# Speed and Flux Estimation for an Induction Motor Using a Parameter Estimation Technique

Gil-Su Lee, Dong-Hyun Lee, Tae-Woong Yoon\*, Kyo-Beum Lee, Joong-Ho Song, and Ick Choy

Abstract: In this paper, an estimator scheme for the rotor speed and flux of an induction motor is proposed on the basis of a fourth-order electrical model. It is assumed that only the stator currents and voltages are measurable, and that the stator currents are bounded. There are a number of common terms in the motor dynamics, and this is utilized to find a simple error model involving some auxiliary variables. Using this error model, the state estimation problem is converted into a parameter estimation problem assuming that the rotor speed is constant. Some stability properties are given on the basis of Lyapunov analysis. In addition, the rotor resistance, which varies with the motor temperature, can also be estimated within the same framework. The effectiveness of the proposed scheme is demonstrated through computer simulations and experiments.

**Keywords:** Induction motor, parameter estimation, sensorless drive.

#### 1. INTRODUCTION

Speed controlled induction motor drives are widespread electromechanical systems suitable for a large spectrum of industrial applications. When high dynamic performance and precision control are required for an induction motor in a wide speed range, the speed must be measured. In contrast, in the case of medium and low performance applications, sensorless control without measuring the motor speed is

Manuscript received September 1, 2004; revised January 9, 2005; accepted January 25, 2005. Recommended by Editorial Board member Jae Weon Choi under the direction of Editor Keum-Shik Hong. This work has been supported by KESRI (01-JI-03), which is funded by MOCIE (Ministry of Commerce, Industry and Energy).

Gil-Su Lee and Tae-Woong Yoon are with the Department of Electrical Engineering, Korea University, Anam-dong 5-ga, Seongbuk-gu, Seoul 136-713, Korea (e-mails: {gslee, twy}@ cello.korea.ac.kr).

Dong-Hyun Lee is with the 3rd SD Center, Agency for Defense Development, P.O.Box 35 Yuseong, Daejeon 305-600, Korea (e-mail: 971192@hanmail.net).

Kyo-Beum Lee is with the Institute of Energy Technology, Aalborg University, Fredrik Bajers Vej 5, P.O.Box 159, DK-9100, Aalborg, Denmark (e-mail: kyl@iet.aau.dk).

Joong-Ho Song is with the Department of Electrical Engineering, Seoul National University of Technology, 172 gongreung 2-dong, Nowon-gu, Seoul 139-743, Korea (e-mail: joongho@snut.ac.kr).

Ick Choy is with the Department of Information Control Engineering, Kwangwoon University, 447-1 Wolgye-Dong, Nowon-Gu, Seoul 139-701, Korea (e-mail: ickchoy@kw.ac. kr).

becoming an industrial standard because of advantages in terms of cost, simplicity and mechanical reliability of the drive. As a consequence of this, a great deal of research has been carried out on sensorless drives over the last few decades [1].

Observer-based methods ([2-6]) seem attractive as they are relatively simple to implement. Among these observer-based methods, for a certain period, the extended Kalman filter appeared to be an ultimate solution for speed-sensorless drives [2]. However, this stochastic observer comes with certain inherent disadvantages, such as sensitivity to characteristics, computation burden, and absence of design and tuning criteria. This has led to a renewed interest in deterministic approaches, where the structure of the standard Luenberger observer for a linear system is enhanced to permit the simultaneous estimation of the rotor flux and speed [3-5]. In the scheme by Kubota et al. [4], the observer gain matrix is related to the unmeasurable rotor speed. This makes the observer inappropriate for practical applications. There are other schemes that employ a time-scale separation property of the induction motor model [6]. schemes, however, the coordinate transformations involved are functions of the rotor speed.

In this paper, the rotor speed and flux are estimated on the basis of a fourth-order electrical model of the induction motor; the resulting scheme is independent of the current value of the rotor speed, which is in contrast with existing methods mentioned in the previous paragraph. The design philosophy of the proposed scheme is similar to that in [8], where the

<sup>\*</sup> Corresponding author.

rotor resistance is estimated. It is assumed that only the stator currents and voltages are measurable, and that the stator currents are assumed to be bounded. The state estimation problem is converted into a parameter estimation problem assuming that the rotor speed is constant. The speed and parameter adaptation laws are designed such that the estimated stator currents converge to the measured ones. Some stability properties are provided on the basis of Lyapunov analysis. In addition, the rotor resistance, which varies with the motor temperature, can also be estimated within the same framework. The effectiveness of the proposed scheme is demonstrated through computer simulations and experiments.

