An Analytical Investigation for Nash Equilibriums of Generation Markets

Jin-Ho Kim*, Jong-Ryul Won† and Jong-Bae Park**

Abstract - In this paper, Nash equilibriums of generation markets are investigated using a game theory application for simplified competitive electricity markets. We analyze the characteristics of equilibrium states in N-company spot markets modeled by uniform pricing auctions and propose a new method for obtaining Nash equilibriums of the auction. We assume that spot markets are operated as uniform pricing auctions and that each generation company submits its bids into the auction in the form of a seal-bid. Depending on the bids of generation companies, market demands are allocated to each company accordingly. The uniform pricing auction in this analysis can be formulated as a non-cooperative and static game in which generation companies correspond to players of the game. The coefficient of the bidding function of company-*n* is the strategy of player-*n* (company-*n*) and the payoff of player-*n* is defined as its profit from the uniform price auction. The solution of this game can be obtained using the concept of the non-cooperative equilibrium originating from the Nash idea. Based on the so called residual demand curve, we can derive the best response function of each generation company in the uniform pricing auction with N companies, analytically. Finally, we present an efficient means to obtain all the possible equilibrium set pairs and to examine their feasibilities as Nash equilibriums. A simple numerical example with three generation companies is demonstrated to illustrate the basic idea of the proposed methodology. From this, we can see the applicability of the proposed method to the real-world problem, even though further future analysis is required.

Keywords: Equilibrium Set Pairs, Generation Markets, Nash Equilibrium, Residual Demand

1. Introduction

The electric power industry is undergoing the transformation into a competitive marketplace for power transactions, and profitability is a primary concern to every market player [1, 2]. Basically, the power transaction revenues of a generation firm come from bilateral contracts or spot markets, which are usually formed as a sealed bid auction with uniform market prices [3, 4]. In this environment, the optimal bidding strategy, to maximize the profit from the spot market, is indispensable to the generation firms. Bidding strategies, however, should be studied based on the basic market characteristics such as market structure, auction rule, and bidding protocol. In recent years, considerable amounts of research have been carried out related to decision-making in competitive power markets, which are focused on the optimal bidding strategies for the competitive generation firms [3-6, 8-10]. Privatized generation firms can decide their bidding strategies based on various theoretical or empirical studies, e.g., forecasts of market prices, evaluation of competitors' bidding behavior, and game theory applications [7]. Price based operations in an auction market structure are analyzed in [3]. In [4], using the matrix game, where bidding strategy is represented with discrete quantities, decision-making processes in deregulated power systems are simulated. In this paper, we analyze the characteristics of equilibrium states in N-company spot markets modeled by uniform pricing auctions and propose a new analytical method to obtain Nash equilibriums of the auction using residual demands. We assume that each generation companies submits its bids into a market in the form of a sealed bid and the market is assumed to operate as uniform price auctions. We consider N generation companies as the players of the spot market. Depending on the bids of generation companies, market demands are allocated to each generation company. The uniform pricing auction in this analysis can be formulated as a non-cooperative and static game in which generation companies correspond to players of the game. The coefficient of the bidding function of company-*n* is the strategy of player-*n* (company-*n*) and the payoff of player-*n* is defined as its profit from the uniform price auction. The solution of this game can be obtained using the concept of the non-cooperative equilibrium based on the Nash idea. According to the so

* Dept. of Electrical Engineering, Pusan National University, Korea. (jinhkim@pusan.ac.kr)
† Corresponding Author: Dept. of Electrical and Electronic Engineering, Anyang University, Korea. (jrwon@aycc.anyang.ac.kr)
** Dept. of Electrical Engineering, Konkuk University, Korea. (jbaepark@konkuk.ac.kr)
Received October 7, 2004 ; Accepted November 11, 2004

called residual demand curve, we can derive the best response function of each generation company in the uniform pricing auction with N companies, analytically. Finally, we present an efficient way to obtain all the possible equilibrium set pairs and to examine their feasibilities as Nash equilibriums. A simple numerical example with three generation companies is demonstrated to show the basic idea of the proposed methodology. From this, we can see the applicability of the proposed method to the real-world problem, even though it requires further future analysis.

