A Retransmission Power Adjustment Scheme for Performance Enhancement in DS/SSMA ALOHA with Packet Combining

Hanbyul Seo, Seongyong Park, and Byeong Gi Lee

Abstract: **In this paper, we present a *retransmission power adjustment* (RPA) scheme for DS/SSMA ALOHA packet radio systems with packet combining. In the proposed RPA scheme, retransmission power is adjusted in such a way that the erroneously-received packet can be recovered with a minimized interference to other user packets. We analyze the performance of the system with the RPA by employing the *equilibrium point analysis* (EPA), and confirm that the results obtained from the EPA are very close to the simulation results in low power cases. Simulation results demonstrate that the RPA scheme brings forth performance gain in the throughput and the average delay while saving a significant amount of transmission power. We also investigate the stability of the system from the EPA results, and conclude that the system becomes stable as the offered load increases or the level of retransmission power decreases.**

Index Terms: **Direct-sequence spread-spectrum multiple access (DS/SSMA), equilibrium point analysis (EPA), hybrid automatic repeat request (HARQ), packet combining, retransmission power adjustment.**

I. INTRODUCTION

ALOHA is a simple *multiple access control* (MAC) protocol that is useful in packet radio systems. In narrow-band ALOHA packet radio systems, when two or more users transmit packets simultaneously, the packets from all users get severely corrupted due to *multiple access interference* (MAI). The information in the erroneously-received packet does not help much for detecting packets through retransmission, and hence, *forward error correction* (FEC) or combining of the erroneously-received packets brings no benefit to narrow-band ALOHA systems [1].

A number of narrow-band channels can be combined to form a *direct-sequence spread-spectrum multiple access* (DS/SSMA) channel. If DS/SSMA is employed in a packet radio system, the number of errors introduced into a packet then depends on the number of packets that are transmitted simultaneously. When the number of such simultaneous users is small, the average number of errors within a packet is small, so the packet contains much useful information about the transmitted data. Moreover, the erroneous bits in an erroneously-received packet can be recovered easily as they are spread by a *pseudo-noise* (PN) sequence. Therefore, it is reasonable to expect that combining erroneous copies of the same packet can improve the performance of the system [1], [2].

There have been several proposals for packet combining techniques based on the above-mentioned idea in DS/SSMA systems: The code combining scheme [1], [3] combines the received packets at the *codeword level*, thereby making a more reliable packet transmission at a lower code rate. The diversity combining scheme [4], [5] combines the received packets at the *bit level*, thus enabling a more reliable packet at the same code rate. Since the maximum-likelihood decoding is applicable to the code combining scheme, it can outperform the diversity combining scheme in terms of packet error probability. However, the diversity combining scheme has the advantage that it is simple to implement and requires less memory.

In the conventional combining schemes, the power used for the retransmitted packets, or retransmission power, is set equal to the initial transmission power. However, in reality, a small amount of additional information may be good enough to recover the erroneous packet. In the DS/SSMA with packet combining, the receiver already contains much useful information taken from the initial transmission, so the amount of information that should be delivered in retransmitted packets is much less than that of the initial transmission. Therefore, the retransmissions in conventional schemes induce unnecessarily high interference to other users, consequently disabling the full utilization of the potential capacity of DS/SSMA channel. In order to resolve this problem, we propose in this paper a *retransmission power adjustment* (RPA) scheme. In this scheme, the level of the retransmission power is adjusted in such a way that only a small amount of information that is adequate to recover the corrupted packet is retransmitted. The RPA scheme prevents inducing unnecessarily high interferences so that the system performance can be improved.

There have been reports on the performance analysis of the DS/SSMA system adopting the *hybrid automatic repeat request* (HARQ) schemes [1], [2], [4]–[8]: Among them, [1] and [5] assume that the number of active users at different times is an independent and identically distributed Poisson random variable, and [4] assumes that the number of active transmitting users is fixed. However, if we consider that the number of transmitting and retransmitting users at a given time instant depends on the previous transmission events, the number of active users should be a stochastic process with time-dependency. This indicates that the above analyses ignored the time-dependent characteris-

---

Manuscript received May 28, 2003; approved for publication by Dong Geun Jeong, Division II Editor, September 23, 2004.

H. Seo and B. G. Lee are with the School of Electrical Engineering #029, Seoul National University, Seoul, 151-742, Korea, email: {shb, blee}@tsp.snu.ac.kr.

S. Park is with Samsung Electronics Co. Ltd., Korea, email: shone.park@samsung.com.

This research was supported by University IT Research Center Project.

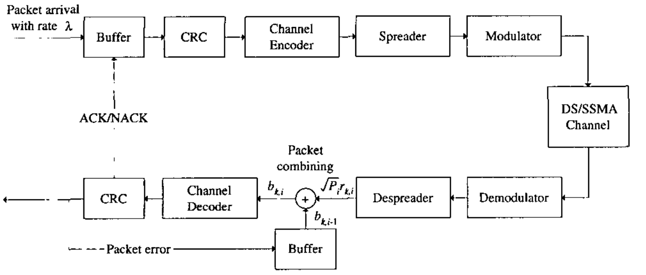


Fig. 1. Block diagram of the DS/SSMA ALOHA system with packet combining for each user.

tics. On the other hand, [2] and [6] consider this characteristic, but the examined HARQ scheme was limited to combining only two recently received packets. [7] and [8] obtained the throughput of multiple access schemes employing HARQ, but both of them are limited to the asymptotic case where the number of users increases to infinity.

In the performance analysis of the RPA-based DS/SSMA system in this paper, we employ the *equilibrium point analysis* (EPA) technique [6], [9]. The dynamics of the system with RPA is very complicated to analyze since the RPA scheme uses various different levels of transmission power, but the EPA technique can render an easy means to determine an approximated system throughput and delay. We can check the accuracy of the EPA by comparing the results given by the EPA with simulation results. We can also investigate the stability of the system from the EPA results.

This paper is organized as follows: First, we describe the system model of the target DS/SSMA ALOHA system in Section II, and present the concept of the RPA scheme in Section III. Then, we analyze the system performance using the EPA in Section IV. Finally, we examine the performance of the RPA scheme through simulations and EPA analyses in Section V.

