A Strategy for the Simulation of Adhesive Layers

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Abstract: The high accurate simulation of very thin glue layers based on the finite element method is still connected to many problems which result from the necessity to construct a complicated mesh of essentially different sizes of elements. This can lead to a loss of accuracy, unstable calculations and even loss of convergence. However, the implementation of special transmission elements along the glue line and special edge-elements in the near-edge region would lead to a dramatic decrease of the number of finite elements in the mesh and thus, prevent unsatisfactory phenomena in numerical analysis and extensive computation time. The theoretical basis for such special elements is the knowledge about appropriate transmission conditions and the edge effects near the free boundary of the adhesive layer. Therefore, recently proposed so-called non-classical transmission conditions and the behavior near the free edge are investigated in the context of the single-lap tensile-shear test of adhesive technology.

Keywords: *adhesive layers, interface, transmission conditions, finite element method, free-edge effects*

1. Introduction

Different approximation procedures for the solution of partial differential equations are known (cf. Figure 1) and each of the method possess its own advantages or disadvantages. The finite element method (FEM) is derived from variation principles or the principle of virtual work and results in a symmetric system of equations with a diagonally dominant matrix. Many commercial codes are available and such codes are widely used for industrial simulations. Even with commercial codes, arbitrary geometries and non-linearities, e.g. plastic or visco-elastic material behavior, can nowadays be considered. The finite difference method (FDM) is derived from differential equations of the corresponding field problem and can result in a non-symmetric and diagonally dominant matrix. This method is easily to transform into computational codes but reveals its disadvantages for complex geometries, singular crack behavior or non-continuous solutions. The boundary element method (BEM) is derived from integral equations and results in a non-symmetric and full matrix. The advantage of this method is that only the boundary needs to be discretisized. The main disadvantage is that arbitrary inhomogeneous structures and non-linearities are difficult to transform completely into integral equations. Therefore, this method is reserved for special applications, e.g. fracture mechanics.

Nevertheless, the application of the finite element method requires a lot of experience and many problems are still unsolved or unsatisfactory with respect to economic requirements. The high-accurate simulation of thin adhesive layers requires in the framework of the finite element simulation the introduction of a huge amount of finite elements. The generation of such computational models is on the one hand difficult to automatize and extremely time-consuming and on the other hand later on, the solving of the resulting system of equations may also take considerable time. Furthermore, complicated meshes with elements of essentially different sizes and deformed transition elements can lead to numerical problems, such as loss of accuracy or even loss of convergence[2,3]. A further problem is connected with the fact that the aspect ratio, i.e. length-width ratio, is limited for classical finite elements to a maximum number of 1:2 in order to avoid numerical instability. The improvement of such calculations is therefore not only an economic requirement but also the necessity for a better dimensioning of structural applications which will lead to maximum utilization of the materials and higher reliability and service life of entire structures and applications.

A simplified adhesively bonded joint under shear load is

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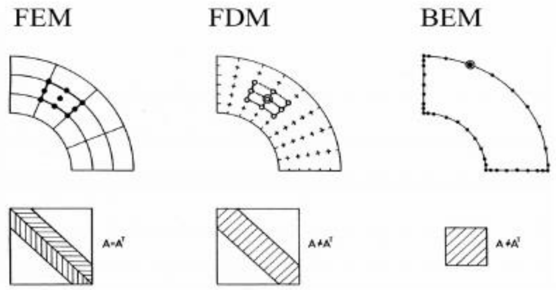

Figure 1. Numerical approximation procedures for partial differential equations.

