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Blind channel equalization using fourth-order cumulants and a neural network

Soowhan Han

Dept. of Multimedia Engineering Dongeui University, Busan, Korea 614-714

Abstract

This paper addresses a new blind channel equalization method using fourth-order cumulants of channel inputs and a three-layer neural network equalizer. The proposed algorithm is robust with respect to the existence of heavy Gaussian noise in a channel and does not require the minimum-phase characteristic of the channel. The transmitted signals at the receiver are over-sampled to ensure the channel described by a full-column rank matrix. It changes a single-input/single-output (SISO) finite-impulse response (FIR) channel to a single-input/multi-output (SIMO) channel. Based on the properties of the fourth-order cumulants of the over-sampled channel inputs, the iterative algorithm is derived to estimate the deconvolution matrix which makes the overall transfer matrix transparent, i.e., it can be reduced to the identity matrix by simple recordering and scaling. By using this estimated deconvolution matrix, which is the inverse of the over-sampled unknown channel, a three-layer neural network equalizer is implemented at the receiver. In simulation studies, the stochastic version of the proposed algorithm is tested with three-ray multi-path channels for on-line operation, and its performance is compared with a method based on conventional second-order statistics. Relatively good results, withe fast convergence speed, are achieved, even when the transmitted symbols are significantly corrupted with Gaussian noise.

Key words : blind channel equalization, fourth-order cumulants, neural network equalizer.

1. Introduction

In digital communication systems, data symbols are transmitted at regular intervals. Time dispersion, which is caused by non-ideal channel frequency response characteristics or multi-path transmission, may create inter-symbol interference (ISI). This has become a limiting factor in many communication environments. Thus, channel equalization is necessary and important with respect to ensuring reliable digital communication links. The conventional approach to channel equalization needs an initial training period with a known data sequence to learn the channel characteristics. In contrast to standard equalization methods, the so-called blind (or self- recovering) channel equalization method does not require a training sequence from the transmitter [1]-[3]. It has two obvious advantages. The first is the bandwidth savings resulting from elimination of training sequences. The second is the self-start capability before the communication link is established or after it experiences an unexpected breakdown. Because of these advantages, blind channel equalization has gained practical interest during the last decade.

Recently, blind channel equalization based on second-order cyclostationary has been receiving increasing interest. The algorithm presented by Tong *et al.* [4] is one of the first subspace-based methods exploiting only second-order statistics for a system with channel diversity that has a single-input/ multi-output (SIMO) discrete-time equivalent model. After their work, a number of different second-order statistical (SOS) methods have been proposed [5]-[10]. However, it should be noted that most SOS methods require a relatively high signal-to-noise ratio (SNR) to achieve reliable performance. In practice, the performance degradation using SOS methods is severe if a received signal is significantly corrupted by noise. In this case, a larger sample size is necessary [4]. To avoid this problem, higher-order statistics (HOS) can be exploited. Several recent works have re-established the robustness of higher-order statistical methods in channel equalization and identification [11]-[13].

In this study, a new iterative algorithm based on the fourth-order cumulants of over-sampled channel inputs is derived to estimate the deconvolution (equalization) matrix which makes the overall transfer matrix transparent, i.e., it can be reduced to the identity matrix by simple reordering and scaling. This solution is chosen so that the fourth-order statistics of the equalized output sequence $\{\hat{s}(k)\}$ is close to the fourth-order statistics of the channel input sequence $\{s(k)\}$. It has a similar formulation with the cumulant-based iterative inversion algorithm which was introduced by Cruces *et al.* [14] for blind separation of independent source signals, but the iterative solution in our algorithm is extended with an additional constraint (a fourth-order statistical relation between the equalized outputs of over-sampled channels) in order to be applied to the blind channel equalization problem. In the experiments, the proposed iterative solution provides more precise estimates of the deconvolution matrix with fast convergence speeds than a method based on second-order

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statistics, even when the outputs of a non-minimum phase channel are corrupted by heavy Gaussian noise. However, this deconvolution matrix may yield to an amplification of the noise at the outputs because of noise-corrupted inputs, even though it can be precisely estimated from the noisy channel outputs. To avoid this limitation, a three-layer neural equalizer, instead of the deconvolution matrix itself, is implemented at the receiver by using the over-sampled channel matrix (inverse of estimated deconvolution matrix). It is known that the equalizer made of neural network structure has a better noise-tolerant characteristic [15]-[17].

This paper is organized as follows: A brief summary of the problem formulation for blind channel equalization is presented in Section 2, the proposed iterative algorithm presented in Section 3, the structure of three-layer neural equalizer in Section 4, extensive computer simulations, including the comparisons with an approach based on second-order statistics, and our conclusions in Section 5 and 6, respectively.