II. SYSTEM DESCRIPTION

Fig. 1 depicts the block diagram of the DS/SSMA ALOHA system with packet combining. We model the system as a finite-user packet radio network of U users transmitting the fixed size packets of L bits.

A. Transmitter

We assume that packets are generated at rate λ (packets/bit time) for each user. Each user has a buffer which can store up to B packets, including the one being transmitted. In order to protect packets from interferences, we use a t-error correcting code for FEC, which can correct up to t-bit errors regardless of the correlation of errors in a packet. We adopt Gilbert-Varshamov lower bound [10] as the code rate r, i.e.,

$$r = 1 + A\log_2 A + (1-A)\log_2(1-A), \quad A \equiv \frac{2t+1}{L}. \qquad (1)$$

The modulation used is *binary phase-shift keying* (BPSK) and each packet is spread by a PN sequence with the spreading factor N. The normalized offered load G is then given by

$$G = \frac{\lambda L U}{N}. \qquad (2)$$

Note that the *offered load* above is defined by the arrival rate of higher-layer packets to the MAC layer, excluding the load caused by retransmissions.

B. Receiver

Each receiver employs the diversity combining scheme to recover erroneous packets. Let $r_{k,i}$ be the despreader output of the k-th bit in the i-th transmission of a packet, and $b_{k,i}$ be the decision variable of the k-th bit after combining i transmitted copies, respectively. We use the update rule of the decision variable in the form (see Fig. 1) of

$$b_{k,i} = b_{k,i-1} + \sqrt{P_i}r_{k,i} \quad \text{for } i \geq 1, \qquad (3)$$

where P_i is the power of the i-th transmission and $b_{k,0} = 0$.

After the combining procedure, the receiver decodes the combined packet and checks if it is correctly received through *cyclic redundancy check* (CRC). A feedback channel is used to report the status of the received packet to the transmitter. If the combined packet is incorrect, the transmitter retransmits the same packet immediately, without any backoff procedure. If the packet is received successfully, then it transmits a new packet in the buffer. Since the objective of the paper is in examining the performance of packet combining techniques, we assume a perfect feedback channel and perfect error detection.

The throughput T is defined as the average number of successfully transmitted packets per packet time. Therefore, the effective throughput which accounts for the redundancy introduced by FEC is given by[1]

$$T_{eff} = \frac{rT}{N}. \qquad (4)$$

C. DS/SSMA Channel Model

With DS/SSMA communications, each user has its own distinct PN sequence. This enables many DS/SS signals to occupy the same bandwidth. At the receiver, the signals from other simultaneous channel users appear as an additive interference. The level of interference varies depending on the number of users transmitting simultaneously. In this paper, we consider an asynchronous system with independent errors at the output of the despreader and use a Gaussian approximation for the interference [11].[2] We generalize the result in [11] to accommodate the cases of varying retransmission power. If there are k_i users whose transmission power is P_i, the *aggregate* transmission power, m, is given by

$$m = \sum_{i=1}^{\infty} P_i k_i. \qquad (5)$$

---

[1] The effective throughput is calculated based on Gilbert-Varshmov lower bound. So (4) gives a lower bound on the effective throughput.

[2] As shown in [12], a Gaussian approximation is valid when the number of simultaneous users is large.

In order to better concentrate on the effects of the interfering users, we assume that MAI is the only source of errors (i.e., the effect of thermal noise is assumed negligible). Since the signal energy of $r_{k,i}$ in (3) is NP_i, the distribution of the despreader output follows the normal distribution [11]

$$r_{k,i} \sim N\Big(\sqrt{NP_i}, \frac{(m-P_i)}{3}\Big), \tag{6}$$

and the *signal-to-noise ratio* (SNR) is given by

$$\mathrm{SNR}(r_{k,i},m) = \frac{[E\{r_{k,i}\}]^2}{2\mathrm{var}(r_{k,i})} = \frac{1.5NP_i}{m-P_i}. \tag{7}$$

III. RETRANSMISSION POWER ADJUSTMENT

In the conventional combining techniques [1]–[7], an erroneous packet is retransmitted at the same power as was used in the initial transmission. However, in reality, a small amount of additional information may be good enough to recover the erroneous packet as it contains much useful information. In this sense, the conventional combining schemes may be said to waste the system capacity, inducing unnecessary interferences. Numerical results in [1] and [5] well demonstrate this shortcoming. According to these results, the system throughput decreases or the slope of the throughput curve reduces to the point where the throughput of the system without combining scheme is maximized.[3] This may be interpreted as follows: If the offered load exceeds the point where the throughput of the system without combining scheme is maximized, the level of interference becomes too high for the initial transmission to succeed, and the number of retransmissions increases. These retransmissions cause the same interference as the initial transmission did, so the overall MAI increases rapidly around this point. However, in the packet combining system, the receiver already contains much useful information taken from the initial transmission even though it is corrupted by interferences, so the amount of information that should be delivered in retransmitted packets is much less than that of the initial transmission. This means that the retransmission power is set too high, so the excessive portion of the transmission power causes unnecessarily high interferences to other packets. In this way, the conventional combining techniques result in an inefficient operation, especially, from the time when the number of retransmissions begins to increase.

In order to overcome the above problem, we propose a *retransmission power adjustment* (RPA) scheme in which the retransmission power is adjusted to an adequate level. We adopt the transmission power to $P_1, P_2, \cdots$, transmitting the i-th copy of the packet with power P_i. If we can determine the adequate level of retransmission power, the system performance can be improved by consuming a minimal amount of resources, thus avoiding unnecessary interferences. Knowing that the required amount of information decreases as the retransmission repeats, we set the retransmission power in a decreasing manner, i.e.,

$$P_i \geq P_j \quad \text{for} \quad i \leq j. \tag{8}$$

We can expect a higher throughput with a smaller retransmission power in most cases. This is because the wasted bandwidth in the system will decrease as the retransmission power decreases. In this case, an adequate amount of information can be delivered in the retransmissions. However, if the retransmission power is set too small, there will be some users who suffer from long delay even though the average delay may be lessened. This is because some severely corrupted packets require several retransmissions when the retransmission power is too low. Therefore, a trade-off relation exists between the system throughput and the number of long-delayed users, and this relation will be governed by the level of the retransmission power. This point should be considered in selecting the level of retransmission power. We further investigate the power adjustment issue through simulations in Section V.

