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Estimators for Parameters Included in Cold Standby Systems with Imperfect Switches

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Abstract. In this paper we derive estimations of the parameters included in the distribution of the lifetime of k-out-of-m cold standby system with imperfect switches. Maximum likelihood and Bayes procedures are followed to get such estimations. Numerical studies, using Monte Carlo simulation method, are given in order to explain how we can utilize the theoretical results derived, and to compare the performance of the two different methods used. The criterion of comparisons is the mean squared errors associated with each estimate.

Key Words : *k-out-of-m standby system, imperfect switches, reliability function, maximum likelihood procedure, Bayes procedure.*

1. INTRODUCTION

In reliability modeling problem many researchers have discussed the problem of modeling reliability for systems with active redundancy property, see for example Chan et al. (1995), Cramer and Kamps (2000), Mokhlis, N. A. (2001), Sarhan and Abouammoh (2001a), Sarhan and Abouammoh (2001b) and the references there in.

In general, the system consists of m components is said to be k-out-of-m system if and only if it works if at least k of its m components are working and fails if $m-k+1$ components fail. Indeed there are many applications for the k-out-of-m systems such as an aircraft with four engines which works if at least two out of its four engines remain functioning, and a satellite which will have enough power to send signals if not more than four out of its ten batteries are discharged.

Many other practical situations can be modeled by using k-out-of-m systems such as quality control problems, inspection procedures and radar detection problems, see Saperstein (1973, 1975) and Nelson (1978).

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Sarhan and Abouammoh (2002) considered a general form of k-out-of-m system with independent and non-identical components, cold standby and imperfect switches. They derived the probability density and reliability functions of this system for the cases $m-k=1$ and $m-k>1$. They deduced the probability density and reliability functions of one, two and three out of four systems in general. Also, they presented some special cases for perfect and imperfect switches with identical and non-identical main and standby components.

Sarhan and El-Gohary (2003) and Sarhan and Tadj (2003) used maximum likelihood and Bayes methods to estimate the parameters included in 1-out-of-2:G repairable system with different repair facilities.

Our objective in this paper is to estimate the unknown parameters included in reliability function of the k-out-of-m system with independent and non-identical components, cold standby and imperfect switches, when $m-k=1$. The maximum likelihood and Bayes procedures are followed to derive such estimators.

The paper is organized as follows. Section 2 presents the notation required and main assumptions. The likelihood function and maximum likelihood estimators for the unknown parameters are given in section 3. Bayes procedure is used in section 4 to derive the Bayes estimators for the unknown parameters. Simulation study and conclusion are given in section 5.

2. THE MODEL ASSUMPTION

The system considered here is a k-out-of-m system, with $k=m-1$. All components are s-independent with constant failure rates. There is only one component in standby. The components are classified into two types: operating and standby. The standby component is connected to the system via an imperfect switch. The system starts operating with $k=m-1$ main components, while the rest component is in cold standby mode. The operating components are identical while the standby one may not identical with the operating ones. Once a component fails among the main components, instantantly the standby component becomes operating. Components do not fail simultaneously and there is no repair. The failure rate of the operating components is λ, while the failure rate of the standby component is μ and the failure rate of the switch is α.

Let X be the lifetime of k-out-of-m system, with $k=m-1$ and $f(x)$ be the probability density function of X. Sarhan and Abouammoh (2001b) derived the function $f(x)$ as on the following form

$$f(x)=\frac{k\lambda[(k-1)\lambda+\mu+\alpha]}{\lambda-(\mu+\alpha)}e^{-k\lambda x}\left(e^{-(\mu+\alpha-\lambda)x}-1\right). \tag{2.1}$$

The reliability function of the system takes the following form, Sarhan and Abouammoh (2001b):

$$R(x)=\frac{e^{-k\lambda x}}{\lambda-(\mu+\alpha)}\left\{k\lambda e^{-(\mu+\alpha-\lambda)x}-[(k-1)\lambda+\mu+\alpha]\right\}. \tag{2.2}$$