2. Simplified Markets Model

2.1 Markets Model

In this study, we analyze the characteristic of equilibrium states in N-company spot markets modeled by uniform pricing auctions and suggest a practical method for obtaining Nash equilibriums using the concept of residual demands. We assume that spot markets are operated as uniform pricing auctions and that each generation company submits its bids into the auction in the form of a seal-bid. Depending on the bids of generation companies, market demands are allocated to each generation company. The generation quantity allocated to generation company-n is denoted by q_n. Generation and marginal costs of generation company-n are denoted by $C_n(q_n)$ and $C_n'(q_n)$, respectively. The bidding function of generation company-n is denoted by $B_n(q_n)$ and the profit of generation company-n is represented by Π_n. The total generation quantity covering all the generation companies, total system demand, and market clearing price are signified by q, d, and p, respectively. Notations defined are summarized as follows:

q_n : Generation quantity allocated to company-$n(0 \le q_n \le \bar{q}_n)$,

$\bar{q}_n$: Maximum generation limit of company-n,

q : Sum of the allocated generation quantity of all the companies such as $q = q_1 + \cdots + q_N$, where N is the number of generation companies,

$C_n(q_n)$: Generation cost of company-n,

$C_n'(q_n) \equiv \frac{d}{dq_n} C_n(q_n)$: Marginal cost of company-n,

$B_n(q_n)$: Bidding function of company-n,

p : Market clearing price,

d : System demand,

$\Pi_n \equiv p \cdot q_n - C_n(q_n)$: Profit of company-n.

We formulate basic assumptions on the market setups and some functions that are defined above. First, the number of generation companies in markets is assumed to be N. Second, system demand is defined by an affine function such that it has an inverse given by $p = d^{-1}(q) = \alpha - \omega q$, where $\alpha > 0$ and $\omega \ge 0$. Third, the generation cost, $C_n(q_n)$, and the marginal cost, $C_n'(q_n)$, of generation company-n, $n \in \Omega$, $\Omega \equiv \{1, 2, \cdots, N\}$, are defined by a quadratic and a linear function as $C_n(q_n) = \frac{1}{2} a_n (q_n)^2$ and $C_n'(q_n) = a_n q_n$, respectively. Finally, the bidding function of generation company-n is assumed to have a linear form by $B_n(q_n) \equiv b_n q_n$, where $b_n > 0$.

2.2 Game Formulation

In this study, spot markets are modeled by uniform pricing auctions and the auction results are based on the interactions among companies in the spot markets. Therefore, the uniform pricing auction in this analysis can be formulated as a non-cooperative and static game in which generation companies correspond to players of the game. The coefficient of the bidding function of company-n, b_n, is the strategy of player-n (company-n) and the payoff of player-n is defined as its profit from the uniform price auction. The solution of this game can be obtained using the concept of the non-cooperative equilibrium based on the Nash idea, and therefore, the solution strategies can be calculated as follows:

$$b_n^{Nash} = \underset{b_n}{argmax}(\Pi_n) \text{ for a given } b_m^{Nash} \quad (1)$$
$$\text{, where } n \ne m \text{ and } n, m \in \Omega$$

3. Analysis of Nash Equilibriums

3.1 Classification of Companies

In any equilibrium, companies can be classified into two distinct sets. Some companies are in the set of unconstrained companies in which every company sells its electricity for less than its maximum generation limit, while the other companies are in the set of constrained companies in which every company sells its electricity up to its maximum limit. Consequently, we define two equilibrium sets of companies according to the generation quantity allocated to the individual companies as follows:

$$\begin{aligned}\Omega^U &= \{n \in \Omega \mid q_n < \bar{q}_n\}: \text{Set of unconstrained companies}\\ \Omega^C &= \{n \in \Omega \mid q_n = \bar{q}_n\}: \text{Set of constrained companies}\end{aligned} \quad (2)$$

This classification of companies covers all companies in the market and the classified sets are disjointed such as $\Omega = \Omega^U \cup \Omega^C$ and $\Omega^U \cap \Omega^C = \phi$, and therefore, each company is included in either Ω^U or Ω^C, depending on its bidding function and given system demands. We can obtain an explicit expression for the generation quantity allocated to each company, depending on the set in which the company is included at equilibriums as follows:

$$q_n = \begin{cases} p/b_n, & n \in \Omega^U \\ \bar{q}_n, & n \in \Omega^C \end{cases} \quad (3)$$