IV. EQUILIBRIUM POINT ANALYSIS

We analyze the performance of the DS/SSMA ALOHA system that employs the RPA scheme. A multi-dimensional Markov chain may be used to model this system. However, this approach is too complex to allow any form of exact analysis [2], [6]. Moreover, the dynamics becomes more complicated in the case of the RPA scheme as it uses different level of transmission power at each retransmission. Therefore, we rather employ the *equilibrium point analysis* (EPA) technique [6], [9] that enables an approximated but simple computation of the system performance. Due to the trade-off relation discussed above, the system should be designed such that it maximizes the overall system performance while satisfying the constraints on the number of long-delayed users. The proposed EPA enables this approach by producing estimated system performances.

In the EPA, it is assumed that the system is at an equilibrium, i.e., the expected number of users leaving a given state is exactly equal to the expected number of users entering the same state. We assume that the system operates about an equilibrium point, and approximate the time-averaged system performance to the performance at the equilibrium point. The number of equilibrium points may be more than one depending on the system parameters, which can disrupt the accuracy of the EPA [9]. Therefore, we consider the number of equilibrium points and the stability of the system in our analysis.

In the analysis, we assume the buffer size $B = 1$, and take the end of each bit time as the time reference.

A. Equilibrium Equations

A user is said to be in state E_i, for $i = 0, 1, \cdots$, when the user tries the i-th transmission of a packet. State E_0 means that a user in this state has no packet to send. We denote the number of users in state E_i by k_i and, define an associated vector $\mathbf{k} \equiv [k_0, k_1, \cdots]$.

Let $\mathbf{K} \equiv [K_0, K_1, \cdots]$ be the value of $\mathbf{k} = [k_0, k_1, \cdots]$ at an equilibrium point. Note that each K_i takes a non-negative real number and $K_0 + K_1 + \cdots = U$. We denote the probability at the equilibrium that the packet is decoded successfully after combining the i transmitted copies by $p_{s,i}$, and define an associated vector $\mathbf{p_s} \equiv [p_{s,1}, p_{s,2}, \cdots]$.

---

[3] For example, see Figs. 6 and 7 in [1].

Now, we relate $\mathbf{K}$ and $\mathbf{p_s}$ by a pair of equations;

$$\mathbf{p_s} = \phi(\mathbf{K}), \tag{9a}$$

$$\mathbf{K} = \psi(\mathbf{p_s}), \tag{9b}$$

which are elaborated in the next two subsections. Then, the two equations form the equilibrium equations, and their solutions, $(\mathbf{K}, \mathbf{p_s})$, become the equilibrium points.

We can solve the equilibrium equations in an iterative manner as follows:

1) Set the initial value $\mathbf{K}^{(0)}$.
2) Calculate $\mathbf{p_s}^{(n)}$ by inserting $\mathbf{K}^{(n)}$ to (9a).
3) Calculate $\tilde{\mathbf{K}}^{(n+1)}$ by inserting $\mathbf{p_s}^{(n)}$ to (9b).
4) Update $\mathbf{K}^{(n+1)}$ such that $\mathbf{K}^{(n+1)} = \mathbf{K}^{(n)} + a(\tilde{\mathbf{K}}^{(n+1)} - \mathbf{K}^{(n)})$ for an a in (0,1).
5) Go back to step 2) and repeat the process until it converges.

B. Derivation of Packet Success Probability $\phi()$

We first consider the packet success probability at each state when the number of users at each state is given. At equilibrium, the MAI from which a transmitted packet suffers is at the fixed level[4]

$$M = \sum_{i=1}^{\infty} P_i K_i. \tag{10}$$

We denote the event that the packet is in error after combining the i transmitted copies by A_i and the number of erroneous bits in the packet after combining the i transmitted copies by $B_{e,i}$, respectively. Then, the successful decoding probability $p_{s,i}$ is given by,

$$\begin{aligned} p_{s,i} &= \Pr(B_{e,i} \le t) = \sum_{k=0}^{t} \binom{L}{k} (p_{be,i})^k (1 - p_{be,i})^{L-k} \\ &\approx 1 - \frac{1}{2}\mathrm{erfc}\left(\frac{t - m_i}{\sqrt{2}\sigma_i}\right), \end{aligned} \tag{11}$$

where $p_{be,i}$ is the bit error probability at state E_i, $m_i \equiv Lp_{be,i}$, $\sigma_i^2 \equiv Lp_{be,i}(1 - p_{be,i})$, and $\mathrm{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} du$. Note that the last term in (11) is obtained by approximating a binomial distribution to a Gaussian distribution.

According to the derivation given in Appendix A, $p_{be,i}$ is given by

$$\begin{aligned} p_{be,1} &= \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{1.5NP_1}{M}}\right), \\ p_{be,i} &\approx \frac{1}{2Lp_{be,i-1}} \mathrm{erfc}\left(\sqrt{\frac{1.5N\sum_{j=1}^{i} P_j}{M}}\right) \\ &\quad \times \mathrm{E}[B_{e,i-1} | A_{i-1}], \text{ for } i \ge 2, \end{aligned} \tag{12}$$

where E[·] stands for the expectation. The conditional expectation in this equation can be calculated numerically as follows:

$$\begin{aligned} \Pr(B_{e,i-1} = k | A_{i-1}) &= \frac{\Pr(B_{e,i-1} = k)}{\sum_{l=t+1}^{L} \Pr(B_{e,i-1} = l)}, \\ \Pr(B_{e,i-1} = k) &\approx \frac{1}{2}\mathrm{erfc}\left(\frac{k - m_{i-1}}{\sqrt{2}\sigma_{i-1}}\right) \\ &\quad - \frac{1}{2}\mathrm{erfc}\left(\frac{k + 1 - m_{i-1}}{\sqrt{2}\sigma_{i-1}}\right), \\ \mathrm{E}[B_{e,i-1} | A_{i-1}] &= \sum_{k=t+1}^{L} k \cdot \Pr(B_{e,i-1} = k | A_{i-1}). \end{aligned} \tag{13}$$

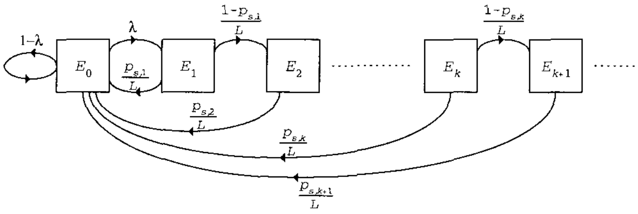


Fig. 2. State transitions for each user at equilibrium.