If $\lambda = \mu$, that is the operating and standby components are identical, then the function $f(x)$ becomes

$$f(x) = \frac{k\lambda(k\lambda + \alpha)}{\alpha} e^{-k\lambda x}\left(1 - e^{-\alpha x}\right), \tag{2.3}$$

and reliability function becomes

$$R(x) = \frac{e^{-k\lambda x}}{\alpha}\left\{(k\lambda + \alpha) - k\lambda e^{-\alpha x}\right\}. \tag{2.4}$$

Our objective in this paper is to estimate the unknown parameters λ, μ . The parameter α is assumed to be known.

3. LIKELIHOOD FUNCTION AND MAXIMUM LIKELIHOOD ESTIMATORS

Given the simple random sample $X_1, X_2, \cdots, X_n$, say $\underline{X}$, from the lifetime of the underlying system, the likelihood function becomes

$$L(\underline{x}) = (k\lambda)^n \left\{\frac{(k-1)\lambda + \mu + \alpha}{\lambda - (\mu + \alpha)}\right\}^n e^{-k\lambda T} \prod_{i=1}^{n}\left(e^{-(\mu+\alpha-\lambda)x_i} - 1\right), \tag{3.1}$$

where $T = \sum_{i=1}^{n} x_i$.

The log-likelihood function is given by the following form:

$$\begin{aligned} l(\underline{x}) = n\{\ln(k\lambda) + \ln((k-1)\lambda + \mu + \alpha) - \ln(\lambda - \mu - \alpha)\} \\ - k\lambda T + \sum_{i=1}^{n} \ln\left(e^{-(\mu+\alpha-\lambda)x_i} - 1\right) \end{aligned} \tag{3.2}$$

The first and second partial derivatives of log-likelihood function can be derived as in the following forms:

$$\frac{\partial l(\underline{x})}{\partial \lambda} = \frac{n}{\lambda} + \frac{n(k-1)}{(k-1)\lambda + \mu + \alpha} - \frac{n}{\lambda - (\mu + \alpha)} + \sum_{i=1}^{n} \frac{x_i\, e^{-(\mu+\alpha-\lambda)x_i}}{e^{-(\mu+\alpha-\lambda)x_i} - 1} - kT, \tag{3.3}$$

$$\frac{\partial l(\underline{x})}{\partial \mu} = \frac{n}{(k-1)\lambda + \mu + \alpha} + \frac{n}{\lambda - \mu - \alpha} - \sum_{i=1}^{n} \frac{x_i\, e^{-(\mu+\alpha-\lambda)x_i}}{e^{-(\mu+\alpha-\lambda)x_i} - 1}, \tag{3.4}$$

$$\frac{\partial^2 l(\underline{x})}{\partial \lambda^2} = -\frac{n}{\lambda^2} - \frac{n(k-1)^2}{[(k-1)\lambda + \mu + \alpha]^2} + \frac{n}{[\lambda - \mu - \alpha]^2} + \sum_{i=1}^{n} \frac{x_i^2\, e^{-(\mu+\alpha-\lambda)x_i}}{\left[e^{-(\mu+\alpha-\lambda)x_i} - 1\right]^2}, \tag{3.5}$$

$$\frac{\partial^2 l(\underline{x})}{\partial \mu^2} = -\frac{n}{[(k-1)\lambda + \mu + \alpha]^2} + \frac{n}{[\lambda - \mu - \alpha]^2} - \sum_{i=1}^{n} \frac{x_i^2 \, e^{-(\mu+\alpha-\lambda)x_i}}{\left[e^{-(\mu+\alpha-\lambda)x_i} - 1\right]^2}, \tag{3.6}$$

$$\frac{\partial^2 l(\underline{x})}{\partial \lambda \, \partial \mu} = -\frac{n(k-1)}{[(k-1)\lambda + \mu + \alpha]^2} + \frac{n}{[\lambda - \mu - \alpha]^2} + \sum_{i=1}^{n} \frac{x_i^2 \, e^{-(\mu+\alpha-\lambda)x_i}}{\left[e^{-(\mu+\alpha-\lambda)x_i} - 1\right]^2}. \tag{3.7}$$