At given market demands, since the generation quantity allocated to each company is determined by its bidding function, we can classify companies into two equilibrium states (i.e., set of unconstrained companies and set of constrained companies) according to the slope of their bidding function, b_n.

$$\begin{cases} n \in \Omega^U, & b_n > \dfrac{p}{\bar{q}_n} = \dfrac{\alpha - \omega q}{\bar{q}_n} \\ n \in \Omega^C, & b_n \le \dfrac{p}{\bar{q}_n} = \dfrac{\alpha - \omega q}{\bar{q}_n} \end{cases} \quad (4)$$

where, $p = \alpha - \omega q$ and q, total sum of the generation quantities allocated to all the companies, can be expressed as $q = \sum_{n\in\Omega} q_n = \sum_{n\in\Omega^U} \frac{p}{b_n} + \sum_{n\in\Omega^C} \bar{q}_n$. Figure 1 shows an example of equilibrium, in which company-n and company-m is in the set of constrained and unconstrained companies, respectively.

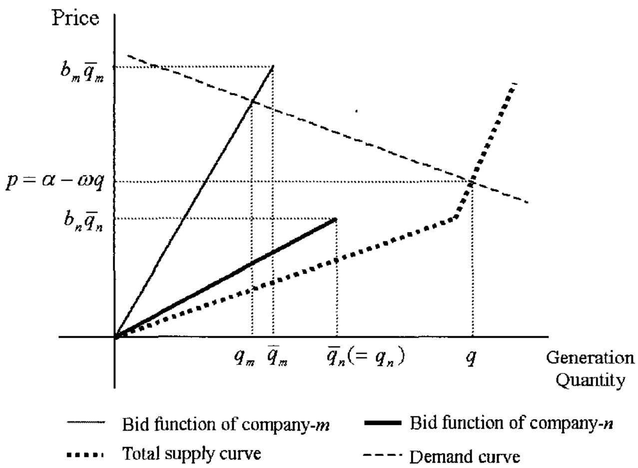

Fig. 1 Classification of companies $(n \in \Omega^C \text{ and } m \in \Omega^U)$

3.2 Best Response of Individual Companies Using Residual Demands

In order to obtain the Nash equilibrium of N-company uniform pricing auctions, the best responses of individual companies should be obtained first. Based on the so called residual demand curve, we can derive the best response function of each generation company in the uniform pricing auction with N companies, analytically. Suppose that an equilibrium set pair, (Ω^U, Ω^C), is given. Then, we can determine the best response of company-n, which is in the equilibrium set of unconstrained companies, that is, $n \in \Omega^U$.

Let us consider the inverse demand curve $p = \alpha - \omega q$ and, in this equation, ω can be interpreted as the parameter that determines how much demand's willingness-to-pay decreases when quantity (demand) increases by one unit. In the same way, we can also interpret strategy b_n, the coefficient of bidding function of company-n, as the parameter that represents how much company-n's willingness-to-earn rises when quantity (supply) increases by one unit. However, it is more convenient to consider the reciprocal of these parameters since both consumers and suppliers respond to the price. That is, $1/\omega$ is the decreasing amount of demand when price increases by one unit and $1/b_n$ is the increasing amount of company-n's supply when price increases by one unit.

Let Ω^U_{-n} denote the equilibrium set of unconstrained companies with the exception of company-n, i.e., $\Omega^U_{-n} \equiv \Omega^U - \{n\}$. Given the strategies of other companies, b_o, $o \in \Omega^U_{-n}$, the market clearing price can be obtained as follows:

$$p = \alpha - \omega \sum_{o\in\Omega} q_o = \alpha - \omega \sum_{o\in\Omega^C} \bar{q}_o - \omega \sum_{o\in\Omega^U_{-n}} \left(\frac{p}{b_o}\right) - \omega q_n \quad (5)$$