C. Derivation of Number of Users in Each State $\psi()$

Next, we consider the number of users in each state at equilibrium when packet success probability at each state is given. Let K_i' be the number of users in state E_i at the next bit time when the system is at equilibrium. At equilibrium, λK_0 users leave state E_0 by beginning the first transmission, and K_i/L users leave state E_i by finishing the i-th transmission for $i \ge 1$ at every bit time. After completing the packet transmission successfully, they go idle with a probability $p_{s,i}$ or transit to state E_{i+1} with a probability $(1 - p_{s,i})$. Fig. 2 shows the resulting state transition diagram for each user.

By considering the expected number of transition to each state, we can find the relation between K_i, and K_i' as follows:

$$K_0' = K_0 - \lambda K_0 + \frac{1}{L} \sum_{j=1}^{\infty} K_i p_{s,i}, \tag{10a}$$

$$K_1' = K_1 - \frac{1}{L} K_1 + \lambda K_0, \tag{10b}$$

$$K_i' = K_i - \frac{1}{L} K_i + \frac{1}{L} K_{i-1}(1 - p_{s,i-1}), \; i \ge 2. \tag{10c}$$

Since K_i' becomes K_i at the equilibrium, we can easily derive

$$K_0 = \frac{1}{\lambda L} \sum_{i=1}^{\infty} K_i p_{s,i},$$

$$K_1 = \lambda L K_0, \tag{11a}$$

$$K_i = \lambda L K_0 \prod_{j=1}^{i-1} (1 - p_{s,j}), \; i \ge 2. \tag{11b}$$

Now, since $K_0 + K_1 + \cdots = U$, we obtain

$$K_0 = \frac{U}{1 + \lambda L(1 + \sum_{j=2}^{\infty} \prod_{k=1}^{j-1} (1 - p_{s,k}))}, \tag{12}$$

---

[4] In effect, $M = \sum_{i=1}^{\infty} P_i K_i - P_j$ for the user trying the j-th transmission. However, we may use the simplified form in (10) instead of this, since $\sum_{i=1}^{\infty} P_i K_i \gg P_j$ in general.

and returning this back to (11a) and (11b), we can determine all K_i's for $i \geq 1$.

D. Performance Parameters

Once the equilibrium point of the system is determined by the method in the previous subsections, we can calculate the system throughput and the average delay at equilibrium.

The system throughput T is equal to the expected inflow to state E_0 for a packet time since a user will go idle when a transmission of the user succeeds. From Fig. 2, we find that

$$T = \sum_{i=1}^{\infty} K_i p_{s,i} = \lambda L K_0. \tag{13}$$

The second equality is obtained by applying the equilibrium equation (11a), and the resulting rightmost term means the average number of new packets generated in a packet time at the equilibrium.

The average delay D is defined by the average number of transmissions required to transmit a data packet successfully. According to this definition, we can calculate the average delay as follows;

$$\begin{aligned} D &= p_{s,1} + 2p_{s,2}(1-p_{s,1}) + 3p_{s,3}(1-p_{s,1})(1-p_{s,2})\cdots \\ &= p_{s,1} + \sum_{i=2}^{\infty} i \cdot p_{s,i} \prod_{j=1}^{i-1}(1-p_{s,j}). \end{aligned} \tag{14}$$

A DS/SSMA ALOHA system can have multiple equilibrium points [6]. In such cases, the system may visit each equilibrium point, and the time-averaged performance takes the form of a weighted average of the performance at each equilibrium point. However, in the EPA, it is assumed that the system is always at a single equilibrium point, so the results obtained via the EPA may be not accurate in the case of multiple equilibrium points. In fact, the multiplicity of equilibrium points are represented as multiple solutions of (9a) and (9b). Therefore, we can check whether or not the system to analyze has a single equilibrium point by applying the EPA technique. In general, the system becomes stable as the level of retransmission power decreases, since the feedback gain of the system reduces accordingly.[5] We investigate the stability issue with some numerical examples in the next section.

V. NUMERICAL RESULTS

We consider some examples to evaluate the performance of the RPA based system through simulations. We also extend the simulations to check the accuracy of the EPA. We assume that each packet consists of $L = 1{,}000$ bits, and FEC is capable of correcting up to $t = 50$ bit errors. Therefore, the code rate becomes $r = 0.5278$ from (1). We set the spreading factor N to 31, the buffer size B to 1, and the total number of users U to 100. All the users are assumed idle at the beginning, and each simulation is conducted for at least 5,000 packet times. For the EPA, we set the iteration step size a to 0.2. In order to examine the bistability of the system, we set the initial value $\mathbf{K}^{(0)}$ to $[U-1, 1, 0, \cdots]$ for the initially idle case and to $[0, U, 0, \cdots]$ for the initially busy case.

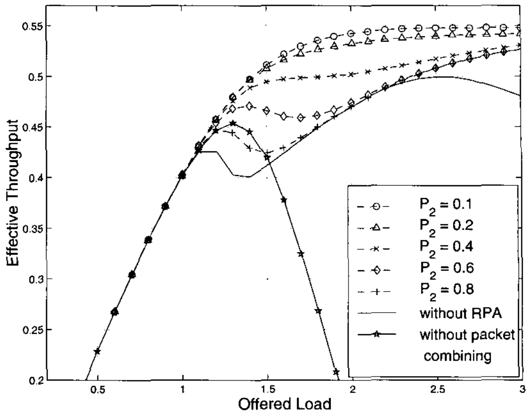


Fig. 3. Throughput performance of DS/SSMA ALOHA with RPA for various transmission powers via simulation.