2. Problem fomulation

In a multi-path digital communication system, a data sequence $\{s(k)\}$, $k=\cdots,-1,0,1,2,\cdots$, is sent over a communication channel with a time interval T. The channel is characterized by a continuous function $h(t)$, and the signals may be corrupted by noise $e(t)$. The received signal $y(t)$ can be expressed as:

$$x(t) = \sum_{-\infty}^{+\infty} s(k)h(t-kT) \tag{1}$$

$$x(t) = x(t) + e(t) \tag{2}$$

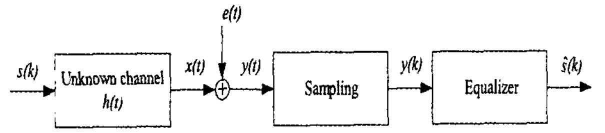

Fig.1. Blind channel equalization in digital communication.

This is shown in Fig. 1. The objective of blind equalization is to recover the transmitted input symbol sequence $\{s(k)\}$ given only the received signal $y(t)$. Instead of choosing the equalizer so that the equalized output sequence $\{\hat{s}(k)\}$ is close to the source symbol sequence $\{s(k)\}$, as in the standard equalization formulation, in blind equalization one chooses the equalizer so that the statistics of the equalized output sequence is close to the statistics of the source symbol sequence. In this study, a robust algorithm with respect to noise is constructed with a higher-order statistical constraint, which makes the fourth-order statistics of $\{\hat{s}(k)\}$ close to the fourth-order statistics of $\{s(k)\}$. For this approach, the following assumption is necessary.

1>The symbol interval T is known and is an integer multiple of the sampling period.

2>The impulse response $h(t)$ has finite support, if the duration of $h(t)$ is L_h, $h(t)=0$ for $t < 0$ or $t \geq L_h$

3>$\{s(k)\}$is zero mean, and is driven from a set of i.i.d. random vaiables, which means the fourth-order zero-lag cumulant or kurtosis of $\{s(k)\}$can be expressed by

$$C^{1,3}_{s(k),s(l)}(0) = cum(s(k),s(l),s(l),s(l)) = E\{s(k)s^*(l)s(l)s^*(l)\} = \alpha\delta(k-l) \tag{3}$$

where α is non-zero constant and $\delta(t)$ is the discrete time impulse function.

4>*$e(t)$ is zero-mean Gaussian noise, and uncorrelated with* $\{s(k)\}$.

In the conventional equalizer, the incoming signal at the receiver $y(t)$ is spaced at the reciprocal of the transmitted symbol interval T. However, in this study, the over-sampling technique is applied to change a finite-impulse response (FIR) channel to a SIMO channel, which requires the incoming signal $y(t)$ to be sampled at least as fast as the Nyquist rate. This is illustrated by way of an example shown in Fig. 2, where the channel lasts for 4 adjacent bauds, and the over-sampling rate is $T/4$.

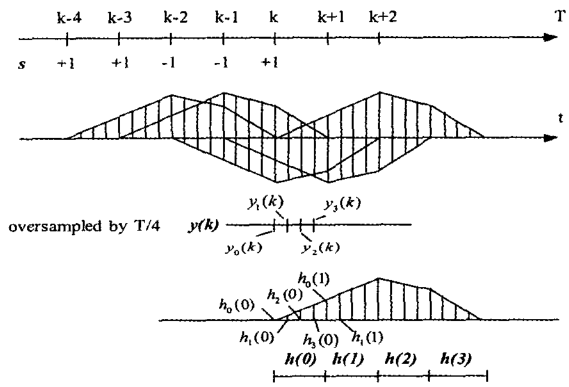

Fig. 2. An over-sampling example of a FIR channel.

With over-sampling at rate $T/4$ during observation interval $L=T$ in Fig. 2, a channel output vector at time index k is given by equation (4). If we define a FIR channel $h(t)$ as in equation (5), $y_0(k)$ and $y_1(k)$ can be expressed as in equations (6) and (7), respectively. In the same way, $y_2(k)$ and $y_3(k)$ can be obtained.

$$\mathbf{y}(k)=[y_0(k),y_1(k),y_2(k),y_3(k)]^T \tag{4}$$

$$\mathbf{h}(0)=[h_0(0),h_1(0),h_2(0),h_3(0)]^T \tag{5}$$

$$y_0(k)=h_0(0)s(k)+h_0(1)s(k-1)+h_0(2)s(k-2)+h_0(3)s(k-3)+e_0(k) \tag{6}$$

$$y_1(k)=h_1(0)s(k)+h_1(1)s(k-1)+h_1(2)s(k-2)+h_1(3)s(k-3)+e_1(k) \tag{7}$$

Then we have

$$y(k)=Hs(k)+e(k) \tag{8}$$

where $s(k)=[s(k),s(k-1),s(k-2),s(k-3)]^T$, $e(k)=[e_0(k),e_1(k),e_2(k),e_3(k)]^T$ and

$$H=\begin{matrix} h_0(0) & h_0(1) & h_0(2) & h_0(3) \\ h_1(0) & h_1(1) & h_1(2) & h_1(3) \\ h_2(0) & h_2(1) & h_2(2) & h_2(3) \\ h_3(0) & h_3(1) & h_3(2) & h_3(3) \end{matrix} = [h(0),h(1),h(2),h(3)] \tag{9}$$