From the equation above, the residual demand of company-n, that is, the amount of demand that company-n is facing, can be obtained as follows:

$$\begin{aligned} p &= \left(\alpha - \omega \sum_{o\in\Omega^C} \bar{q}_o\right) \cdot \left(1 + \omega \sum_{o\in\Omega^U_{-n}} \frac{1}{b_o}\right)^{-1} - \left(1 + \omega \sum_{o\in\Omega^U_{-n}} \frac{1}{b_o}\right)^{-1} \omega q_n \\ &= \alpha_n^{rs} - \omega_n^{rs} q_n \end{aligned} \quad (6)$$

where $\alpha_n^{rs} = \left(\alpha - \omega \sum_{o\in\Omega^C} \bar{q}_o\right) \cdot \left(1 + \omega \sum_{o\in\Omega^U_{-n}} \frac{1}{b_o}\right)^{-1}$ and $\omega_n^{rs} = \left(\frac{1}{\omega} + \sum_{o\in\Omega^U_{-n}} \frac{1}{b_o}\right)^{-1}$.

The equation above can be interpreted in the same way as we do in the original demand function. That is, $1/\omega_n^{res}$ is the amount by which demand decreases when price increases by one unit. We can see that this amount is related to the parameter in the original demand curve and strategies of generation companies. From the interpretation of the demand curve above, we know that if price increases by one unit, original demand decreases by $1/\omega$ and supply of each company-o ($o \in \Omega_{-n}^U$) is augmented by $1/b_o$. Therefore, the residual demand is decreased by the sum of the two amounts when price increases by one unit. This implies that $1/\omega_n^{res}$ can be represented as the sum of these two factors such as $\frac{1}{\omega_n^{res}} = \frac{1}{\omega} + \sum_{o \in \Omega_{-n}^U} \frac{1}{b_o}$. For this residual demand, company-n can be regarded as being monopolist and its best response can be determined as the strategy of a monopolistic company as follows [Appendix]:

$$b_n^{BR} = \gamma_n + \left(\frac{1}{\omega} + \sum_{o \in \Omega_{-n}^U} \frac{1}{b_o} \right)^{-1} \tag{7}$$

In (7), we can see that the best response of company-n takes a value between γ_n and $\gamma_n + \omega$. In the first place, when all the other companies choose their strategies, b_n's, as taking almost a zero value, market clearing prices are fixed and company-n becomes a price taker who can maximize its profit by bidding at its marginal price. Alternatively, if all the other companies bid with infinitely large value, then company-n covers the entire demand and it meets a monopoly situation. Another observation is that the best response of a company in the equilibrium set of unconstrained companies is determined only by the strategies of the companies in that equilibrium set of unconstrained companies.

Next, the best response of company-m, $m \in \Omega^C$, which is in the equilibrium set of constrained companies, can be obtained as follows. From Figure 1, we can see that, for company-m in Ω^C, the best response at equilibrium can be any value less than $p/\bar{q}_m$, where p is the market clearing price at the equilibrium. Therefore, from (5), market clearing price and the best response of company-m can be obtained as follows:

$$p = \left(\alpha - \omega \sum_{o \in \Omega^C} \bar{q}_o \right) \left(1 + \omega \sum_{o \in \Omega^U} \frac{1}{b_o} \right)^{-1} \tag{8}$$

$$b_m^{BR} = \left\{ b_m \mid b_m < \frac{p}{\bar{q}_m} = \frac{1}{\bar{q}_m} \left(\alpha - \omega \sum_{o \in \Omega^C} \bar{q}_o \right) \left(1 + \omega \sum_{o \in \Omega^U} \frac{1}{b_o} \right)^{-1} \right\} \tag{9}$$

3.3 Obtaining Candidates of Nash Equilibrium and Their Feasibilities Check

In the previous section, best responses of individual companies at a given equilibrium set pair (Ω^U, Ω^C) are derived. In this section, we show a practical way to obtain all the possible equilibrium set pairs and to examine their feasibilities as Nash equilibriums. That is, we show how we can define a feasible space for equilibrium set pairs and can check the feasibilities of the individual equilibrium set pairs in the feasible space as Nash equilibrium.