A. Performance of the RPA Scheme

Fig. 3 shows the system throughput obtained through simulations for various transmission powers. Here, we vary only P_2 with other transmission powers fixed at $P_1 = 1, P_3 = P_4 = \cdots = 0.1$. We observe that the throughput of the system without packet combining begins its decrease to zero as the offered load exceeds 1.2. Contrary to this, the throughput of the systems with packet combining is maintained at some level when the offered load is heavy. In the case of the system with conventional packet combining there appears a dent around the offered load of 1.2, where the throughput of the system without packet combining is maximized. This dent is consistent with the shortcoming of conventional combining schemes discussed in Section III. The dent in the throughput curve disappears as the value of P_2 decreases in the case of the systems with RPA. We find that the throughput of the system with RPA scheme exhibits more improvements at heavier offered load. We observe about 10% enhancement in the maximum available throughput for the case of $P_2 = 0.1$. In addition, we see the system throughput increases as the level of P_2 decreases, as was expected in Section III. This tendency is consistent in a wide range of offered loads.

Fig. 4 shows the average delay obtained through simulations for various different transmission powers. We observe that the average delay of the system with RPA is smaller than that of the system with conventional packet combining over all range of the offered load. In the curve for the conventional packet combining, we observe that the average delay rapidly increases to 2 at the point of offered load that exhibited a throughput dent in Fig. 3. This supports the explanation in Section III. Moreover, we see the average delay decreases as P_2 decreases.

Fig. 5 shows the average transmission power required to transmit a data packet successfully. We observe that the average required power reduces significantly as the level of P_2 decreases. This is because both the level of the transmission power and the average number of transmissions are reduced by employing the RPA scheme. From the above results, we can confirm that the

---

[5] This is consistent with the results in [6] where the system becomes stable as the retransmission probability decreases.

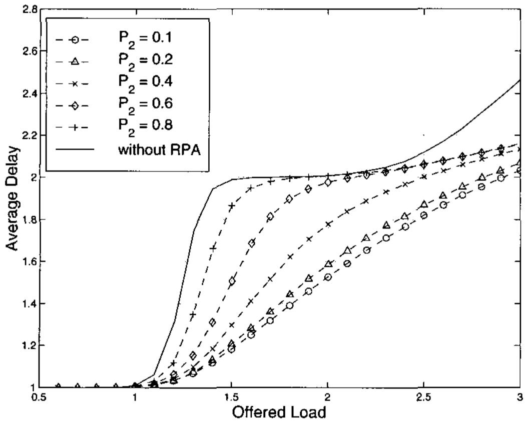


Fig. 4. Average delay for DS/SSMA ALOHA with RPA for various transmission powers via simulation.

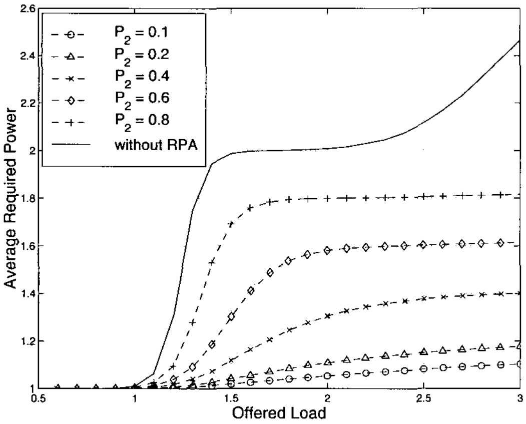


Fig. 5. Average transmission power required to transmit a data packet successfully via simulation.

RPA scheme is advantageous both in system throughput and in power-saving effect.

Fig. 6 shows the ratio of long-delayed (by more than four packet times) packets out of all the successful transmissions. In this figure, we observe that the ratio is high for the case of small P_2, as was expected in Section III. Therefore we can confirm a trade-off relation between the system throughput and the number of long-delayed users.

B. Accuracy of EPA and System Stability

Fig. 7 compares the system throughput obtained through simulation and that obtained through EPA for various transmission powers. From Fig. 7(a), we observe that the results obtained through EPA with the two initial conditions—initially idle and initially busy—are the same. This means that the system has a unique equilibrium point for the cases of small retransmission power as discussed in Section IV-D. In addition, the results obtained through EPA are very close to the simulation results in the case of RPA system with small P_2. However, from Fig. 7(b), we observe that the EPA result deviates from the simulated result in the case of a larger P_2 and without RPA: First, the EPA results with the two initial conditions exhibit a large deviation when the system does not employ RPA and the offered load is smaller than 1.5. This happens because the system has two different stable equilibrium points. In this case, the simulation result lies between the two EPA curves, as can be observed in the figure. Second, there are some cases when the EPA result does not coincide with the simulation result even if the system has one unique equilibrium point. This, in fact, happens due to the asymmetry of the "potential well" around the equilibrium point [9] as will be discussed in the following.

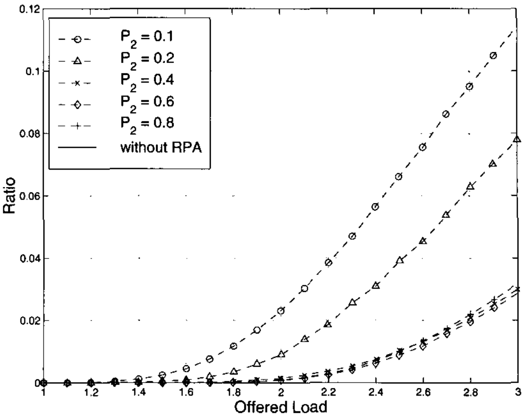


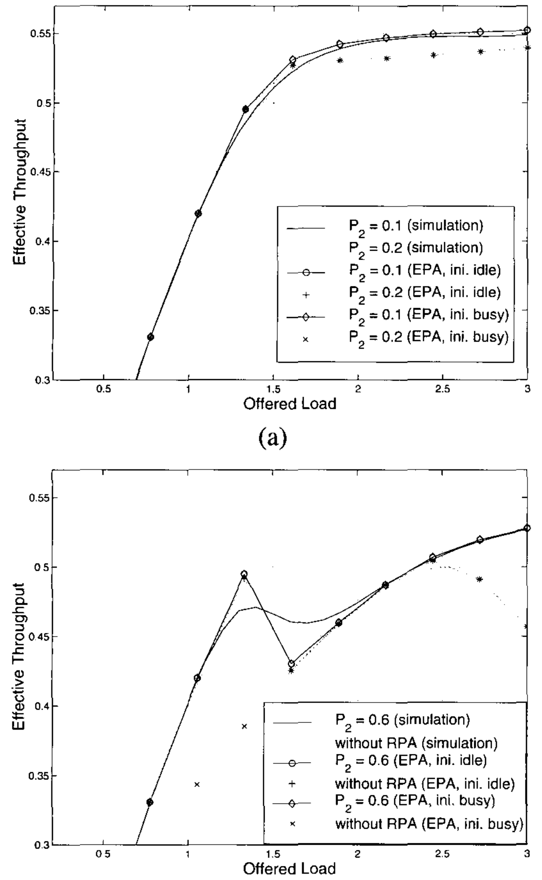
Fig. 6. Ratio of the long-delayed packets (more than four packet times) out of all successful transmissions via simulation.

