (f_s(\mu_A), f_i(\gamma_A))$ is an intuitionistic fuzzy $\circ$-subalgebra of Y. □

Let $f : X \to Y$ be a $\circ$-homomorphism of BCK-algebras with condition (S). For any IFS $A = (\mu_A, \gamma_A)$ in Y, we define an IFS $A^f = (\mu_A^f, \gamma_A^f)$ in X by

$$\mu_A^f(x) := \mu_A(f(x)),\ \gamma_A^f(x) := \gamma_A(f(x)),$$

for all $x \in X$.

THEOREM 3.11. *Let $f : X \to Y$ be a $\circ$-homomorphism of BCK-algebras with condition (S). If an IFS $A = (\mu_A, \gamma_A)$ in Y is an intuitionistic fuzzy $\circ$-subalgebra of Y, then the IFS $A^f = (\mu_A^f, \gamma_A^f)$ in X is an intuitionistic fuzzy $\circ$-subalgebra of X.*

Proof. Let $x, y \in X$. Then

$$\begin{aligned}
\mu_A^f(x \circ y) &= \mu_A(f(x \circ y))\\
&= \mu_A(f(x) \circ f(y))\\
&\geq \min\{\mu_A(f(x)), \mu_A(f(y))\}\\
&= \min\{\mu_A^f(x), \mu_A^f(y)\}
\end{aligned}$$

and

$$\begin{aligned}
\gamma_A^f(x \circ y) &= \gamma_A(f(x \circ y))\\
&= \gamma_A(f(x) \circ f(y))\\
&\leq \max\{\gamma_A(f(x)), \gamma_A(f(y))\}\\
&= \max\{\gamma_A^f(x), \gamma_A^f(y)\}.
\end{aligned}$$

Hence, $A^f = (\mu_A^f, \gamma_A^f)$ is an intuitionistic fuzzy $\circ$-subalgebra of X. This completes the proof. □

THEOREM 3.12. *Let $f : X \to Y$ be an epimorphism of BCK-algebras with condition (S) and let $A = (\mu_a, \gamma_A)$ be an IFS in Y. If $A^f = (\mu_A^f, \gamma_A^f)$ is an intuitionistic fuzzy $\circ$-subalgebra of X, then $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy $\circ$-subalgebra of Y.*

Proof. Let $y_1, y_2 \in Y$. Then there exist $x_1, x_2 \in X$ such that $f(x_i) = y_i$, for $i = 1, 2$. Then

$$\begin{aligned} \mu_A(y_1 \circ y_2) &= \mu_A(f(x_1) \circ f(x_2)) \\ &= \mu_A(f(x_1 \circ x_2)) \\ &= \mu_A^f(x_1 \circ x_2) \\ &\geq \min\{\mu_A^f(x_1), \mu_A^f(x_2)\} \\ &= \min\{\mu_A(f(x_1)), \mu_A(f(x_2))\} \\ &= \min\{\mu_A(y_1), \mu_A(y_2)\} \end{aligned}$$

and

$$\begin{aligned} \gamma_A(y_1 \circ y_2) &= \gamma_A(f(x_1) \circ f(x_2)) \\ &= \gamma_A(f(x_1 \circ x_2)) \\ &= \mu_A^f(x_1 \circ x_2) \\ &\leq \max\{\gamma_A^f(x_1), \gamma_A^f(x_2)\} \\ &= \max\{\gamma_A(f(x_1)), \gamma_A(f(x_2))\} \\ &= \max\{\gamma_A(y_1), \gamma_A(y_2)\}. \end{aligned}$$

Thus, $A = (\mu_A, \gamma_A)$ is an intuitionistic fuzzy $\circ$-subalgebra of Y. This completes the proof. □

REFERENCES

[1] K. T. Atanassov, *An intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20** (1986), 87–96.
[2] D. Çoker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems **88** (1997), 137–142.
[3] S. M. Hong and H. G. Kim, *Cartesian product of fuzzy subalgebras in BCK-algebras with condition (S)*, Far East J. Math. Sci. **1** (1999), 33–48.

[4] S. M. Hong, K. H. Kim and Y. B. Jun, *Intuitionistic fuzzy subalgebras of BCK/BCI-algebras*, Korean J. Comput. & Appl. Math. **8** (2001), 261–272.
[5] K. Iséki, *A special class of BCK-algebras*, Math. Seminar Notes **5** (1977), 191–198.
[6] Y. Imai and K. Iséki, *On axiom systems of proposition calculi*, Proc. Japan. Acad. **42** (1966), 19–22.
[7] Y. B. Jun, S. M. Hong, S. J. Kim and S. Z. Song, *Fuzzy ideals and fuzzy subalgebras of BCK-algebras*, J. Fuzzy Math. **7** (1999), 411–418.
[8] Y. B. Jun and K. H. Kim, *Intuitionistic fuzzy ideals of BCK-algebras*, Internat. J. Math. & Math. Sci. **24** (2000), 839–849.
[9] K. H. Kim and Y. B. Jun, *T-fuzzy circled subalgebras of BCK-algebras*, Korean J. Comput. & Appl. Math. **7** (2000), 685–692.
[10] J. Meng and Y. B. Jun, *BCK-algebras*, Kyungmoonsa, Seoul, Korea 1994.
[11] O. G. Xi, *Fuzzy BCK-algebra*, Math. Japon. **36** (1991), 935–942.
[12] L. A. Zadeh, *Fuzzy sets*, Inform and Control. **8** (1965), 378–353.

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