We can see that, at any given equilibrium set pair, a condition is needed to guarantee the existence of an equilibrium set of constrained companies at that pair. In the uniform pricing auction model described above, the lowest equilibrium market clearing price, $\underline{p}$, can be obtained when all companies are in the equilibrium set of constrained companies, while the highest equilibrium market clearing price, $\bar{p}$, can be obtained when all companies are in the equilibrium set of unconstrained companies. However, it should be noted that company-n cannot be in the equilibrium set of constrained companies at any given equilibrium set pair if market clearing price at that equilibrium, p, is less than $\gamma_n \bar{q}_n$. That is, for an example, suppose that company-n is in the equilibrium set of constrained companies and market clearing price p is less than $\gamma_n \bar{q}_n$. Then, this assumption gives a relationship as $b_n \bar{q}_n \le p < \gamma_n \bar{q}_n$ and this inequality implies that the marginal revenue is less than the marginal cost of company-n. Therefore, company-n loses its money by selling its marginal generation unit. In this case, the company will try to decrease its generation, which can only be done by choosing b_n large enough to transfer its state from the set of constrained companies to unconstrained companies. Therefore, the best response of company-n cannot exist in the equilibrium set of constrained companies and this implies that any Nash equilibrium cannot be reached with company-n's constrained equilibrium state. Therefore, the following condition is needed to guarantee the existence of an equilibrium set of constrained companies at a given equilibrium set pair:

$$p \ge \gamma_n \bar{q}_n, \quad \text{where } n \in \Omega^C \tag{10}$$

Let Ω^R denote a set of companies that can remain in the equilibrium set of constrained companies in a Nash equilibrium, in the sense of marginal profit (i.e., p is greater than $\gamma_n \bar{q}_n$). Then, we can obtain a set of candidates of equilibrium set pairs, Λ , and this set implies a feasible space for equilibrium set pairs for the problem. For each equilibrium set pair in the set Λ, we will check whether it satisfies the condition in (10) and equilibrium set pairs satisfying (10) can be confirmed as Nash equilibriums subsequent to another examination that is presented in the following section. The number of elements of Λ, that is, the total number of candidates of equilibrium set pair, is identical to the number of subsets of Ω^R (i.e., $2^{|\Omega^R|}$, where $|\Omega^R|$ denotes the number of elements of the set Ω^R).

We can check the possibility of a candidate equilibrium set pair as being a Nash equilibrium using the condition described in (10). That is, for a given equilibrium set pair (Ω^U, Ω^C), we can check whether there is a possibility that this equilibrium set pair will be a Nash equilibrium, by comparing $\max_{k \in \Omega^C}(\gamma_k \bar{q}_k)$ and the corresponding equilibrium market clearing price. Therefore, if market clearing price at the given equilibrium set pair is less than $\max_{k \in \Omega^C}(\gamma_k \bar{q}_k)$, the equilibrium set pair cannot result in Nash equilibrium and therefore will not be considered in further examinations. The procedure to obtain a set of feasible equilibrium set pairs and to check for the possibility of Nash equilibrium is summarized as follows:

1) Without loss of generality, we can assume that companies are ordered by the increasing order of the parameter $\gamma_n \bar{q}_n$, $n \in \Omega$, that is, $\gamma_n \bar{q}_n < \gamma_{n+1} \bar{q}_{n+1}$, since otherwise we can order companies.
2) Obtain the maximum market clearing price $\bar{p}$.
3) Find an index R such as $\gamma_R \bar{q}_R \leq \bar{p} < \gamma_{R+1} \bar{q}_{R+1}$.
4) Obtain a set of companies, $\Omega^R = \{1, \cdots, R\}$.
5) Obtain a set of candidates of equilibrium set pairs, $\Lambda = \{(\Omega^U, \Omega^C) \mid \Omega^U, \Omega^C \in 2^{\Omega^R}, \Omega^U \cap \Omega^C = \emptyset, \Omega^U \cup \Omega^C = \Omega\}$.
6) Check the feasibility of each candidate equilibrium set pair, by comparing $\max_{k \in \Omega^C}(\gamma_k \bar{q}_k)$ and market clearing price at the equilibrium set pair.

4. Numerical Examples

4.1 Market Data

We consider three generation companies and the inverse demand function given by $p = 25 - 0.01q$, where q is the total supply such as $q = q_1 + q_2 + q_3$. The parameter for each company is illustrated in Table 1.