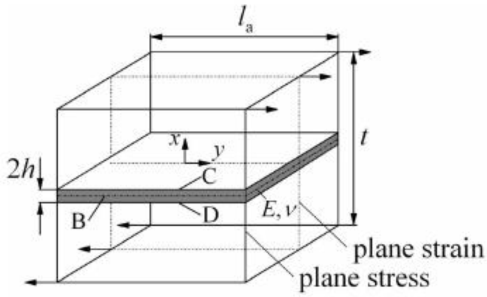

Figure 2. Schematic representation of an adhesively bonded joint.

shown in Figure 2 with its geometrical dimensions. The real three-dimensional behavior can be approximated by two-dimensional limiting cases in order to reduce the dimensionality (this leads to significant less finite elements in the computational model): the plane strain case which holds inside the joint and the plane stress case which holds at the free surface. In the following, we are going to present and to investigate so called non-classical transmission conditions which are the theoretical basis for the introduction of novel finite elements for adhesive layers. It should be mentioned here that the overall deformation behavior is determined by the plane strain case. However, the total description based on two-dimensional models requires also the consideration of the plane stress case.

2. Transmission Conditions

The idea of transmission conditions can easily be introduced based on simple one-dimensional structural elements, such as springs or rods (cf. Figure 3). The relative displacements of both ends for symmetric loading, $[u_x] = u_1\text{-}(-u_1)$, can be related to the acting force F in the spring or the stress σ in the rod according to Eq. (1) (k: spring stiffness; E: Young's modulus; A: cross-sectional area; l: length).

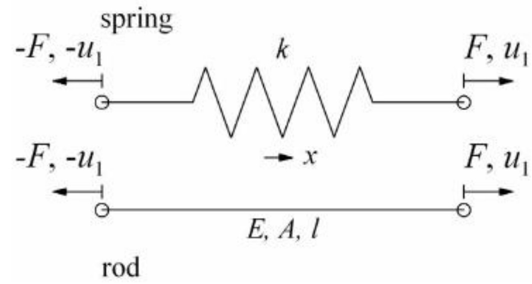

Figure 3. Simple one-dimensional structural elements.

Table 1. Possible sets of transmission conditions along the line $x=0$ depending on the relative properties of the thin intermediate layer. 2D case

Interface	Transmission conditions			
soft	$[u_y]\text{-}a_2\sigma_{xy}=0$	$[u_x]\text{-}a_1\sigma_x=0$	$[\sigma_{xy}]=0$	$[\sigma_x]=0$
comparable	$[u_y]=0$	$[u_x]=0$	$[\sigma_{xy}]=0$	$[\sigma_x]=0$
stiff	$[u_y]=0$	$[u_x]=0$	$[\sigma_{xy}]+\partial/\partial x(a_3\,\partial u_y/\partial y)$	$[\sigma_x]=0$

Table 2. Parameters $a_i(y)$ for the plane strain and plane stress case

Case	a_1	a_2	a_3
plane strain	$\frac{2h(1+v)(1-2v)}{E(1-v)}$	$\frac{4h(1+v)}{E}$	$\frac{2hE}{1-v^2}$
plane stress	$\frac{2h(1-v^2)}{E}$	$\frac{4h(1+v)}{E}$	$2hE$

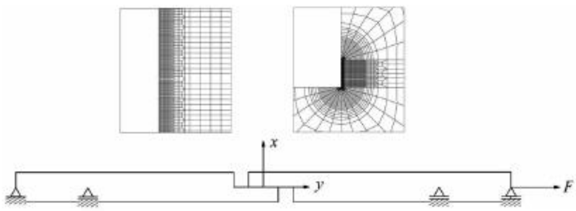

Figure 4. Two-dimensional mesh (details) and boundary conditions.

$$\text{(spring)};\ [u_x]=2u_1=\frac{F}{k}\ \ [u_x]=2u_1\frac{l}{E}\cdot\sigma\ \text{(rod)} \quad (1)$$

Recently, so-called non-classical transmission conditions for two-dimensional problems were proposed which relate the difference of displacements $[u]$ and stresses $[\sigma]$ at the adhesive/adherend interface (cf. line C, D in Figure 2) to the behavior in the middle of the adhesive layer (cf. line B in Figure 2, $x=0$). Table 1 and 2 summarize these non-classical transmission conditions for isotropic elastic material behavior[4,5]. The elastic constants E and v are related to the adhesive layer of thickness $2h$.