### 2. INDUCTION MOTOR MODEL

An electrical model of the induction motor in the fixed stator reference frame (a-b) is given in [7] as follows:

$$\begin{split} &\dot{i}_{a}=-\beta\left(-\alpha\phi_{a}-n_{p}\omega\phi_{b}+\alpha Mi_{a}\right)-\frac{R_{s}}{\sigma L_{s}}i_{a}+\frac{1}{\sigma L_{s}}u_{a},\\ &\dot{i}_{b}=-\beta\left(-\alpha\phi_{b}+n_{p}\omega\phi_{a}+\alpha Mi_{b}\right)-\frac{R_{s}}{\sigma L_{s}}i_{b}+\frac{1}{\sigma L_{s}}u_{b},\\ &\dot{\phi}_{a}=-\alpha\phi_{a}-n_{p}\omega\phi_{b}+\alpha Mi_{a},\\ &\dot{\phi}_{b}=-\alpha\phi_{b}+n_{p}\omega\phi_{a}+\alpha Mi_{b}, \end{split} \tag{1}$$

where  $i_a$ ,  $i_b$  are the stator current,  $\phi_a$ ,  $\phi_b$  the rotor flux linkages,  $u_a$ ,  $u_b$  the stator voltages,  $\omega$  the rotor speed,  $R_s$   $(R_r)$ ,  $L_s$   $(L_r)$  the stator (rotor) resistance and inductance, M the stator-rotor mutual inductance,  $n_p$  the number of pole pairs,

$$\sigma = 1 - \left(M^2 / L_s L_r\right), \quad \alpha = R_r / L_r$$
, and  $\beta = M / \sigma L_s L_r$ .

On the basis of this model, a new method of estimating the rotor speed  $(\omega)$  and flux  $(\phi_a, \phi_b)$  is proposed. It is assumed that only the stator currents  $(i_a, i_b)$  and voltages  $(u_a, u_b)$  are measurable, and also that the stator currents are bounded.

## 3. SPEED AND FLUX ESTIMATOR DESIGN

In order to obtain a speed and flux estimator, we proceed as in [8], where the rotor resistance is estimated. Using the same structure in (1), the proposed estimator takes the form:

$$\begin{split} \dot{\hat{i}}_{a} &= -\beta \left( -\alpha \hat{\phi}_{a} - n_{p} \hat{\omega} \hat{\phi}_{b} + \alpha M i_{a} \right) - \frac{R_{s}}{\sigma L_{s}} i_{a} + \frac{1}{\sigma L_{s}} u_{a} - v_{a}, \\ \dot{\hat{i}}_{b} &= -\beta \left( -\alpha \hat{\phi}_{b} + n_{p} \hat{\omega} \hat{\phi}_{a} + \alpha M i_{b} \right) - \frac{R_{s}}{\sigma L_{s}} i_{b} + \frac{1}{\sigma L_{s}} u_{b} - v_{b}, \end{split}$$

$$\dot{\hat{\phi}}_{a} = -\alpha \hat{\phi}_{a} - n_{p} \hat{\omega} \hat{\phi}_{b} + \alpha M i_{a} - v_{c}, 
\dot{\hat{\phi}}_{b} = -\alpha \hat{\phi}_{b} + n_{p} \hat{\omega} \hat{\phi}_{a} + +\alpha M i_{b} - v_{d},$$
(2)

where  $v_a$ ,  $v_b$ ,  $v_c$ ,  $v_d$  are compensation terms to be designed, and  $\hat{i}_a$ ,  $\hat{i}_b$ ,  $\hat{\phi}_a$ ,  $\hat{\phi}_b$ ,  $\hat{\omega}$  are the estimates of  $i_a$ ,  $i_b$ ,  $\phi_a$ ,  $\phi_b$ ,  $\omega$ .

Introduce the error variables as follows:

$$\begin{split} \tilde{i}_{a} &= i_{a} - \hat{i}_{a}, & \tilde{\phi}_{a} &= \phi_{a} - \hat{\phi}_{a}, \\ \tilde{i}_{b} &= i_{b} - \hat{i}_{b}, & \tilde{\phi}_{b} &= \phi_{b} - \hat{\phi}_{b}, \\ \tilde{\omega} &= \omega - \hat{\omega}. \end{split} \tag{3}$$

It ensues from (1) and (2) that

$$\begin{split} &\dot{\tilde{t}}_{a} = -\beta \left\{ -\alpha \tilde{\phi}_{a} - n_{p} \left( \omega \tilde{\phi}_{b} + \hat{\phi}_{b} \tilde{\omega} \right) \right\} + v_{a}, \\ &\dot{\tilde{t}}_{b} = -\beta \left\{ -\alpha \tilde{\phi}_{b} + n_{p} \left( \omega \tilde{\phi}_{a} + \hat{\phi}_{a} \tilde{\omega} \right) \right\} + v_{b}, \\ &\dot{\tilde{\phi}}_{a} = -\alpha \tilde{\phi}_{a} - n_{p} \left( \omega \tilde{\phi}_{b} + \hat{\phi}_{b} \tilde{\omega} \right) + v_{c}, \\ &\dot{\tilde{\phi}}_{b} = -\alpha \tilde{\phi}_{b} + n_{p} \left( \omega \tilde{\phi}_{a} + \hat{\phi}_{a} \tilde{\omega} \right) + v_{d}. \end{split} \tag{4}$$

The adaptation law for  $\omega$  and other design variables should be devised such that  $\tilde{i}_a$  and  $\tilde{i}_b$  tend asymptotically to zero. However, it is not a straightforward process to derive such an estimator directly from (4) due to unknown terms like  $\omega \tilde{\phi}_a$ . To cope with this problem, we observe that

$$\dot{\tilde{i}}_a + \beta \dot{\tilde{\phi}}_a = v_a + \beta v_c, 
\dot{\tilde{i}}_b + \beta \dot{\tilde{\phi}}_b = v_b + \beta v_d.$$
(5)