Table 1 Cost data of generation companies

Company	1	2	3
Cost coefficient γ	0.0219	0.0173	0.0111
Maximum Generation	400	600	1000

4.2 Solution Procedure

Step 1. The given company index has already been ordered.

Step 2. Λ is obtained as follows: $\Lambda = \{\lambda_1, \cdots, \lambda_8\}$, where $\lambda_i = \{\Omega^U, \Omega^C\}$ is a candidate equilibrium set pair such as $\lambda_1 = \{\phi, \{1,2,3\}\}$, $\lambda_2 = \{\{1\}, \{2,3\}\}$, $\lambda_3 = \{\{2\}, \{1,3\}\}$, $\lambda_4 = \{\{3\}, \{1,2\}\}$, $\lambda_5 = \{\{1,2\}, \{3\}\}$, $\lambda_6 = \{\{1,3\}, \{2\}\}$, $\lambda_7 = \{\{2,3\}, \{1\}\}$, and $\lambda_8 = \{\{1,2,3\}, \phi\}$.

Step 3. We examine Nash conditions for each candidate Nash equilibrium set pair as follows:

1) $\lambda_1 = \{\phi, \{1,2,3\}\}$

Since every company is in the constrained state, the corresponding generation is its maximum generation. Therefore, total generation is the sum of maximum generations and the corresponding value is 2000. By way of the demand function, we can acquire the market clearing price as follows:

$$p = 25 - 0.01 \times 2000 = 5 \tag{11}$$

The maximum marginal cost at the maximum generation level in Ω^C is acquired as company-3's marginal cost at the maximum generation level of 11.057. Based on (9), this candidate cannot be Nash, because the company-3's marginal cost at the maximum generation level is greater than the market clearing price p.

2) $\lambda_2 = \{\{1\}, \{2,3\}\}$

Company-1's strategy is determined as follows:

$$b_1 = \gamma_1 + \left(\frac{1}{\omega} + \sum_{o \in \Omega_1^U} \frac{1}{b_o} \right)^{-1} = 0.0219 + 0.01 = 0.0319 \tag{12}$$

The corresponding market clearing price is obtained as follows:

$$p=\left(\alpha-\omega\sum_{o\in\Omega^{f}}\bar{q}_o\right)\cdot\left(1+\omega\sum_{o\in\Omega^{U}}\frac{1}{b_o}\right)^{-1}=6.5549 \quad (13)$$

The maximum marginal cost in Ω^C is 11.057 and this is greater than the price above. Therefore, this cannot be a Nash equilibrium because it does not meet condition (9).

3) Other candidate equilibrium set pairs except for λ_8 also cannot result in Nash equilibrium by similar analysis.

4) $\lambda_8=\{\{1,2,3\},\phi\}$

Each company's strategy is determined as follows:

$$b_1=\gamma_1+\left(\frac{1}{\omega}+\sum_{o\in\Omega^{U}_{-1}}\frac{1}{b_o}\right)^{-1}=0.026809 \quad (14)$$

$$b_2=\gamma_2+\left(\frac{1}{\omega}+\sum_{o\in\Omega^{U}_{-2}}\frac{1}{b_o}\right)^{-1}=0.022339 \quad (15)$$

$$b_3=\gamma_3+\left(\frac{1}{\omega}+\sum_{o\in\Omega^{U}_{-3}}\frac{1}{b_o}\right)^{-1}=0.01655 \quad (16)$$

The corresponding market clearing price is obtained as follows:

$$p=\left(\alpha-\omega\sum_{o\in\Omega^{f}}\bar{q}_o\right)\cdot\left(1+\omega\sum_{o\in\Omega^{U}}\frac{1}{b_o}\right)^{-1}=10.31 \quad (17)$$

There is no company in Ω^C, so this equilibrium meets condition (9). The generation quantity allocated to each generation company can be obtained from (3) as follows:

$$(q_1,q_2,q_3)=(384.5723,\ 461.5247,\ 622.9607)$$

Since we know each price and generation, we can calculate each company's payoff. The results are $\Pi_1=2342.8$, $\Pi_2=2917.6$, and $\Pi_3=4277.0$.