The likelihood equations for λ, μ are

$$\frac{n}{\lambda} + \frac{n(k-1)}{(k-1)\lambda + \mu + \alpha} - \frac{n}{\lambda - (\mu + \alpha)} + \sum_{i=1}^{n} \frac{x_i \, e^{-(\mu+\alpha-\lambda)x_i}}{e^{-(\mu+\alpha-\lambda)x_i} - 1} - kT = 0, \tag{3.8}$$

$$\frac{n}{(k-1)\lambda + \mu + \alpha} + \frac{n}{\lambda - \mu - \alpha} - \sum_{i=1}^{n} \frac{x_i \, e^{-(\mu+\alpha-\lambda)x_i}}{e^{-(\mu+\alpha-\lambda)x_i} - 1} = 0. \tag{3.9}$$

Using equations (3.8) and (3.9), we get

$$\frac{n}{\lambda} + \frac{nk}{(k-1)\lambda + \mu + \alpha} - kT = 0$$

which may be written as

$$k(k-1)T\lambda^2 - \{n(2k-1) - kT(\mu + \alpha)\}\lambda + n(\mu + \alpha) = 0$$

Solving the above relation with respect to μ, we get

$$\mu + \alpha = w(\lambda), \tag{3.10}$$

where

$$w(\lambda) = \left\{ \frac{(3k-2)n}{kT\lambda + n} - (k-1) \right\} \lambda$$

the MLE of λ, can be derived by solving the following equation, which can be derived by substituting from (3.10) into (3.9), with respect to λ,

$$g(\lambda) = \lambda, \tag{3.11}$$

where

$$g(\lambda) = \lambda + \frac{n}{(k-1)\lambda + w(\lambda)} + \frac{n}{\lambda - w(\lambda)} - \sum_{i=1}^{n} \frac{x_i \, e^{-(w(\lambda)-\lambda)x_i}}{e^{-(w(\lambda)-\lambda)x_i} - 1}. \tag{3.12}$$

Therefore, a simple iteration scheme may be used to compute the fixed point solution of equation (3.11). From the i-th iteration $\lambda_{(i)}$, the (i+1)-th iteration $\lambda_{(i+1)}$ can be obtained as $g(\lambda_{(i)})$. The iterative process should be stopped when the pre-assigned 'stopping criterion' is met. Once $\hat{\lambda}$, the MLE of λ, is obtained, then MLE of μ can be obtained from (3.10) as $\hat{\mu} = w(\hat{\lambda}) - \alpha$.

Asymptotic confidence intervals

The confidence intervals of the unknown parameters λ, μ are derived in this subsection based on the asymptotic distribution of the MLEs $\hat{\lambda}, \hat{\mu}$. The 2×2 Fisher information of λ, μ is $\mathbf{I}(\lambda, \mu) = (I_{ij}(\lambda, \mu))$ for $i, j = 1$ and 2. Here

$$I_{ij}(\theta_1, \theta_2) = -E\left(\frac{\partial^2 l(\lambda, \mu)}{\partial \theta_i \partial \theta_j}\right), \theta_1 = \lambda, \theta_2 = \mu$$

and

$$I_{11} = \frac{n}{\lambda^2} + \frac{n(k-1)^2}{[(k-1)\lambda + \mu + \alpha]^2} - \frac{n}{[\lambda + \mu + \alpha]^2} + n\eta,$$

$$I_{22} = \frac{n}{[(k-1)\lambda + \mu + \alpha]^2} - \frac{n}{[\lambda + \mu + \alpha]^2} + n\eta,$$

$$I_{21} = I_{12} = \frac{n(k-1)}{[(k-1)\lambda + \mu + \alpha]^2} - \frac{n}{[\lambda + \mu + \alpha]^2} + n\eta,$$

where

$$\eta = E\left[\frac{X_i^{\,2} \exp\{-(\mu + \alpha - \lambda)X_i\}}{[\exp\{-(\mu + \alpha - \lambda)X_i\} - 1]^2}\right].$$