Define the first order filters

$$\dot{\zeta}_1 = v_a + \beta v_c, \qquad \zeta_1(0) = 0, 
\dot{\zeta}_2 = v_b + \beta v_d, \qquad \zeta_2(0) = 0,$$
(6)

with

$$\beta \tilde{\phi}_a = A + \zeta_1 - \tilde{i}_a, \beta \tilde{\phi}_b = B + \zeta_2 - \tilde{i}_b,$$
(7)

where A and B are given by

$$A = \tilde{i}_a(0) + \beta \tilde{\phi}_a(0),$$
  

$$B = \tilde{i}_b(0) + \beta \tilde{\phi}_b(0).$$
(8)

Substituting the expressions  $\beta \tilde{\phi}_a$  and  $\beta \tilde{\phi}_b$  in (7)

into the first two equations in (4), we obtain

$$\begin{split} \ddot{\tilde{i}}_{a} &= -\alpha \tilde{i}_{a} - n_{p} \omega \tilde{i}_{b} + n_{p} \beta \hat{\phi}_{b} \tilde{\omega} + \alpha A \\ &+ n_{p} \omega B + \alpha \zeta_{1} + n_{p} \omega \zeta_{2} + v_{a}, \\ \ddot{\tilde{i}}_{b} &= n_{p} \omega \tilde{i}_{a} - \alpha \tilde{i}_{b} - n_{p} \beta \hat{\phi}_{a} \tilde{\omega} + \alpha B \\ &- n_{p} \omega A - n_{p} \omega \zeta_{1} + \alpha \zeta_{2} + v_{b}. \end{split} \tag{9}$$

In order to partially compensate the right-hand side of (9), we set  $v_a$  and  $v_b$  to

$$v_{a} = -\left(\rho_{a}\tilde{i}_{a} + \alpha\zeta_{1} + n_{p}\hat{\omega}\zeta_{2} + \hat{\xi}_{1}\right),$$

$$v_{b} = -\left(\rho_{b}\tilde{i}_{b} + \alpha\zeta_{2} - n_{p}\hat{\omega}\zeta_{1} + \hat{\xi}_{2}\right),$$
(10)

in which  $\rho_a$  and  $\rho_b$  are positive design parameters,  $\hat{\xi}_1$  and  $\hat{\xi}_2$  are estimates of the terms  $\alpha A + n_p \omega B$  and  $\alpha B - n_p \omega A$  in (9). Substituting (10) into (9) results in

$$\begin{split} & \ddot{\tilde{t}}_{a} = -\left(\rho_{a} + \alpha\right)\tilde{t}_{a} - n_{p}\omega\tilde{t}_{b} + n_{p}\left(\zeta_{2} + \beta\hat{\phi}_{b}\right)\tilde{\omega} + \tilde{\xi}_{1}, \\ & \ddot{\tilde{t}}_{b} = n_{p}\omega\tilde{t}_{a} - \left(\rho_{b} + \alpha\right)\tilde{t}_{b} - n_{p}\left(\zeta_{1} + \beta\hat{\phi}_{a}\right)\tilde{\omega} + \tilde{\xi}_{2}. \end{split} \tag{11}$$

Now consider the following positive definite function

$$V = \frac{1}{2} \left( \tilde{i}_a^2 + \tilde{i}_b^2 + \frac{1}{\lambda_1} \tilde{\omega}^2 + \frac{1}{\lambda_2} \tilde{\xi}_1^2 + \frac{1}{\lambda_3} \tilde{\xi}_2^2 \right), \tag{12}$$

where  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  are positive. The time derivative of this function is obtained as

$$\dot{V} = -(\rho_a + \alpha)\tilde{i}_a^2 - (\rho_b + \alpha)\tilde{i}_b^2 
+ \tilde{\omega} \left[ \frac{1}{\lambda_1} \dot{\tilde{\omega}} + n_p \left\{ \left( \zeta_2 + \beta \hat{\phi}_b \right) \tilde{i}_a - \left( \zeta_1 + \beta \hat{\phi}_a \right) \tilde{i}_b \right\} \right] 
+ \tilde{\xi}_1 \left( \frac{1}{\lambda_2} \dot{\tilde{\xi}}_1 + \tilde{i}_a \right) 
+ \tilde{\xi}_2 \left( \frac{1}{\lambda_3} \dot{\tilde{\xi}}_2 + \tilde{i}_b \right).$$
(13)

For the purpose of speed estimator design, we now assume that the speed is constant, i.e.,  $\dot{\omega}=0$ , which in turn leads to  $\dot{\xi}_1=\dot{\xi}_2=0$ . (Note that this assumption, commonly employed in sensorless control, can be justified because the speed is varying much more slowly than the other electrical variables.) Under this assumption of constant speed, we propose the estimation laws as follows:

$$\dot{\hat{\omega}} = \lambda_1 n_p \left\{ \left( \zeta_2 + \beta \hat{\phi}_b \right) \tilde{i}_a - \left( \zeta_1 + \beta \hat{\phi}_a \right) \tilde{i}_b \right\}, 
\dot{\hat{\xi}}_1 = \lambda_2 \tilde{i}_a, 
\dot{\hat{\xi}}_2 = \lambda_3 \tilde{i}_b.$$
(14)

It then ensues from (13) and (14) that

$$\dot{V} = -(\rho_a + \alpha)\tilde{i}_a^2 - (\rho_b + \alpha)\tilde{i}_b^2. \tag{15}$$

From this, we have the following theorem.