If the observation interval L is greater than T, for example $L=2T$ in Fig. 2, $y(k)=[y_0(k), y_1(k), y_2(k), y_3(k), y_4(k), y_5(k), y_6(k), y_7(k)]^T$, $s(k)=[s(k+1), s(k), s(k-1), s(k-2), s(k-3)]^T$, $e(k)=[e_0(k), e_1(k), e_2(k), e_3(k), e_4(k), e_5(k), e_6(k), e_7(k)]^T$, and H becomes as a 8×5 channel matrix shown in equation (10).

$$H=\begin{bmatrix} \mathbf{0}, h(0), h(1), h(2), h(3) \\ h(0), h(1), h(2), h(3), \mathbf{0} \end{bmatrix} \tag{10}$$

where $\mathbf{0}=[0,0,0,0]^T$.

In our approach to recover the transmitted input symbol sequence $\{s(k)\}$, a deconvolution matrix G in equation (11) is derived to transform the overall transfer function $W=GH$ into the identity matrix by using the observed channel output $y(k)$ only. For the solvability of blind equalization problem, an additional assumption is made throughout, i.e., the over-sampling rate T/N or the length of the observation interval L, qT, is selected to make the over-sampled channel matrix H full column rank. This means if a channel $h(t)$ has p taps, H can described by a $Nq\times(p+q-1)$ matrix, and N or q should be chosen for $Nq\geq(p+q-1)$

$$\hat{s}(k)=Gy(k)=GHs(k)=Ws(k) \tag{11}$$

3. Iterative solution based on fourth-order cumulants

The aim in blind equalization is to select G in equation (11) that recovers the original source sequence $\{s(k)\}$ only from the observations of the sampled channel output $y(k)$. This is obtainable when the overall transfer system W is transparent (or reduced to an identity). Here, for notational simplicity, we consider a special reordering and scaling so that W will always be an identity matrix. If the over-sampled channel H is a $Nq\times(p+q-1)$ matrix and full column rank, its input sequences can be expressed as in equation (12).

$$s=\begin{bmatrix} s_{p+q-2} \\ \mathrm{M} \\ s_1 \\ s_0 \end{bmatrix}=\begin{bmatrix} s(p+q-2) & s(p+q-1) & \Lambda & s(M-1) \\ \mathrm{M} & \mathrm{M} & \Lambda & \mathrm{M} \\ s(1) & s(2) & \Lambda & s(M-(p+q-2)) \\ s(0) & s(1) & \Lambda & s(M-(p+q-1)) \end{bmatrix} \tag{12}$$

where M is the total number of transmitted sequences and $s_0, s_1, \Lambda, s_{p+q-2}$ are the shifted input vectors by time interval T for each of $p+q-1$ over-sampled FIR channels. Then, for the noise-free case, equation (11) can be rewritten as

$$\hat{s}=\begin{bmatrix} \hat{s}_{p+q-2} \\ \mathrm{M} \\ \hat{s}_1 \\ \hat{s}_0 \end{bmatrix}=GH\begin{bmatrix} s_{p+q-2} \\ \mathrm{M} \\ s_1 \\ s_0 \end{bmatrix}=Ws \tag{13}$$

The identifiability of system W can be guaranteed because the channel H has full column rank and its input vectors, $s_0, s_1, \Lambda, s_{p+q-2}$, are mutually independent [18]. Equation (13) can be considered as a blind source separation (BSS) problem. If we properly scale channel input s such that the kurtosis of each of $s_0, s_1, \Lambda, s_{p+q-2}$ is equal to +1 or -1 (scaled to $\alpha=1$ in equation (3)), its BSS solution by using a preconditioned iteration [19], is given by equation (14) [14].