Step 4. Now we examine the deviation of each company. For company-1, the reasonable deviation is to decrease its slope of bid curve and to be included in Ω^C. In this case, the deviated price is 10.2345, the deviated quantity is its maximum quantity of 400 and the corresponding deviated payoff is 2339, which is less than the original payoff of 2342.8. For company-2, the reasonable deviation is to decrease its slope of bid curve and to be included in Ω^C. In this case, the deviated price is 9.6092, the deviated quantity is its maximum generation 600 and the corresponding deviated payoff is 2645.8, which is less than the original payoff of 2917.6. Company-3's reasonable deviation is also to decrease its slope of bid curve and to be included in Ω^C. In this case, the deviated price is 8.2388, the deviated quantity is its maximum generation 1000 and the corresponding deviated payoff is 2710.3, which is less than the original payoff of 4277. Since this candidate satisfies all Nash conditions, Nash equilibrium will result. The following are the quantities of this Nash equilibrium:

$$\left(b_1^{Nash},b_2^{Nash},b_3^{Nash}\right)=(0.026809,0.022339,0.01655) \quad (18)$$

$$\left(q_1^{Nash},q_2^{Nash},q_3^{Nash}\right)=(384.5723,\ 461.5247,\ 622.9607) \quad (19)$$

Since we explored every possible candidate for`Nash equilibrium, this Nash equilibrium is a unique Nash equilibrium.

4.3 Results

From the analysis above, unique Nash equilibrium can be obtained. The corresponding Nash quantities for this numerical example are illustrated in Table 2.

5. Conclusion

In this paper, we analyze the characteristics of equilibrium states in N-company spot markets modeled by uniform pricing auctions. We propose a new method for obtaining Nash equilibriums of the auction. In this study, we assume spot markets to be operated as uniform pricing auctions and each generation company to submit its bids into the auction in the form of a seal-bid. Market demands are allocated to each generation company, depending on the bids of generation companies. The strategy of player-*n* (company-*n*) and the payoff of player-*n* are defined by the coefficient of the bidding function of company-*n* and by the profit of company-*n* from the uniform price auction, respectively. The solution of the problem can be obtained using the concept of the non-cooperative equilibrium based on the Nash idea. As well, based on the residual demand curve, we can analytically derive the best response function of each generation company in the N-company uniform pricing auctions. Consequently, we present an efficient way to obtain all the possible equilibrium set pairs and to examine their feasibilities as Nash equilibriums. A simple numerical example with three generation companies is demonstrated to show the basic idea of the proposed theory. In theory, as this paper is focused on the analytical study of the equilibrium of N-company generation markets, we can see the applicability of the proposed method to the real-world problem.

Table 2 Nash Equilibrium Quantities

Company	1	2	3
b^{Nash}	0.026809	0.022339	0.01655
q^{Nash}	384.5723	461.5247	622.9607
Π^{Nash}	2342.8	2917.6	4277.0
p^{Nash}	10.31		

However, in order to apply the proposed method to the real-world problem, further investigations on the realistic constraints in electricity markets are needed. It is a limitation of this paper and can be studied more in the future study.

Appendix

Let a subscript mc denote a monopolistic company. If only one company is in the market, the following equality is satisfied since supply and demand must be balanced:

$$b_{mc}q_{mc} = \alpha - \omega q_{mc} \quad \text{(A-1)}$$

Now the monopolist's bidding strategy will be determined by maximizing its profit $\Pi_{mc}(b_{mc})$ as follows:

$$\begin{aligned}\max_{b_{mc}}\Pi_{mc}(b_{mc}) &= \max_{b_{mc}}\left\{b_{mc}(q_{mc})^2 - \frac{1}{2}\gamma_{mc}(q_{mc})^2\right\} \\ &= \max_{b_{mc}}\left(\frac{\alpha}{b_{mc}+\omega}\right)^2\left(b_{mc}-\frac{1}{2}\gamma_{mc}\right)\end{aligned} \quad \text{(A-2)}$$

By the necessary condition for the optimality, the following equality must be satisfied at the optimal b_{mc}^*:

$$\frac{d\Pi_{mc}}{db_{mc}}\left(b_{mc}^*\right) = \alpha^2\frac{\omega+\gamma_{mc}-b_{mc}^*}{\left(b_{mc}^*+\omega\right)^3} = 0 \quad \text{(A-3)}$$

Since $b_{mc}^*+\omega>0$, the monopolistic company's optimal decision for bidding is determined as $b_{mc}^*=\omega+\gamma_{mc}$.