**Theorem 1:** Assume that the speed is constant, and also that the stator currents are bounded. Consider the proposed estimator in (2), (6), (10) and (14), and set  $v_c = k_1 \tilde{i}_a$ ,  $v_d = k_2 \tilde{i}_b$  with  $k_1$  and  $k_2$  being design parameters. Then the following holds true.

i) 
$$\tilde{i}_a, \tilde{i}_b, \tilde{\omega}, \tilde{\xi}_1, \tilde{\xi}_2, \tilde{\phi}_a, \tilde{\phi}_b \in L_{\infty}$$

$$\begin{split} &\text{ii)} \quad \tilde{i}_a \in L_2 \;, \quad \tilde{i}_b \in L_2 \;, \\ &\quad \lim_{t \to \infty} \tilde{i}_a = 0 \;, \quad \lim_{t \to \infty} \tilde{i}_b = 0 \;. \end{split}$$

iii) If, in addition, the persistent excitation (PE) condition is satisfied,

$$\begin{split} \lim_{t\to\infty}\tilde{\omega} &= 0 \;,\;\; \lim_{t\to\infty}\tilde{\xi}_1 = 0 \;,\;\; \lim_{t\to\infty}\tilde{\xi}_2 = 0 \;,\\ \lim_{t\to\infty}\tilde{\phi}_a &= 0 \;,\;\; \lim_{t\to\infty}\tilde{\phi}_b = 0 \;. \end{split}$$

**Proof:** i) (15) guarantees the boundedness of all the error variables ( $\tilde{i}_a$ ,  $\tilde{i}_b$ ,  $\tilde{\omega}$ ,  $\tilde{\xi}_1$ ,  $\tilde{\xi}_2$ ). To show the boundedness of the flux estimation errors  $\tilde{\phi}_a$  and  $\tilde{\phi}_b$ , we define the error signals  $z_1 = A + \zeta_1$  and  $z_2 = B + \zeta_2$ . From (7), we attain

$$\beta \tilde{\phi}_a = z_1 - \tilde{i}_a, \beta \tilde{\phi}_b = z_2 - \tilde{i}_b.$$
(16)

The boundedness of the flux estimation errors  $\tilde{\phi}_a$  and  $\tilde{\phi}_b$  can then follow from that of the signals  $z_1$  and  $z_2$ . We now derive the dynamics for  $(z_1, z_2)$  from (6) and (10), and the definitions of  $v_c$  and  $v_d$  as follows:

$$\begin{bmatrix} \dot{z}_{1} \\ \dot{z}_{2} \end{bmatrix} = \begin{bmatrix} -\alpha & -n_{p}\hat{\omega} \\ n_{p}\hat{\omega} & -\alpha \end{bmatrix} \begin{bmatrix} z_{1} \\ z_{2} \end{bmatrix}$$

$$+ \begin{bmatrix} -n_{p}B & 1 & 0 \\ n_{p}A & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{\omega} \\ \tilde{\xi}_{1} \\ \tilde{\xi}_{2} \end{bmatrix}$$

$$+ \begin{bmatrix} \beta k_{1} - \rho_{a} & 0 \\ 0 & \beta k_{1} - \rho_{a} \end{bmatrix} \begin{bmatrix} \tilde{i}_{a} \\ \tilde{i}_{b} \end{bmatrix}.$$

$$(17)$$

As can be shown easily, the system in (17) is bounded-input bounded-output stable; the

boundedness of the signals  $\tilde{i}_a$ ,  $\tilde{i}_b$ ,  $\tilde{\omega}$ ,  $\tilde{\xi}_1$ ,  $\tilde{\xi}_2$  thus guarantees that of the signals  $z_1$  and  $z_2$ , which in turn ensures that the flux estimation errors  $\tilde{\phi}_a$  and  $\tilde{\phi}_b$  are bounded in view of (16).

ii) By integrating both sides of equation (15), we have  $\tilde{i}_a \in L_2$  and  $\tilde{i}_b \in L_2$ . Note also that  $\dot{\tilde{i}}_a$ ,  $\dot{\tilde{i}}_b \in L_\infty$  in view of (9). The convergence of  $\tilde{i}_a$  and  $\tilde{i}_b$  to zero is then obtained by applying Barbalat's lemma [9, pp. 85-86].

iii) The proof of convergence of all the error signals under the PE condition can be obtained in a standard manner as in [9, pp. 72-75], and thus is not given here.

**Remark 1:** If the design parameters  $k_1$  and  $k_2$  are selected as follows:

$$k_1 = \frac{\rho_a}{\beta},$$

$$k_2 = \frac{\rho_b}{\beta},$$
(18)

then the dynamic equation in (17) becomes independent of the current estimation errors  $\tilde{i}_a$  and  $\tilde{i}_b$ .