Fig. 8 shows the probability distribution of the number of idle users obtained from simulations for (a) bistable case (without RPA and $G = 1.2$), (b) asymmetric case ($P_2 = 0.6$ and $G = 1.6$), and (c) symmetric case ($P_2 = 0.2$ and $G = 2.2$), and Fig. 9 shows the corresponding EPA iteration convergence. Note that Fig. 8 shows the tendency staying at each state, so this can be interpreted as an illustration of the potential well.[6] First, in the bistable case, we find that there are two modal points around 60 and 70, which correspond to the two equilibrium points. We observe that the two points coincide with the two converged points of K_0 in Fig. 9(a). This demonstrates that the two different start points of iteration lead to two different points of convergence due to the bistability of the system. In this case, the time-averaged throughput obtained through simulation lies between the two values obtained via EPA for those two points. Second, in the asymmetric case, we observe that the distribution is heavy-tailed toward the idle side and this asymmetry is also observable in the EPA convergence graph in Fig. 9(b), which indicates that the convergence speed is much slower for the initially idle case. This happens because the system leans towards the idle side so is more probable to move toward the idle side. In this case, the throughput obtained via EPA is lower than the simulation result because EPA underestimates the average number of idle users.[7] Third, in the symmetric case, we observe that the distribution is symmetric around the maximum point and the convergence speed is similar for the two different start points. Therefore, from this simulation, we learn that the bistability or asymmetry of the system can be determined by checking the

---

[6] We can interpret a high probability of a state as a low system potential.

[7] Note that the throughput is proportional to the number of idle users K_0 in (13).



(a)

(b)

Fig. 7. Throughput performance for DS/SSMA ALOHA for via both analysis and simulation; (a) small retransmission power, (b) large retransmission power.

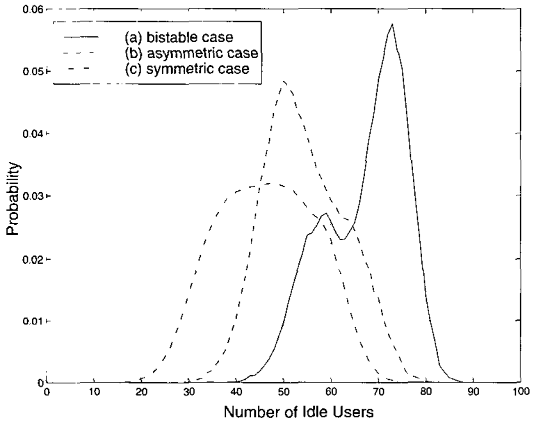
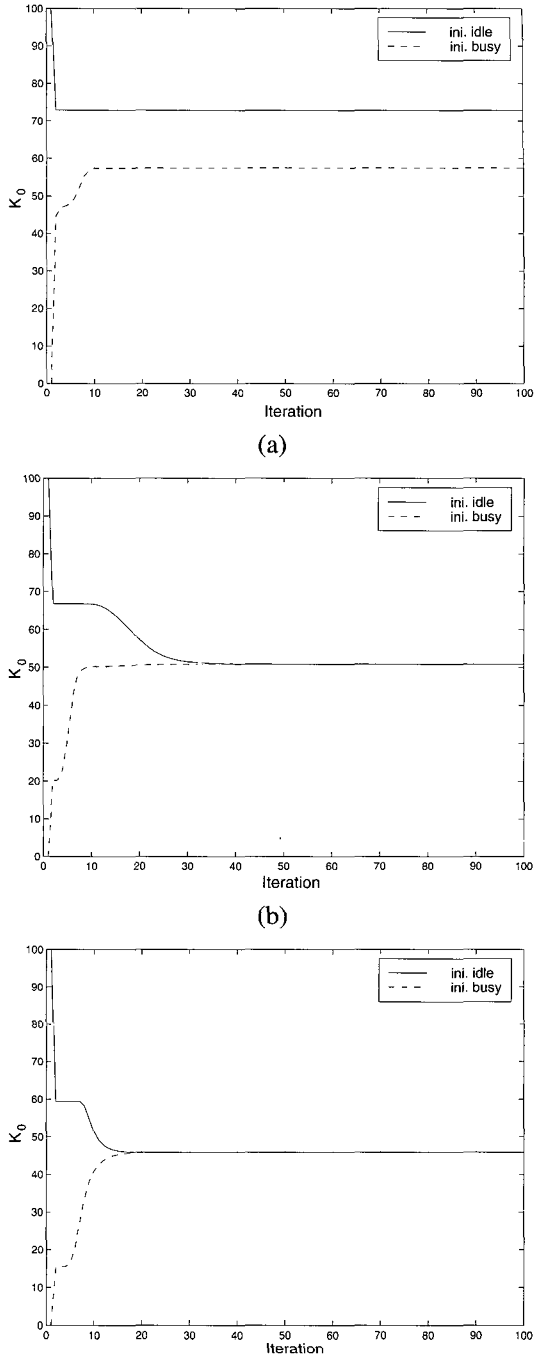


Fig. 8. Probability distribution of the number of idle users for (a) bistable case (without RPA and G=1.2), (b) asymmetric case ($P_2 = 0.6$ and $G = 1.6$), and (c) symmetric case ($P_2 = 0.2$ and $G = 2.2$).

convergence speed of the iteration of EPA and that the system gets stable and symmetric as the offered load increases or the level of retransmission power decreases.



(a)

(b)

(c)

Fig. 9. Convergence of the iteration of EPA for (a) bistable case (without RPA and $G = 1.2$), (b) asymmetric case ($P_2 = 0.6$ and $G = 1.6$), and (c) symmetric case ($P_2 = 0.2$ and $G = 2.2$).