$$G^{(n+1)}=G^{(n)}-\mu^{(n)}(C^{1,3}_{\hat{s}_k,\hat{s}_l}(0)S^3_{\hat{s}}-I)G^{(n)} \tag{14}$$

where $C^{1,3}_{\hat{s}_k,\hat{s}_l}(0)=cum(\hat{s}_k,\hat{s}_l,\hat{s}_l,\hat{s}_l)=E\{\hat{s}_k\hat{s}^*_l\hat{s}_l\hat{s}^*_l\}$: the fourth-order zero-lag cumulant or kurtosis matrix of $\hat{s}(k,l=0,1,\cdots,p+q-2)$, $S^3_{\hat{s}}=diag(sign(diag(C^{1,3}_{\hat{s}_k,\hat{s}_l}(0))))$ in the Matlab convention, μ =a step-size of iteration, and I is an identity matrix. The fundamental idea of this solution is based on the fact that the fourth-order statistics of equalizer output $\hat{s}$ should be close enough to the fourth-order statistics of channel input s. However, in order to apply the BSS method in equation (14) to the blind channel equalization problem, an additional constraint must be considered. It is as follows.

The channel input $s=[s_{p+q-2}, \Lambda, s_1, s_0]^T$ is constructed by shifting the same sequences with a time interval T, which is shown in equation (12). It means the fourth-order cumulant matrix of s with lag 1 always satisfies the following expression

$$C^{1,3}_{s_k,s_l}(1)J^TS^3_{sJ}=JJ^T \tag{15}$$

where $C^{1,3}_{s_k,s_l}(1)=cum(s_k,s_{l+1},s_{l+1},s_{l+1})=E\{s_ks^*_{l+1}s_{l+1}s^*_{l+1}\}$, J is a shifting matrix denoted by equation (16), and $S^3_{sJ}=diag(sign(diag(C^{1,3}_{s_k,s_l}(1)J^T)))$.

$$J=\begin{bmatrix} 0 & 0 & \Lambda & 0 & 0 \\ 1 & 0 & \Lambda & 0 & 0 \\ 0 & 1 & \Lambda & 0 & 0 \\ \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{M} \\ 0 & 0 & \Lambda & 1 & 0 \end{bmatrix} \tag{16}$$

Thus, the fourth-order cumulant matrix of equalizer output $\hat{s}$ with lag 1 should be forced to satisfy equation (15), and its iterative solution can be written as

$$G^{(n+1)} = G^{(n)} - \beta^{(n)}(C^{1,3}_{\hat{s}_k,\hat{s}_l}(1)J^T S^3_{\hat{s}J} - JJ^T)G^{(n)} \tag{17}$$

where β=a step-size of iteration. Based on the above analysis, a new iterative solution combining equation (14) with equation (17) is derived for blind channel equalization, which is shown in equation (18).

$$G^{(n+1)} = G^{(n)} - \mu^{(n)}(C^{1,3}_{\hat{s}_k,\hat{s}_l}(0)S^3_{\hat{s}} - I)G^{(n)} - \beta^{(n)}(C^{1,3}_{\hat{s}_k,\hat{s}_l}(1)J^T S^3_{\hat{s}J} - JJ^T)G^{(n)} \tag{18}$$

If the formulation of equation (18) is based on the second-order statistics of equalizer output and the channel input s is scaled to have a unity power, the iterative solution is reduced as

$$G^{(n+1)} = G^{(n)} - \mu^{(n)}(C^{1,1}_{\hat{s}_k,\hat{s}_l}(0) - I)G^{(n)} - \beta^{(n)}(C^{1,1}_{\hat{s}_k,\hat{s}_l}(1)J^T - JJ^T)G^{(n)} \tag{19}$$

where $C^{1,1}_{\hat{s}_k,\hat{s}_l}(0)=cum(\hat{s}_k,\hat{s}_l)=E\{\hat{s}_k\hat{s}_l^*\}$ and $C^{1,1}_{\hat{s}_k,\hat{s}_l}(1)=cum(\hat{s}_k,\hat{s}_{l+1})=E\{\hat{s}_k\hat{s}_{l+1}^*\}$: zero-lag and lag 1 correlation function of $\hat{s}$, respectively. These two iterative solutions have been implemented in a batch manner in order to obtain an accurate comparison, and tested with three-ray multi-path channels. In our experiments, their stochastic versions, which are shown in equation (20) for the fourth-order statistics and in equation (21) for the second-order statistics, are evaluated for possible use on-line. These are accomplished by estimating continuously the fourth-order cumulants in equation (18) and the second-order correlations in equation (19) with the over-sampled channel outputs coming in at time interval T. Thus, G gets updated at time interval T. By applying these stochastic versions of algorithm, it is not necessary to wait until a whole block of the sample is received to estimate G. The stochastic version based on second-order statistics in equation (21) is the same as the one used by Fang *et al.* [5] for their two-layer neural network equalizer. It is compared with our proposed algorithm based on the fourth-order statistics shown in equation (20).