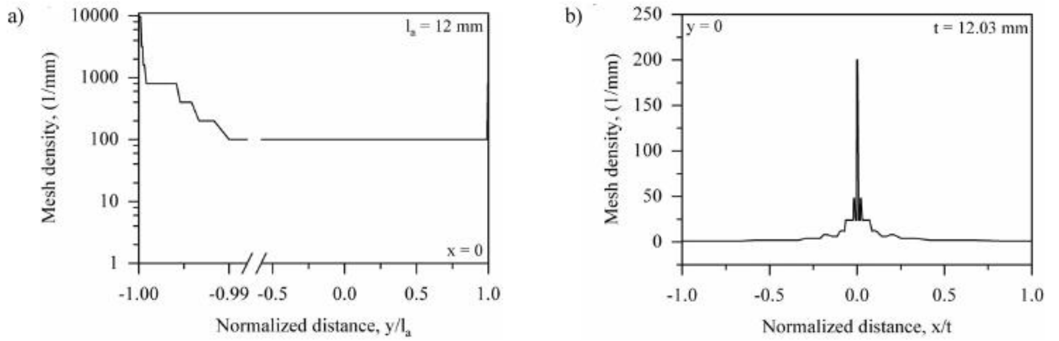

Figure 5. Mesh density: a) along the glue line ($x = 0$); b) transverse to the glue line ($y = 0$).

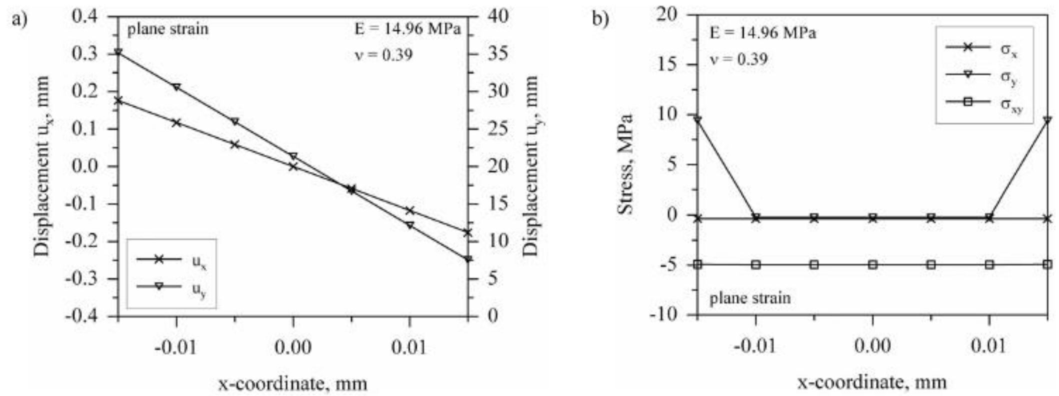

Figure 6. Stress orthogonal the bond line for an adhesive with $v = 0.39$ and $E = 14.96$ MPa.

The classification soft, comparable and stiff relates to the relationship between the stiffness of the adhesive and the adherend (E/E_s). The formulae in Table 2 have been found under the assumption that the material parameters of the adhesive layer do not change perpendicular to the glue line. General expressions which incorporate any functional dependency can be found in[6].

The knowledge about the validity of these conditions will enable the derivation of novel elements.

3. Finite Element Modeling

The validity of the indicated non-classical transmission conditions will be numerically investigated in the framework of the single-lap tensile-shear test of adhesive technology (cf. Figure 4)[7]. Later on, this procedure will be used for the experimental verification of the novel computation method. Figure 5 shows the high mesh density which is required for accurate solutions especially near the free boundary of the adhesive layer. Special elements with reduced integration using an assumed strain formulation written in natural coordinates which insures good representation of the shear strains in the element were used. A commercial finite element software (MSC.Marc) was used for simulating the mechanical behavior of the thin adhesive layer with a thickness of $2h = 30$ mm. In the following, the results are presented for stepped brass adherends ($E_s = 119704$ MPa, $\nu_s = 0.3395$, length 106 mm, width 25 mm, total depth 12 mm) and an adhesive ($E = 14.96$ MPa; $\nu = 0.39$) which can be classified as soft according to Table 1. We assumed for these calculations that the adhesive layer is isotropic and homogeneous. Both cases, i.e. the plane strain and plane stress case were investigated. However, to reduce the amount of presented results, only the plane strain case is presented here. It should be mentioned here that the plane stress case reveals similar results.