VI. CONCLUDING REMARKS

In this paper, we have introduced the RPA scheme as a means for performance enhancement in the DS/SSMA packet radio system with packet combining. In the RPA scheme, we adjust the retransmission power to some fixed values in such a way that the erroneous packet can be recovered with a minimized interference to other user packets. Simulation results demonstrated that the RPA scheme with a small retransmission power can bring forth performance gain in throughput and average delay. Aside from such performance gain, the RPA scheme has the advantage that it saves a significant amount of transmission power, and does not require additional overhead for information conveyance or accurate load estimation. For a maximized performance of the RPA scheme, however, we should carefully consider the trade-off relation that exists between the system throughput and the number of long-delayed users.

We employed the EPA technique as a simple means to analyze the system performance. We have confirmed that the results obtained via the EPA are very close to the simulation results in the low retransmission power cases, but exhibit deviation in the high power cases due to the bistability and asymmetry of the system. We investigated the effects of the system stability on the EPA results, and learned that system stability is indicated by the convergence speed of iteration of EPA. From the numerical results, we confirmed that the system becomes stable as the offered load increases or the level of retransmission power decreases.

APPENDIX

A. *Derivation of Error Probability in* (12)

For convenience, we use a and b to denote the variables $\sqrt{P_i} r_{k,i}$ and $b_{k,i-1}$ in (3), respectively. We also use $\hat{A}_i$ to denote the intersection of the events $A_1, A_2, \cdots, A_i$. Note that a is a Gaussian random variable but b is not if there occured any error in the previous transmissions.

We first consider the probability of the event A_i assuming that it is independent of the previous events A_1, A_2, $\cdots$, A_{i-1}. In this case $a+b$ becomes a Gaussian random variable with the mean $\sum_{j=1}^{i} P_j\sqrt{N}$ and the variance $\sum_{j=1}^{i} P_j M/3$, because b also becomes a Gaussian random variable and is independent of a. If we assume that all-1 data are transmitted, without loss of generality, then we get

$$p_{be,i} = \Pr(a+b<0|\hat{A}_{i-1}) = \Pr(a+b<0) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{1.5N\sum_{j=1}^{i}P_j}{M}}\right). \quad (15)$$

In reality, however, the event A_i depends on the previous event A_{i-1}, with A_{i-1} depending on A_{i-2}, and so forth, so (15) turns into an approximated relation, i.e., $\Pr(a+b<0|\hat{A}_{i-1}) \approx \Pr(a+b<0)$.

We now consider the case when A_i is dependent on A_{i-1}. Then

$$\begin{aligned} p_{be,i} &= \Pr(a+b<0|\hat{A}_{i-1}) \\ &= \sum_{k=t+1}^{L} \Pr(a+b<0|\hat{A}_{i-1}, B_{e,i-1}=k)\Pr(B_{e,i-1}=k|\hat{A}_{i-1}) \\ &= \sum_{k=t+1}^{L} \Pr(B_{e,i-1}=k|\hat{A}_{i-1}) \\ &\quad \times \left\{ \frac{k}{L}\Pr(a+b<0|b<0,\hat{A}_{i-1},B_{e,i-1}=k) \right. \\ &\quad \left. + \frac{L-k}{L}\Pr(a+b<0|b>0,\hat{A}_{i-1},B_{e,i-1}=k) \right\}. \end{aligned} \quad (16)$$

The first term in the right-hand side, $\Pr(a+b<0|b<0,\hat{A}_{i-1},B_{e,i-1}=k)$, is independent of $B_{e,i-1}$ and A_{i-1} due to the condition $b<0$. So this conditional probability reduces to $\Pr(a+b<0|b<0,\hat{A}_{i-2})$. The second term, which refer to the probability of getting an incorrect bit after adding the retransmitted information to the previous correct bit, becomes negligibly small if the retransmission power P_i for $i \geq 2$ is very small compared with P_1. So (16) may be simplified to

$$\begin{aligned} p_{be,i} &\approx \frac{1}{L}\Pr(a+b<0|b<0,\hat{A}_{i-2}) \\ &\quad \times \mathrm{E}[B_{e,i-1}|\hat{A}_{i-1}]. \end{aligned} \quad (17)$$

We consider the conditional probability $\Pr(a+b<0|\hat{A}_{i-2})$ and expand it in terms of b (i.e., $b>0$ and $b<0$). Then we apply the same reasoning as used above to simplify the second term to

$$\begin{aligned} \Pr(a+b<0|\hat{A}_{i-2}) &\approx \Pr(a+b<0|b<0,\hat{A}_{i-2}) \\ &\quad \times \Pr(b<0|\hat{A}_{i-2}), \end{aligned} \quad (18)$$

which yields the relation

$$\begin{aligned} \Pr(a+b<0|b<0,\hat{A}_{i-2}) &\approx \frac{\Pr(a+b<0|\hat{A}_{i-2})}{\Pr(b<0|\hat{A}_{i-2})} \\ &\approx \frac{1}{2p_{be,i-1}}\operatorname{erfc}\left(\sqrt{\frac{1.5N\sum_{j=1}^{i}P_j}{M}}\right). \end{aligned} \quad (19)$$

The last approximation above utilized the approximation $\Pr(a+b<0|\hat{A}_{i-2}) \approx \Pr(a+b<0)$, similar to the case of (15).[8]

The event $\hat{A}_{i-1}$ is equivalent to the event A_{i-1} in effect, since A_{i-1} can occur on the condition that A_{i-2} has occurred. Thus we may substitute $\mathrm{E}[B_{e,i-1}|\hat{A}_{i-1}]$ in (17) by $\mathrm{E}[B_{e,i-1}|A_{i-1}]$. Inserting (19) to (17), we finally get the $p_{be,i}$ in (12).[9]

REFERENCES

[1] A. M. Y. Bigloo, T. A. Gulliver, and V. K. Bhargava, "Maximum-likelihood decoding and code combining for DS/SSMA slotted ALOHA," *IEEE Trans. Commun.*, vol. 45, pp. 1602–1612, Dec. 1997.

[2] Q. Zhang, T. F. Wong, and J. S. Lehnert, "Performance of a type-II hybrid ARQ protocol in slotted DS-SSMA packet radio systems," *IEEE Trans. Commun.*, vol. 47, pp. 281–290, Feb. 1999.