$$G^{(n+1)} = G^{(n)} - \mu^{(n)}(f(\hat{s}_i^{(n-1)})(\hat{s}_i^{(n-1)})^T S^3_{f\hat{s}} - I)G^{(n)} - \beta^{(n)}(f(\hat{s}_i^{(n)})(\hat{s}_i^{(n-1)})^T J^T S^3_{fJ} - JJ^T)G^{(n)} \tag{20}$$

$$G^{(n+1)} = G^{(n)} - \mu^{(n)}(\hat{s}_i^{(n-1)}(\hat{s}_i^{(n-1)})^T - I)G^{(n)} - \beta^{(n)}(\hat{s}_i^{(n)}(\hat{s}_i^{(n-1)})^T J^T - JJ^T)G^{(n)} \tag{21}$$

where $\hat{s}_i^{(n)} = [\hat{s}^{(n)}_{p+q-2}, \Lambda, \hat{s}_1^{(n)}, \hat{s}_0^{(n)}]^T$: a $(p+q-1)\times 1$ output vector of $G^{(n)}$, $f(\hat{s}_i^{(n)}) = (\hat{s}_i^{(n)})^3 - 3\hat{s}_i^{(n)}\sigma^2_{\hat{s}_i}$, $\sigma^2_{\hat{s}_i}$: adaptively estimated power of $\hat{s}_i$ at each iteration, $S^3_{f\hat{s}} = diag(sign(diag(f(\hat{s}_i^{(n-1)})(\hat{s}_i^{(n-1)})^T)))$ and $S^3_{fJ} = diag(sign(diag(f(\hat{s}_i^{(n)})(\hat{s}_i^{(n-1)})^T J^T)))$.

4. Neural network-based equalizer

In the absence of noise, the deconvolution matrix G perfectly recovers the source symbols at the output because of the overall transfer function $W=GH=I$. However, when there is noise, this deconvolution matrix may yield to an amplification of the noise at its outputs, even though it can be precisely estimated from the noisy channel outputs y by using our proposed algorithm. To avoid this limitation, a three-layer neural equalizer is employed at the receiver because of its noise robust characteristic [15]-[17]. This is done by using the estimated over-sampled channel as a reference system to train the neural equalizer. It consists of an input layer, a hidden layer, and an output layer of processing elements called neurons [15][16], as shown in Fig. 3.

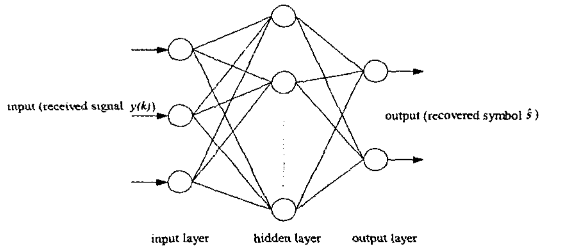

Fig. 3. The structure of three-layer neural equalizer.

Once the deconvolution matrix G is estimated, which means the over-sampled channel H is available, the training sequences based on H are generated at the receiver. The three-layer neural equalizer is trained with these sequences by using the back-propagation algorithm. In back-propagation, the output value is compared with the desired output. This results in the generation of an error signal, which is fed back through layers in the network, and the weights are adjusted to minimize the error. More details on the back-propagation algorithm can be found in [15][16]. A sample of equalized binary (+1,-1) source symbols under 15dB SNR by this neural equalizer, one by the deconvolution matrix G itself, and one

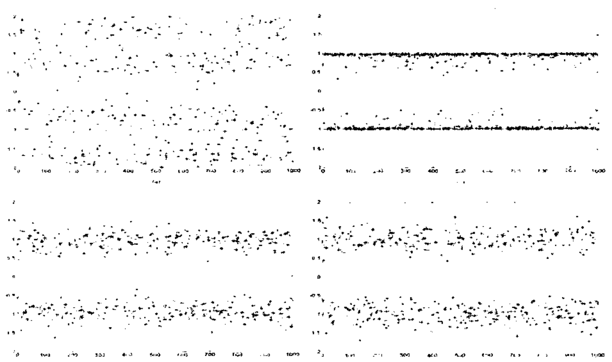

Fig. 4. Samples of received and equalized symbols under 15db SNR: (a) 1000 received symbols, (b) equalization by a neural equalizer, (c) by G *itself derived from eq. (20), and (d) by the optimal inverse of* H.

by the optimal inverse of over-sampled channel $\boldsymbol{H}$ are shown in Fig. 4. The deconvolution matrix $\boldsymbol{G}$ used in Fig. 4 is derived from the proposed algorithm in equation (20). The outputs of neural equalizer can be more densely placed onto the transmitted symbols (+1,-1) even in heavy noise environments.