4. Results

First of all, Figure 6 shows the displacement and stress distribution perpendicular to the glue line, this means along the line where $y = 0$ holds, in order to verify some basic

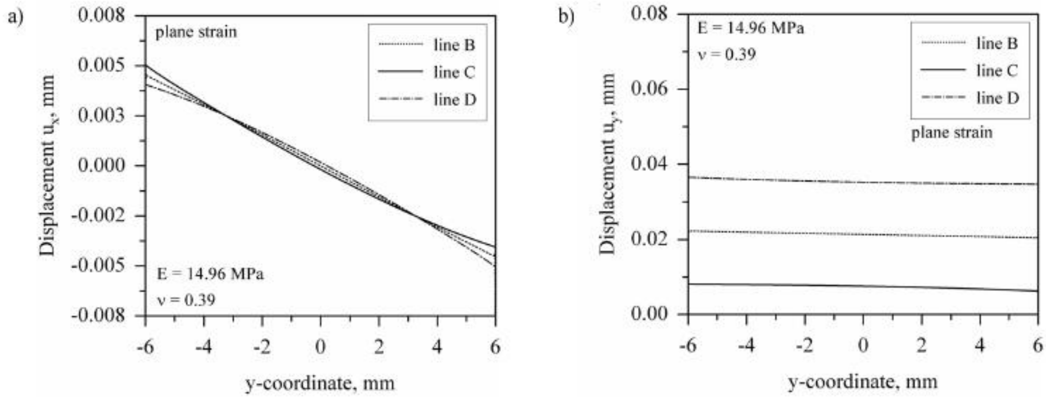

Figure 7. Displacement distribution for lines B, C and D.

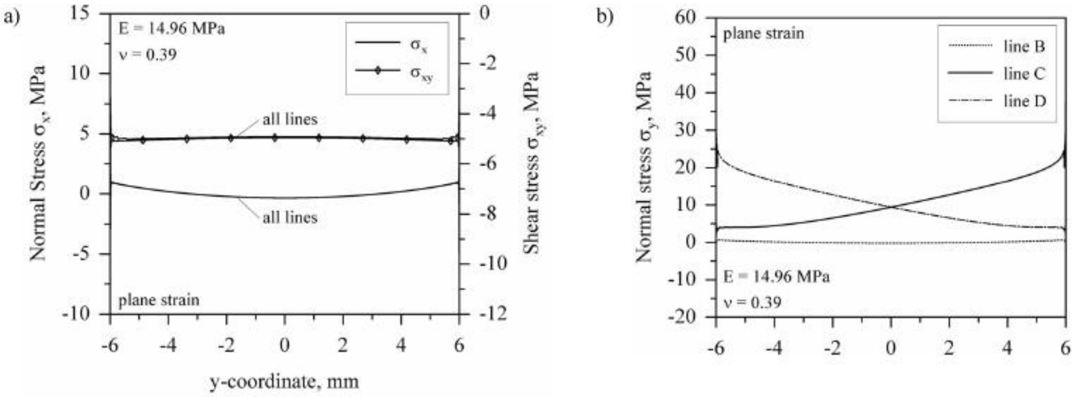

Figure 8. Normal and shear stress distribution for lines B, C and D.