Finally, the resulting algorithm for the speed and flux estimation is summarized below.

$$\begin{split} &\dot{\hat{i}}_{a} = -\beta \left( -\alpha \hat{\phi}_{a} - n_{p} \hat{\omega} \hat{\phi}_{b} + \alpha M i_{a} \right) \\ &- \frac{R_{s}}{\sigma L_{s}} i_{a} + \frac{1}{\sigma L_{s}} u_{a} - v_{a}, \\ &\dot{\hat{i}}_{b} = -\beta \left( -\alpha \hat{\phi}_{b} + n_{p} \hat{\omega} \hat{\phi}_{a} + \alpha M i_{b} \right) \\ &- \frac{R_{s}}{\sigma L_{s}} i_{b} + \frac{1}{\sigma L_{s}} u_{b} - v_{b}, \\ &\dot{\hat{\phi}}_{a} = -\alpha \hat{\phi}_{a} - n_{p} \hat{\omega} \hat{\phi}_{b} + \alpha M i_{a} - v_{c}, \\ &\dot{\hat{\phi}}_{b} = -\alpha \hat{\phi}_{b} + n_{p} \hat{\omega} \hat{\phi}_{a} + \alpha M i_{b} - v_{d}, \\ &\dot{\zeta}_{1} = v_{a} + \beta v_{c}, \qquad \zeta_{1}(0) = 0, \\ &\dot{\zeta}_{2} = v_{b} + \beta v_{d}, \qquad \zeta_{2}(0) = 0, \\ &\dot{\hat{\omega}} = \lambda_{1} n_{p} \left\{ \left( \zeta_{2} + \beta \hat{\phi}_{b} \right) \tilde{i}_{a} - \left( \zeta_{1} + \beta \hat{\phi}_{a} \right) \tilde{i}_{b} \right\}, \\ &\dot{\hat{\xi}}_{1}^{2} = \lambda_{2} \tilde{i}_{a}, \\ &\dot{\hat{\xi}}_{2} = \lambda_{3} \tilde{i}_{b}, \\ &v_{a} = -\left( \rho_{a} \tilde{i}_{a} + \alpha \zeta_{1} + n_{p} \hat{\omega} \zeta_{2} + \hat{\xi}_{1} \right), \\ &v_{b} = -\left( \rho_{b} \tilde{i}_{b} + \alpha \zeta_{2} - n_{p} \hat{\omega} \zeta_{1} + \hat{\xi}_{2} \right), \\ &v_{c} = k_{1} \tilde{i}_{a}, \\ &v_{d} = k_{2} \tilde{i}_{b}. \end{split}$$

# 4. HANDLING UNCERTAINTY OF THE ROTOR RESISTANCE

The rotor resistance, which varies with the motor temperature, can also be estimated within the same framework. The uncertainty of the rotor resistance is handled by considering

$$\alpha := \frac{R_r}{L_r} = \frac{R_{rN} + \Delta R_r}{L_r} =: \alpha_N + \theta, \tag{20}$$

where  $R_{rN}$  is the nominal value of  $R_r$ ,  $\Delta R_r$  represents the uncertainty of  $R_r$ , and  $\theta = \Delta R_r / L_r$  is the parameter to be estimated.

By taking the same steps as in section 3, we derive the following algorithm for estimating  $\theta$  as well as  $\omega$ ,  $\phi_a$ , and  $\phi_b$ :

$$\begin{split} \hat{i}_{a} &= -\beta \left( -\alpha_{N} \hat{\phi}_{a} - n_{p} \hat{\omega} \hat{\phi}_{b} + \alpha_{N} M i_{a} + \hat{\theta} \left( -\hat{\phi}_{a} + M i_{a} \right) \right) \\ &- \frac{R_{s}}{\sigma L_{s}} i_{a} + \frac{1}{\sigma L_{s}} u_{a} - v_{a}, \\ \hat{i}_{b} &= -\beta \left( -\alpha_{N} \hat{\phi}_{b} + n_{p} \hat{\omega} \hat{\phi}_{a} + \alpha_{N} M i_{b} + \hat{\theta} \left( -\hat{\phi}_{b} + M i_{b} \right) \right) \\ &- \frac{R_{s}}{\sigma L_{s}} i_{b} + \frac{1}{\sigma L_{s}} u_{b} - v_{b}, \\ \hat{\phi}_{a} &= -\alpha_{N} \hat{\phi}_{a} - n_{p} \hat{\omega} \hat{\phi}_{b} + \alpha_{N} M i_{a} + \hat{\theta} \left( -\hat{\phi}_{a} + M i_{a} \right) - v_{c}, \\ \hat{\phi}_{b} &= -\alpha_{N} \hat{\phi}_{b} + n_{p} \hat{\omega} \hat{\phi}_{a} + \alpha_{N} M i_{b} + \hat{\theta} \left( -\hat{\phi}_{b} + M i_{b} \right) - v_{d}, \\ \dot{\zeta}_{1} &= v_{a} + \beta v_{c}, \qquad \zeta_{1} \left( 0 \right) = 0, \\ \dot{\zeta}_{2} &= v_{b} + \beta v_{d}, \qquad \zeta_{2} \left( 0 \right) = 0, \\ \hat{\phi}_{a} &= \lambda_{\omega} n_{p} \left\{ \left( \zeta_{2} + \beta \hat{\phi}_{b} \right) \tilde{i}_{a} - \left( \zeta_{1} + \beta \hat{\phi}_{a} \right) \tilde{i}_{b} \right\}, \\ \hat{\theta} &= \lambda_{\theta} \left[ \left\{ \zeta_{1} - \tilde{i}_{a} + \beta \left( \hat{\phi}_{a} - M i_{a} \right) \right\} \tilde{i}_{a} + \left\{ \zeta_{2} - \tilde{i}_{b} + \beta \left( \hat{\phi}_{b} - M i_{b} \right) \right\} \tilde{i}_{b} \right], \\ \dot{\xi}_{1}^{2} &= \lambda_{1} \tilde{i}_{a}, \\ \dot{\xi}_{2}^{2} &= \lambda_{2} \tilde{i}_{b}, \\ v_{a} &= - \left( \rho_{a} \tilde{i}_{a} + \alpha_{N} \zeta_{1} + n_{p} \left( \zeta_{2} - \tilde{i}_{b} \right) \hat{\omega} + \left( \zeta_{1} - \tilde{i}_{a} \right) \hat{\theta} + \hat{\xi}_{1} \right), \\ v_{b} &= - \left( \rho_{a} \tilde{i}_{b} + \alpha_{N} \zeta_{2} - n_{p} \left( \zeta_{1} - \tilde{i}_{a} \right) \hat{\omega} + \left( \zeta_{2} - \tilde{i}_{b} \right) \hat{\theta} + \hat{\xi}_{2} \right), \\ v_{c} &= k_{1} \tilde{i}_{a}, \\ v_{d} &= k_{2} \tilde{i}_{b}, \\ \end{split}$$