Next, we consider a market with perfect competition. Let a superscript pc denote this perfect competition situation. In this case, since every company can be regarded as a price-taker, the market clearing price under perfect competition, denoted by $\overline{p}$, can be assumed as constant. Therefore, from the optimality condition on the profit of company-n, the optimal bidding strategy of company-n in a perfectly competitive market, b_n^{pc*}, can be obtained as follows:

$$\frac{d\Pi_n^{pc}}{dq_n^{pc}} = \frac{d}{dq_n^{pc}}\left(\overline{p}\cdot q_n^{pc} - \frac{1}{2}\gamma_n(q_n^{pc})^2\right) = \overline{p} - \gamma_n q_n^{pc} = 0$$

$$b_n^{pc*} = \frac{\overline{p}}{q_n^{pc}} = \frac{\gamma_n q_n^{pc}}{q_n^{pc}} = \gamma_n \quad \text{(A-4)}$$

Acknowledgements

This work was financially supported through the young professor award program of MOCIE in 2003.

References

[1] J. Schwarz, "Overview of the EU Electricity Directive", IEEE Power Engineering Review, Vol. 20, No. 4, April. 2000.

[2] F. Nishimura, R. D. Tabors, M. D. Ilic, and J. R. Lacalle-Melero, "Benefit Optimization of Centralized and Decentralized Power Systems in a Multi-Utility Environment", IEEE Trans. on Power Systems, Vol. 8, No. 3, pp. 1180-1186, August 1993.

[3] H.Singh, S.Hao, and A.Papalexopoulos, "Power auctions and network constraints", Proceedings of the Thirtieth Hawaii International Conference on System Sciences, Vol.5, pp. 608-614, 1997.

[4] F.A.Rahimi and A.Vojdani, "Meet the emerging transmission market segment", IEEE Computer Applications in Power, Vol.12, No.1, pp. 26-32, January 1999.

[5] G.B.Sheble, "Price based operation in an auction market structure", IEEE Trans. on Power Systems, Vol.11, No.4, pp. 1770-1777, November 1996.

[6] R.W.Ferrero, S.M.Shahidehpour, and V.C.Ramesh, "Transaction analysis in deregulated power systems using game theory", IEEE Trans. on Power Systems, Vol.12, No.3, pp. 1340-1347, November 1997.

[7] IEEE, IEEE Tutorial on Game Theory Applications in Electric Power Markets, 99 TP136-0, 1999.

[8] Jong-Bae Park, Balho H. Kim, Jin-Ho Kim, Manho Joung, and Jong-Keun Park, "A Continuous Strategy Game for Power Transactions Analysis in Competitive Electricity Markets", IEEE Trans. on Power Systems, Vol. 16, No. 4, pp. 847-855, November 2001.

[9] A. Rudkevich, M. Duckworth and R. Rosen, "Modeling electricity pricing in a deregulated

generation industry: the potential for oligopoly pricing in a poolco" , Energy Journal, Vol. 19, No. 3, pp. 19-48, August 1998.
[10] Jin-Ho Kim, Jong-Bae Park, Jong-Keun Park, and Balho H. Kim, "A New Approach to Maintenance Scheduling Problems Based on Dynamic Game Theory", KIEE International Transactions on Power Engineering, Vol. 12A, No. 2, pp. 73-79, June 2002.

Jin-Ho Kim
He received his B..S, M.S., and Ph.D. degrees in Electrical Engineering from Seoul National University. Currently, he is an Assistant Professor at Pusan National University. His main research interest is in the area of power system economics.

Jong-Ryul Won
He received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Seoul National University. Currently, he is an Assistant Professor at Anyang University. His research interests are in the areas of power system economics and the operation and planning of power systems.

Jong-Bae Park
He received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Seoul National University. Currently, he is an Assistant Professor at Konkuk University. His research interests are in the areas of power system economics and the operation and planning of power systems.