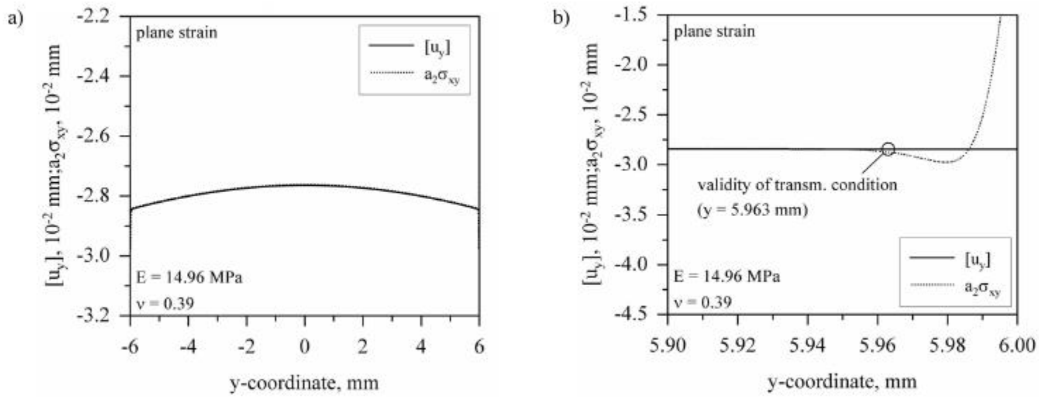

Figure 9. Verification of the first transmission condition along the imperfect interface (plane strain).

assumptions used for the derivation of the transmission conditions. It can be seen that the justified linear behavior for the displacements and the constant behavior for the stresses inside the adhesive layer are fulfilled. Furthermore, this result holds for any line $y = const$ (except the region near the free boundary). The behavior of the σ_y component results from the averaging of the adhesive and adherend values at the interface node. However, the correct extra-

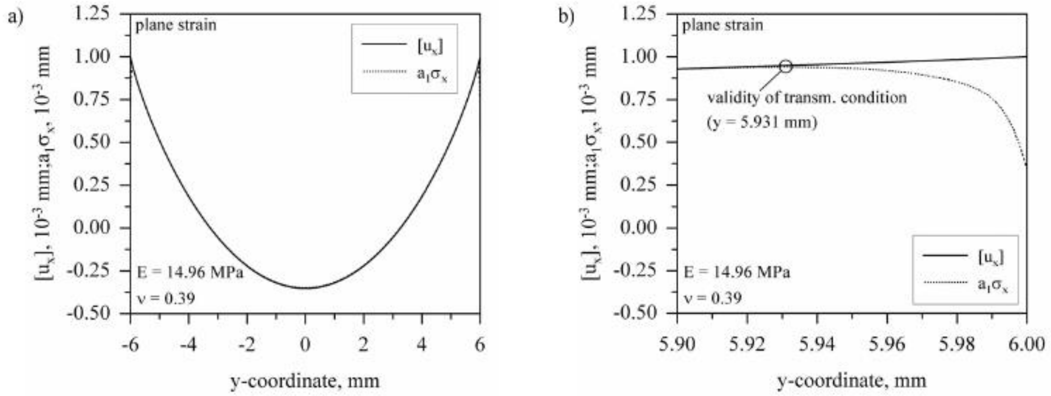

Figure 10. Verification of the second transmission condition along the imperfect interface (plane strain).

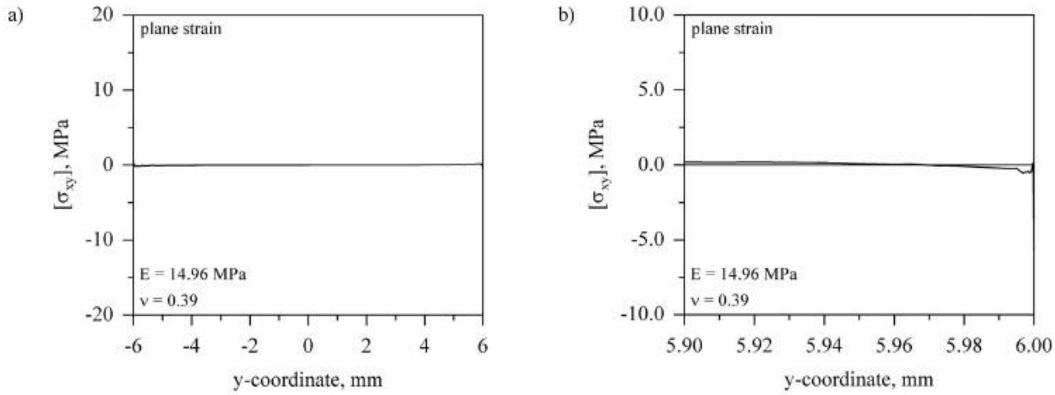

Figure 11. Verification of the third transmission condition along the imperfect interface (plane strain).