where  $\hat{\theta}$  is the estimate of  $\theta$ . The stability operties of this estimator are presented below.

**Theorem 2:** Assume that the speed is constant, and also that the stator currents are bounded. Then the proposed estimator in (21) guarantees that

- i)  $\tilde{i}_a, \tilde{i}_b, \tilde{\omega}, \tilde{\theta}, \tilde{\xi}_1, \tilde{\xi}_2, \tilde{\phi}_a, \tilde{\phi}_b \in L_{\infty}$
- $$\begin{split} \text{ii)} \quad & \tilde{i}_a \in L_2 \;, \quad \tilde{i}_b \in L_2 \;, \\ & \lim_{t \to \infty} \tilde{i}_a = 0 \;, \; \lim_{t \to \infty} \tilde{i}_b = 0 \;. \end{split}$$
- iii) If, in addition, the persistent excitation (PE) condition is satisfied,

$$\begin{split} &\lim_{t\to\infty}\tilde{\omega}=0\;,\;\;\lim_{t\to\infty}\tilde{\theta}=0\;,\;\;\lim_{t\to\infty}\tilde{\xi}_1=0\;,\\ &\lim_{t\to\infty}\tilde{\xi}_2=0\;,\lim_{t\to\infty}\tilde{\phi}_a=0\;,\lim_{t\to\infty}\tilde{\phi}_b=0\;. \end{split}$$

**Remark 2:** The proof of Theorem 2 parallels that of Theorem 1, and thus is not provided here.

# 5. SIMULATIONS

Using the C language, the proposed estimation algorithm has been simulated; the motor data used in simulations are specified in the Appendix. It is assumed that the induction motor is initially at rest and is required to follow the desired speed and flux references. As the controller, the direct torque control (DTC) scheme is employed; however, the estimated flux and speed resulting from the proposed algorithm are used instead of the actual measurements. Fig. 1 depicts a block diagram of the DTC control scheme. The continuous-time algorithm is discretized using the Euler approximation with a sampling period of 50 µs The initial estimation errors are given as follows:  $\tilde{i}_a(0) = 1$ ,  $\tilde{i}_b(0) = -1$ ,  $\tilde{\phi}_a(0) = -0.1$ ,  $\tilde{\phi}_b(0) = 0.1$ ,  $\tilde{\omega}(0) = 10$ . The estimator design parameters are selected as  $\rho_a = \rho_b = 1000$ ,  $k_1 = k_2 = \rho_a / \beta, \lambda_1 = 1$ , and  $\lambda_2 = \lambda_3 = 40000$ . Simulation results are indicated

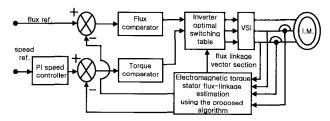


Fig. 1. Block diagram of stator-flux-based DTC induction motor drive with Voltage Source Inverter (VSI).

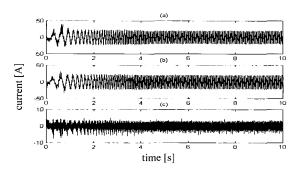


Fig. 2. Current trajectories; (a)  $i_a$  (b)  $\hat{i}_a$  (c)  $\tilde{i}_a$ .

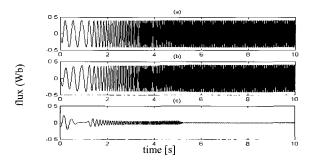


Fig. 3. Flux trajectories; (a)  $\phi_a$  (b)  $\hat{\phi}_a$  (c)  $\tilde{\phi}_a$ .

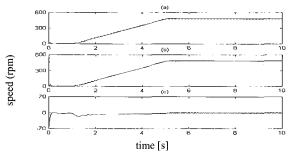


Fig. 4. Speed trajectories; (a)  $\omega_{ref}$ : --,  $\hat{\omega}$ : -- (b)  $\omega_{ref}$ : --,  $\omega$ : -- (c)  $\tilde{\omega}$ .

in Figs. 2, 3, and 4 where  $\omega_{ref}$  denotes the speed reference.

It is seen from the simulation results that the speed estimate converges to the actual value as the estimated currents converge to their actual values. Also, the flux estimation errors tend to zero as the speed estimate approaches the actual speed.

#### 6. EXPERIMENTS

The proposed algorithm has also been implemented for experimentation. The experimental set-up is presented in Fig. 5.