Therefore, if $\theta = (\lambda, \mu)$ and $\hat{\theta} = (\hat{\lambda}, \hat{\mu})$, then we have, see Bain (1978),

$$(\hat{\theta} - \theta) \xrightarrow{d} N(\mathbf{0}, \mathbf{I}^{-1}(\lambda, \mu)).$$

Thus, the approximate $(1-\gamma)100\%$ confidence intervals for λ, μ are given respectively by

$$\hat{\lambda} \pm Z_{\gamma/2}\sqrt{I_{11}^{-1}(\lambda, \mu)}, \quad \hat{\mu} \pm Z_{\gamma/2}\sqrt{I_{22}^{-1}(\lambda, \mu)}, \tag{3.13}$$

Here, $Z_{\gamma/2}$ is the upper $\gamma/2$th percentile of a standard normal distribution.

4. BAYES ESTIMATORS

Here the Bayes procedure is applied to derive the Bayes estimators for the unknown parameters λ, μ. So, the following more assumptions are needed:

1. The parameters λ, μ behave as independent random variables.
2. The random variables λ, μ having uniform prior distributions on non-negative intervals $[a_1, b_1]$ and $[a_2, b_2]$, respectively. Here, $a_i > 0, b_i > 0$, $i = 1,2$.
3. The loss incurred when the estimators $\tilde{\lambda}$ and $\tilde{\mu}$ are used respectively for λ and μ is quadratic.

Based on the assumptions 1 and 2, the joint prior probability density function (pdf) of λ, μ becomes

$$\pi(\lambda,\mu) = \begin{cases} \dfrac{1}{(b_2 - a_2)(b_1 - a_1)}, (\lambda,\mu) \in [b_1 - a_1] \times [b_2 - a_2], \\ 0, \qquad\qquad \text{otherwise.} \end{cases} \tag{4.1}$$

The likelihood function (4.1) can be rewritten as in the following form

$$L(\underline{x}) = (-k\lambda)^n \left\{1 - \frac{k\lambda}{\lambda - (\mu + \alpha)}\right\}^n e^{-k\lambda T} \sum_{l=0}^{n} (-1)^l \tau_l, \tag{4.2}$$

where $\tau_l = \sum_{1 \le i_1 < \cdots < i_l \le n} e^{-(\mu+\alpha-\lambda)\chi_l}$, $\tau_0 = 1$ and $\chi_l = \sum_{j \in \{i_1, \cdots, i_l\}} x_j$.

The joint posterior pdf of (λ, μ) is related with the joint prior pdf of (λ, μ) and the likelihood function (λ, μ) by the following form, see Martz and Waller (1982),

$$p(\lambda, \mu \mid \underline{x}) = \frac{\pi(\lambda,\mu) L(\lambda,\mu)}{\int_{a_2}^{b_2} \int_{a_1}^{b_1} \pi(\lambda,\mu) L(\lambda,\mu) d\lambda\, d\mu}.$$