5. Simulation results and performance assessments

The blind equalizations with three-ray multi-path channels are taken into account to show the effectiveness of the proposed algorithm. Performances under different SNRs, varied from 5 to 15dB with 2.5 dB increments, are averaged after 50 independent simulations. The proposed algorithm and the solution based on the second-order statistics are implemented in a batch manner in order to achieve the accurate comparison. In the first experiment, a three-ray multi-path channel truncated up to 2 symbol periods *(p=2)* is tested with 1000 randomly generated binary transmitted symbols (taken from $\{\pm 1\}$). The delays of this channel are *0.5T* and *1.1T*, and its waveform is a raised-cosine pulse with 11% roll-off. It has a *zero* outside unit circle, which indicates a non-minimum phase characteristic. The channel outputs are sampled twice as fast as the symbol rate, which means the over-sampling rate is *T/2 (N=2)*, and the observation interval used for this channel is *T (q=1)*. Thus, the over-sampled channel $\boldsymbol{H}$ becomes a 2×2 *(Nq×(p+q-1))* matrix. For each simulation, the initial matrix for $\boldsymbol{G}$ and both of step size(μ, β) in equations (20) and (21) are set to an identity matrix $\boldsymbol{I}$ and 0.001, respectively, and the numbers of iteration is limited to 50 epochs. The normalized root-mean square error for overall transfer system $\boldsymbol{W=GH}$ is measured in terms of the index $NRMSE_w$,

$$NRMSE_w = \frac{1}{\|\boldsymbol{I}\|}\sqrt{\frac{1}{NS}\sum_{j=1}^{NS}\left\|\boldsymbol{W}^{(j)} - \boldsymbol{I}\right\|^2} \tag{22}$$

where $\boldsymbol{W}^{(j)} = \boldsymbol{G}^{(j)}\boldsymbol{H}$ is the estimation of overall system at the j^{th} simulation and NS is the number of independent simulations (*NS=50* in this study). The $NRMSE_w$ for the proposed algorithm and the one based on second-order statistics are shown in Fig. 5 with different noise levels, and their averaged convergences within 50 epochs for 5 dB and 15dB SNR are demonstrated in Figs. 6 and 7, respectively.

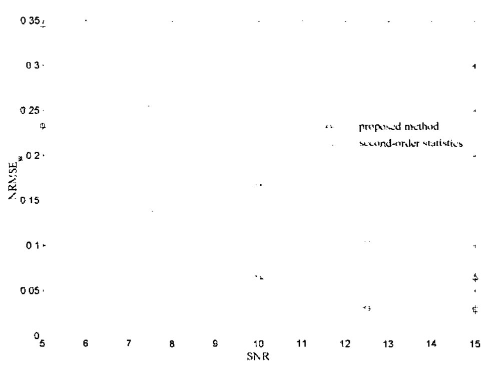

Fig. 5. $NRMSE_w$ with different SNR levels in experiment 1.

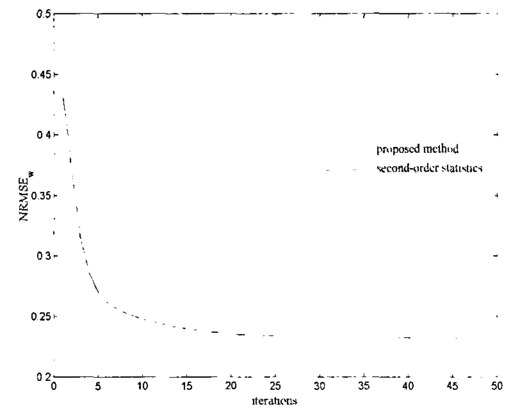

Fig. 6. $NRMSE_w$ vs. iterations for 5 dB SNR in experiment 1.

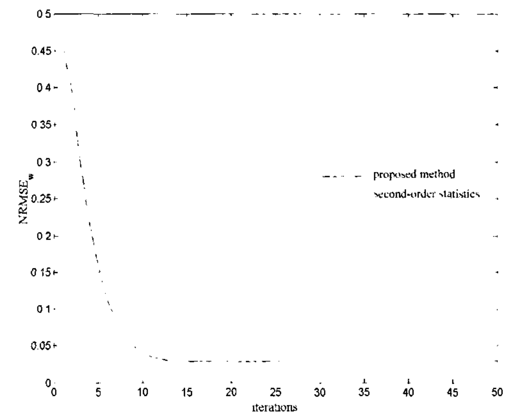

Fig. 7. $NRMSE_w$ vs. iterations for 15 dB SNR in experiment 1

Once $\boldsymbol{G}$ is available, the three-layer neural equalizer is trained with 1000 training sequences which have been generated at the receiver. It has 2 inputs, 4 hidden neurons and 2 outputs, and the learning rate is set to 0.05. The maximum number of iterations for the training process is set to 50 epochs. A portion of the mean-square-error for training is shown in Fig. 8. The output of this neural equalizer is the estimation of transmitted symbols, and its performance measure is defined as follows.

$$NRMSE_s = \frac{1}{\|\boldsymbol{s}\|}\sqrt{\frac{1}{NS}\sum_{j=1}^{NS}\left\|\hat{\boldsymbol{s}}^{(j)} - \boldsymbol{s}\right\|^2} \tag{23}$$

where $\hat{\boldsymbol{s}}^{(j)}$ is the estimate of the channel input $\boldsymbol{s}$ at the j^{th} trial. The $NRMSE_s$ by the neural equalizer with the proposed algorithm and with the one based on second-order statistics are shown in Fig. 9, and their bit error rates (BER) are compared in Fig. 10.