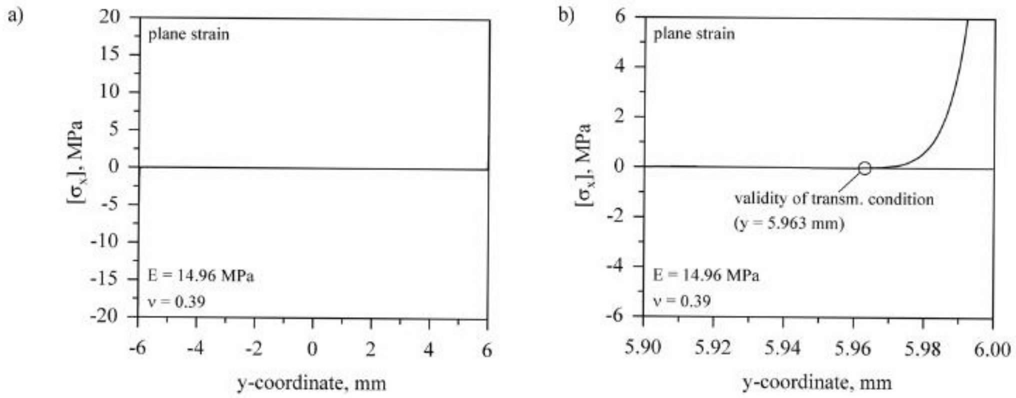

Figure 12. Verification of the fourth transmission condition along the imperfect interface (plane strain).

polation of this value (i.e. constant value in the whole adhesive layer) can be done without any loss of generality of the presented results. To avoid this behavior it would be necessary to refine significantly the finite element mesh also in this region. However, it would increase the total amount of unknowns in such a way that it would be difficult to compute the 2D problem on a standard PC with 1.5 GB of RAM.

Figures 7 and 8 illustrate the displacement and stress distribution along the glue line ($x = 0$) and the interface lines C and D (cf. Figure 1). These values which were derived from FEM analysis will be evaluated according to the relationships given in Table 1 and 2 in order to investigate the validity of the given conditions.

Figures 9 ~ 12 show the evaluation of all four transmission conditions given in Table 1 along the whole glue line and additionally, for magnifications near the free surface. It can be seen in Figures 9 and 10 that the transmission conditions are fulfilled along a very long range of the glue line and that only very near the free surface the conditions fail. In this region, also the influence of the stress singularity becomes visible. Thus, special singularity elements need to be derived in order to offer a complete set of special adhesive elements for the whole range of the glue line. The validity of the transmission conditions is based in our evaluation on a 1% criterion for the deviation between the left and right hand side of the equations presented in Table 1.

However, the application of the 1% criterion is difficult to realize for the jump $[\sigma_{xy}]$ and $[\sigma_x]$ shown in Figures 11 and 12 because the values should be equal to zero. Nevertheless, it can be seen that the conditions are fulfilled in the same range as indicated in Figures 9 and 10.

5. Conclusions and Outlook

In the present work, non-classical transmission conditions were presented and their validity investigated in the framework of the single-lap tensile-shear test of adhesive technology. It could be shown that the proposed transmission conditions are valid over a very long range of the glue line. Only near the free surface, the conditions fail and the size of this zone is obtained more or less independently of the evaluated transmission condition.

The knowledge about the validity region makes it possible to drastically decrease the number of finite elements in the constructed mesh by introducing special transmission elements instead of the thin intermediate zone between the different materials and also to prevent unsatisfactory phenomena in the numerical analysis. The development of such special elements and the implementation into a finite element code is the topic of our future research work. Furthermore, non-linear material, i.e. plastic and visco-elastic, will be investigated and corresponding transmission conditions will be derived and their validity examined.

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