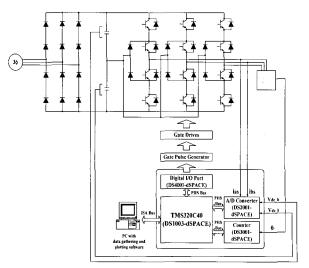


Fig. 5. Experimental set-up.

The main components of the system include the following: a DSP system (dSPACE: main controller board DS1003 containing TMS320C40, I/O board DS4001, counter board DS3001, and A/D board DS2001.); an induction motor (whose data are provided in the Appendix) and a voltage source inverter; an optical encoder attached to the motor shaft for assessing the speed estimation performance; and AC current sensors. The dSPACE system including interface boards was programmed in the C language, interfaced to a standard PC.

The continuous-time algorithm is discretized using the Euler approximation with a sampling period of  $50 \mu s$ .

At each sampling instant, the DSP receives the measurements of the stator currents and voltages, and also the actual position to compute the motor speed (for comparison purposes), and then runs the proposed estimation algorithm and the DTC scheme.

Note that the design parameters are the same as in the simulations above. The initial estimates are all set to 0. The estimation algorithm was tested in various speed regions: normal speed (Figs. 6 and 7), high speed (Figs. 8 and 9), low speed (Figs. 10 and 11) and forward-reverse (Figs. 12 and 13) operations.

As shown in Figs. 6-13 indicating the experimental results, the estimator was seen to work well under various circumstances. In all the cases presented here, the assumption  $\dot{\omega} = 0$  did not appear to be problematic. Hence the proposed estimator is expected to result in satisfactory performance when used for a sensorless scheme.

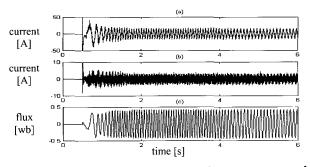


Fig. 6. Currents and flux; (a)  $i_a : -$ ,  $\hat{i}_a : -$  (b)  $\tilde{i}_a$  (c)  $\hat{\phi}_a$ .

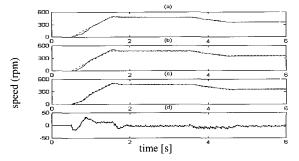


Fig. 7. Speed trajectories; (a)  $\omega_{ref}$ : --,  $\hat{\omega}$ : -- (b)  $\omega_{ref}$ : --,  $\omega$ : -- (c)  $\omega$ : --,  $\hat{\omega}$ : -- (d)  $\tilde{\omega}$ .

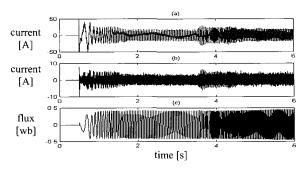


Fig. 8. Currents and flux; (a)  $i_a : -, \hat{i}_a : -$  (b)  $\tilde{i}_a$  (c)  $\hat{\phi}_a$ .

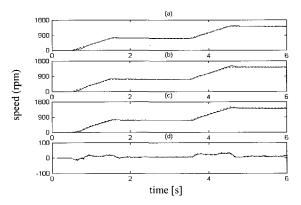


Fig. 9. Speed trajectories; (a)  $\omega_{ref}:$  --,  $\hat{\omega}:$  -- (b)  $\omega_{ref}:$  --,  $\omega:$  -- (c)  $\omega:$  --,  $\hat{\omega}:$  -- (d)  $\tilde{\omega}$ .

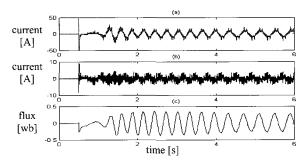


Fig. 10. Currents and flux; (a)  $i_a:$  --,  $\hat{i}_a:$  -(b)  $\tilde{i}_a$  (c)  $\hat{\phi}_a$ .

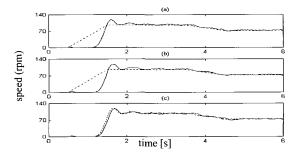


Fig. 11. Speed trajectories; (a)  $\omega_{ref}$ :-,  $\hat{\omega}$ :- (b)  $\omega_{ref}$ :-,  $\omega$ :- (c)  $\omega$ :-,  $\hat{\omega}$ :-.

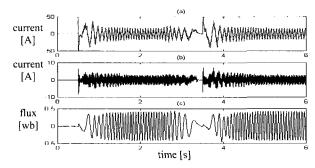


Fig. 12. Currents and flux; (a)  $i_a : -$ ,  $\hat{i}_a : -$  (b)  $\tilde{i}_a$  (c)  $\hat{\phi}_a$ 

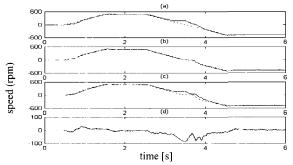


Fig. 13. Speed trajectories; (a)  $\omega_{ref}$ :--,  $\hat{\omega}$ :- (b)  $\omega_{ref}$ :--,  $\omega$ :- (c)  $\omega$ :--,  $\hat{\omega}$ :-.

#### 7. CONCLUSION

A new method for estimating the rotor speed and flux has been derived on the basis of a fourth-order electrical model of the induction motor. It is assumed that only the stator currents and voltages are measurable, and that the stator currents are bounded. Then we convert the state estimation problem into a parameter estimation problem with the assumption that the rotor speed is constant. The speed and flux estimator thus obtained is shown to result in the boundedness of all the error variables and the convergence of the stator current estimation errors to zero. Moreover, if the persistent excitation condition is satisfied, then all the estimation errors are shown to converge exponentially to zero. Furthermore, the rotor resistance, which varies with the motor temperature, can also be estimated within the same framework. Simulations and experimental results demonstrate the effectiveness of the proposed estimation scheme.