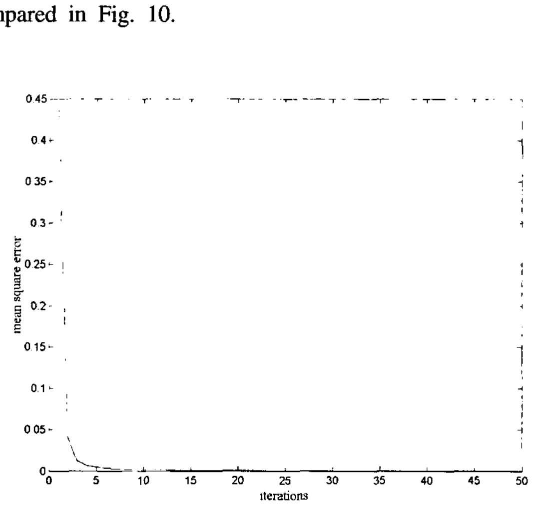

Fig. 8. A mean square error of neural equalizer in training procedure.

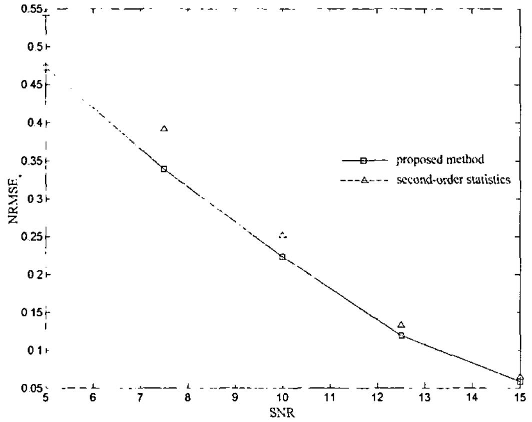

Fig. 9. $NRMSE_s$ with different SNR levels in experiment 1.

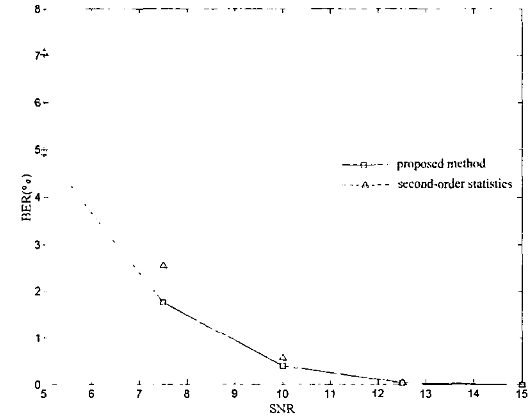

Fig. 10. Averaged BER(%) in experiment 1.

In the second experiment, the same simulation environment is used, such as the step size(μ, β), the learning rate for the neural equalizer, the maximum number of iterations, and the over-sampling rate *(N=2)*. The exceptions are the length of channel, its delays and the observation interval. The three-ray mult-path channel tested at this time is truncated up to 3 symbol periods $(p=3)$, and its delays are T and $1.5T$. It has one *zero* outside unit circle and the other inside. The observation interval used for this non-minimum phase channel is two times longer than one symbol period, $2T$ $(q=2)$, and thus, the over-sampled channel $\boldsymbol{H}$ becomes a 4×4 $(Nq\times(p+q-1))$ matrix. The neural equalizer used to recover the transmitted symbols in this experiment has 4 inputs, 8 neurons in the hidden layer, and 4 outputs. The performance measures, $NRMSE_w$, $NRMSE_s$ after 50 independent simulations, and the averaged BER, are presented in Figs. 11-13, respectively.