[3] D. Chase, "Code combining - a maximum-likelihood decoding approach for combining an arbitrary number of noisy packets," *IEEE Trans. Commun.*, vol. COM-33, no. 5, pp. 385–393, May 1985.

[4] S. Souissi and S. B. Wicker, "A diversity combining DS/CDMA system with convolutional encoding and Viterbi decoding," *IEEE Trans. Veh. Technol.*, vol. 44, pp. 304–312, May 1990.

[5] A. Annamalai, R. S. C. Wong, and V. K. Bhargava, "Throughput enhancement of a slotted DS-CDMA ALOHA with packet combining," in *Proc. IEEE Pacific Rim Conf. Communications, Computers and Signal processing*, Aug. 1997, pp. 506–510.

[6] Q. Zhang, T. F. Wong, and J. S. Lehnert, "Stability of a type-II hybrid ARQ protocol for slotted DS-SSMA packet radio systems," in *Proc. IEEE INFOCOM'98*, San Fransisco, CA, Apr. 1998, vol 3, pp. 1301–1308.

[7] D. Tuninetti and G. Caire, "The throughput of some wireless multiaccess systems," *IEEE Trans. Inform. Theory*, vol. 48, pp. 2773–2785, Oct. 2002.

---

[8] Notice that this approximation is more accurate than the case of (15) since it takes the event A_{i-1} into consideration, ignoring the events $A_1, A_2, \cdots, A_{i-2}$, whereas (15) ignored all the events $A_1, A_2, \cdots, A_{i-1}$.

[9] Note that the approximated $p_{be,i}$ in (12) is a scale-down of (15). In the equation, $Lp_{be,i-1} = \mathrm{E}[B_{e,i-1}] \leq \mathrm{E}[B_{e,i-1}|A_{i-1}]$. Thus (12) is a product of the approximation in (15) and a constant which is larger than one.

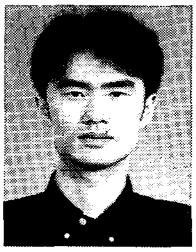
[8] X. Cai, Y. Sun, and A. N. Akansu, "Performance of CDMA random access systems with packet combining in fading channels," *IEEE Trans. Commun.*, vol. 2, pp. 413–419, May 2003.

[9] S. Tasaka, "Stability and performance of the R-ALOHA packet broadcast system," *IEEE Trans. Comput.*, vol. 32, no. 8, pp. 717–726, Aug. 1983.

[10] F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes*, New York: North-Holland Publishing Company, 1977.

[11] M. B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communications - Part I: System analysis," *IEEE Trans. Commun.*, vol. COM-25, pp. 795–799, Aug. 1977.

[12] R. K. Morrow, Jr., and J. S. Lehnert, "Bit-to-bit error dependence in slotted DS/SSMA packet systems with random signature sequences," *IEEE Trans. Commun.*, vol.37, pp. 1052–1061, Oct. 1989.



Hanbyul Seo received the B.S. and M.E. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 2001 and 2003, respectively. He is currently working toward the Ph.D. degree in electrical engineering and computer science at Seoul National University. His current research interests include radio resource allocation, wireless MAC protocol, and wireless sensor networks.

Seongyong Park was born in Taegu, Korea. He received the B.S. and M.E. degrees from Seoul National University, in 1990 and 1992, respectively. From 1992–1998, he was with the Samsung Electronics Corporation, Korea, where he was involved in ATM switching system development. He is currently a Principal Engineer in the Network System Division of the Samsung Electronics Cooperation and is also in Ph.D. course from Seoul National University. His research interests include systems design for wideband wireless access and CDMA access networks, as well as high speed network.

Byeong Gi Lee received the B.S. and M.E. degrees from Seoul National University, Seoul, Korea, and Kyungpook National University, Daegu, Korea, both in electronics engineering, and the Ph.D. degree in electrical engineering from the University of California, Los Angeles. He was with the Electronics Engineering Department of ROK Naval Academy as an Instructor and Naval Officer in active service from 1974 to 1979, and worked for Granger Associates, Santa Clara, CA, as a Senior Engineer responsible for applications of digital signal processing to digital transmission from 1982 to 1984, and for AT&T Bell Laboratories, North Andover, MA, as a Member of Technical Staff responsible for optical transmission system development along with related the standards works from 1984 to 1986. He joined the faculty of Seoul National University in 1986 and served as the Vice Chancellor for Research Affairs from 2000 to 2002.

Dr. Lee was the founding chair of the Joint Conference of Communications and Information (JCCI), the chair of the Steering Committee of the Asia Pacific Conference on Communications (APCC), and the chair of the founding committee of the Accreditation Board for Engineering Education of Korea (ABEEK). He was the editor of the *IEEE Global Communications Newsletter* and an associate editor of the *IEEE Transactions on Circuits and Systems for Video Technology*, and was the Editor-in-Chief of the *Journal of Communications and Networks* (JCN). He served as the Director of Membership Programs Development, the Director of Asia Pacific Region, and a Member-at-Large of IEEE Communications Society and is currently the Director of Magazines of the IEEE Communications Society. He is a Member of the National Academy of Engineering of Korea, a Member of the Board of Governors of IEEE Communications Society, and a Member of Sigma Xi. He is the President of Korea Society of Engineering Education and Technology Transfer (KSEETT) and a Vice President of Korea Institute of Communication Sciences (KICS).

Dr. Lee is a co-author of *Broadband Telecommunication Technology*, 1st & 2nd editions, (Artech House, 1996), *Scrambling Techniques for Digital Transmission* (New York: Springer Verlag, 1994), *Scrambling Techniques for CDMA Communications* (Norwell, MA: Kluwer Publisher, 2001), and *Integrated Broadband Networks* (Norwood, MA: Artech House, April 2002). He holds seven U.S. patents with four more patents pending. His current fields of interest include broadband networks, wireless networks, communication systems, and signal processing. He received the 1984 Myril B. Reed Best Paper Award from the Midwest Symposium on Circuits and Systems, Exceptional Contribution Awards from AT&T Bell Laboratories, a Distinguished Achievement Award from Korea Institute of Communication Sciences (KICS), and the 2001 National Academy of Science (of Korea) Award.