# APPENDIX Induction Motor Data

7460 W Rated power

1740 rpm	Rated speed
40 Nm	Rated torque
220 V	Rated voltage
0.4 Wb	Rated flux
$R_s = 0.1695\Omega$	Stator resistance
$R_r = 0.161\Omega$	Rotor resistance
$L_s = 0.02397 \text{ H}$	Stator inductance
$L_r = 0.02456 \; \mathbf{H}$	Rotor inductance
M = 0.02277  H	Mutual inductance
$J = 0.08 \ kg \cdot m^2$	Motor-load inertia
$n_p = 2$	Number of pole pairs

#### REFERENCES

- [1] P. Vas, Sensorless *Vector and Direct Torque Control*, Oxford Univ. Press, 1998.
- [2] R. Kim, S. K. Sul, and M. H. Park, "Speed sensorless vector control of induction motor using extended Kalman filter," *IEEE Trans. on Industrial Applications*, vol. 30, no. 5, pp. 1225-1233, Sept./Oct. 1994.
- [3] C. Schauder, "Adaptive speed identification for vector control of induction motors without rotational transducers," *Proc. of Conf. Rec. IEEE-IAS Annu. Meeting*, pp. 493-499, 1989.
- [4] H. Kubota, K. Matsuse, and T. Nakano, "DSP-based speed adaptive flux observer of induction motor," *IEEE Trans. on Industrial Applications*, vol. 29, no. 2, pp. 344-348, Mar./Apr. 1993.
- [5] S. Sangwongwanich, S. Doki, T. Furu-hashi, and S. Okuma, "Adaptive sliding mode observers for induction motor control," *Trans. Soc. Instrum. Contr. Eng.*, vol. 27, no. 5, pp. 569-576, May 1991.
- [6] V. A. Bondarko and A. T. Zaremba, "Speed and flux estimation for an induction motor without position sensor," *Proc. Amer. Contr. Conf.*, San Diego, Califonia, America, pp.3890-3894, 1999.
- [7] R. Marino, S. Peresada, and P. Valigi, "Adaptive input-output linearizing control of induction motors," *IEEE Trans. on Automatic Control*, vol. 38, no. 2, pp. 208-221, 1993.
- [8] R. Marino, S. Peresada, and P. Tomei, "Exponentially convergent rotor resistance estimation for induction motors," *IEEE Trans. on Industrial Electronics*, vol. 42, no. 5, pp. 508-515, 1995.
- [9] K. S. Narendra and A. M. Annaswamy, *Stable Adaptive Systems*, Prentice-Hall, 1989.



**Gil-Su Lee** received his M.S. degree in Electrical Engineering from Korea University in 2003. His research interests include nonlinear control and adaptive control.



Dong-Hyun Lee received his M.S. degree in Electrical Engineering from Korea University in 2003. He is currently with the 3rd SD Center, Agency for Defense Development, Korea. His research interests include nonlinear control and adaptive control.



Joong-Ho Song received his B.S. and M.S. degrees in Electrical Engineering from Seoul National University in 1980 and 1982, and his Ph.D. degree from KAIST, Korea, in 1993. He worked as an Engineer at the E-hwa Electrical Co., Korea from 1982 to 1985. From 1985 to 2002, he was with the Intelligent System Control

Research Center, Korea Institute of Science and Technology. Since 2002, he has been with the Department of Electrical Engineering, Seoul National University of Technology. He was a Visiting Scholar at the WEMPEC, University of Wisconsin-Madison in 1995-1996. His primary research interests are switching converters, electric machine drives, and servo control technologies.



Tae-Woong Yoon received his B.S. and M.S. degrees, both in Control Engineering, from Seoul National University in 1984 and 1986. He received his D.Phil. degree in Engineering Science from Oxford University in 1994. From 1986 to 1995, he worked as a Researcher at the Korea Institute of Science and

Technology (KIST). In 1995, he joined the faculty of Electrical Engineering, Korea University, where he is now a Professor. He was a Visiting Professor at Yale University in 2001, whilst on sabbatical leave from Korea University. His research interests include model predictive control, adaptive and switched systems, and control applications.



Ick Choy received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from Seoul National University, Seoul Korea, in 1979, 1981, and 1990, respectively. From 1981 to 2003, he was with the Intelligent System Control Research Center, Korea Institute of Science and Technology, Seoul. Since 2003, he has been with Kwangwoon

University, Seoul, where he is currently an Associate Professor in the Department of Information and Control Engineering. His main research interests include microprocessor applications, high-performance drives, and emerging technologies.



**Kyo-Beum Lee** received his B.S. and M.S. degrees in Electrical and Electronic Engineering from Ajou University, Korea, in 1997 and 1999, respectively. He received his Ph.D. degree in Electrical Engineering from Korea University, Korea in 2003. During 2003-2004 he was a Post doc. Researcher at Aalborg University,

Aalborg, Denmark. Since 2004, he has been with the Institute of Energy Technology as an Assistant Professor at Aalborg University. His research interests include electric machine drives and power electronics.