From the simulation results for $NRMSE_w$, which are shown in Fig. 5 for experiment 1 and in Fig. 11 for experiment 2, the proposed solution is proved highly effective to estimate $\boldsymbol{G}$, the inverse of unknown channel $\boldsymbol{H}$, which makes the overall system $\boldsymbol{W}$=$\boldsymbol{GH}$ an identity even when the observed symbols are heavily corrupted by noise. The difference in performance between the proposed solution and the one based on the

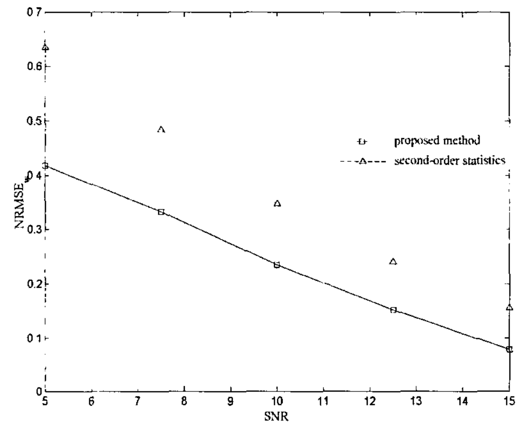

Fig. 11. $NRMSE_w$ with different SNR levels in experiment 2.

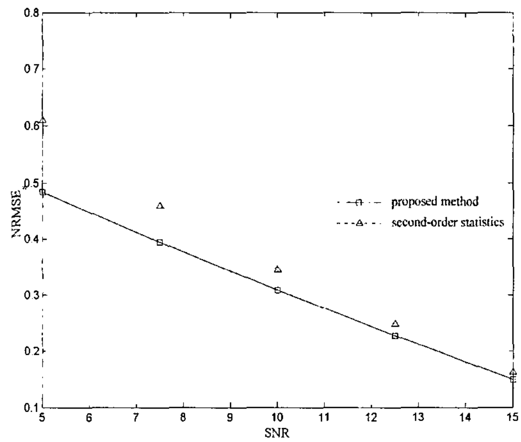

Fig. 12. $NRMSE_s$ with different SNR levels in experiment 2.

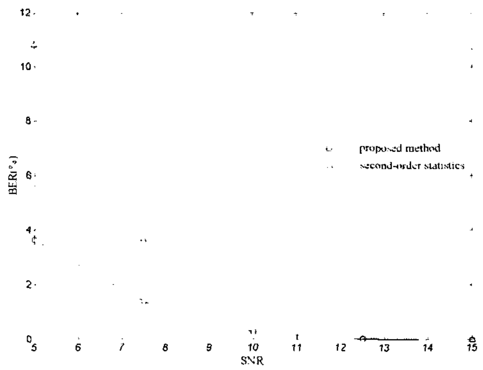

Fig. 13. Averaged BER(%) in experiment 2.

second-order statistics is not severe if the noise(signal) level is as low(high) as 15dB SNR in our experiments. However, it is observed that, if the noise level is getting higher such as to 10 or 5 db SNR, the proposed algorithm performs relatively well, and the performance difference becomes more serious. It results from the fact that our approach is based on the fourth-order cumulant of the received symbols and it always goes to zero for Gaussian noise. This phenomenon can also be found for $NRMSE_s$ in Figs. 9 and 12, and the averaged BER in Figs. 10 and 13, because the neural equalizer trained with more accurate estimation of $\boldsymbol{H}$ produces the lower symbol estimation error. Therefore, the proposed algorithm in our study can be implemented for on-line operation in a *heavy noise* communication environment.

6. Conclusions

In this paper, a new iterative solution based on the fourth-order cumulants of over-sampled channel inputs is presented for blind channel equalization. It does not require the minimum phase characteristic, and shows relatively high performance results even when the observed symbols are significantly corrupted by Gaussian noise. In addition, it can be implemented for on-line operation for channel estimation without waiting for a whole block of the symbols. The proposed algorithm could possibly be used for heavy noise communication environments. In future work, the proposed iterative solution will be further investigated and applied as a learning algorithm for a neural network so that the transmitted symbols can be directly recovered from the output of a neural-based equalizer without the estimation procedure of the deconvolution matrix, $\boldsymbol{G}$.

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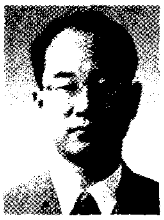

Han, Soowhan
1986 : Dept. of Electronics, Yonsei Univ.(B.S.)
1990 : Dept. of Electrical and Computer Eng., Florida Institute of Technology (M.S.)
1993 : Dept. of Electrical and Computer Eng., Florida Institute of Technology(Ph.D.)
1994~1997 : Assistant Prof. in Dept. of Computer Eng., Kwandong Univ.
1997~present : Associate Prof. in Dept. of Multimedia Eng., Dongeui Univ.
2003~2004 : Visiting Prof., University of Alberta, Canada
Research interest : Digital Signal & Image Processing, Pattern Recognition, Fuzzy Logic & Neural Networks

Phone : +82-51-890-1950
Fax : +82-51-890-1954
E-mail : swhan@deu.ac.kr