Substituting from (4.2) with (4.3) into the above relation, the joint posterior pdf of (λ, μ) becomes

$$p(\lambda, \mu \mid \underline{x}) = \frac{1}{I_0} \lambda^n \left\{1 - \frac{k\lambda}{\lambda - (\mu + \alpha)}\right\}^n e^{-k\lambda T} \sum_{l=0}^{n} (-1)^l \tau_l, \tag{4.3}$$

where

$$I_0 = \int_{a_2}^{b_2}\int_{a_1}^{b_1} \lambda^n \left\{1 - \frac{k\lambda}{\lambda - (\mu+\alpha)}\right\}^n e^{-k\lambda T}\, d\lambda\, d\mu +$$
$$+ \sum_{l=1}^{n} (-1)^l \sum_{1\le i_1 < \cdots < i_l \le n} \int_{a_2}^{b_2}\int_{a_1}^{b_1} \lambda^n \left\{1 - \frac{k\lambda}{\lambda - (\mu+\alpha)}\right\}^n e^{-(\mu+\alpha)\chi_l} e^{-\lambda(kT-\chi_l)}\, d\lambda\, d\mu$$

Under the assumption (3) and using the (4.4), the Bayes estimators for λ, μ are respectively given by

$$\tilde{\lambda} = \frac{I_\lambda}{I_0}, \quad \tilde{\mu} = \frac{I_\mu}{I_0} \tag{4.4}$$

where I_λ, I_μ are given by

$$I_\lambda = \int_{a_2}^{b_2}\int_{a_1}^{b_1} \lambda^{n+1} \left\{1 - \frac{k\lambda}{\lambda - (\mu+\alpha)}\right\}^n e^{-k\lambda T}\, d\lambda\, d\mu$$
$$+ \sum_{l=1}^{n} (-1)^l \sum_{1\le i_1 < \cdots < i_l \le n} \int_{a_2}^{b_2}\int_{a_1}^{b_1} \lambda^{n+1} \left\{1 - \frac{k\lambda}{\lambda - (\mu+\alpha)}\right\}^n e^{-(\mu+\alpha)\chi_l} e^{-\lambda(kT-\chi_l)}\, d\lambda\, d\mu,$$

and

$$I_\mu = \int_{a_2}^{b_2}\int_{a_1}^{b_1} \mu\lambda^n \left\{1 - \frac{k\lambda}{\lambda - (\mu+\alpha)}\right\}^n e^{-k\lambda T}\, d\lambda\, d\mu$$
$$+ \sum_{l=1}^{n} (-1)^l \sum_{1\le i_1 < \cdots < i_l \le n} \int_{a_2}^{b_2}\int_{a_1}^{b_1} \mu\lambda^n \left\{1 - \frac{k\lambda}{\lambda - (\mu+\alpha)}\right\}^n e^{-(\mu+\alpha)\chi_l} e^{-\lambda(kT-\chi_l)}\, d\lambda\, d\mu.$$

As it seems, the above integrals have no closed form solutions. One can use numerical method techniques to derive such integrals.

Two sided Bayesian probability intervals

The two sided $(1-\gamma)100\%$ Bayesian probability interval, shortly $(1-\gamma)100\%$ TBPI, of λ and μ can be obtained using the joint posterior pdf of λ and μ, see Martz and Waller (1982). First, the $(1-\gamma)100\%$ TBPI of λ, say (u_1, v_1), can be obtained by solving the following two equations with respect to u_1, v_1:

$$\frac{1}{I_0} P(a_1, u_1, a_2, b_2) = \gamma/2, \quad \frac{1}{I_0} P(a_1, v_1, a_2, b_2) = 1 - \gamma/2,$$

where

$$P(a_1, w, a_2, b_2) = \int_{a_2}^{b_2}\int_{a_1}^{w} \lambda^n \left\{1 - \frac{k\lambda}{\lambda - (\mu+\alpha)}\right\}^n e^{-k\lambda T}\, d\lambda\, d\mu$$

$$+\sum_{l=1}^{n}(-1)^{l}\sum_{1\le i_1<\cdots<i_l\le n}\int_{a_2}^{b_2}\int_{a_1}^{w}\lambda^{n}\left\{1-\frac{k\lambda}{\lambda-(\mu+\alpha)}\right\}^{n}e^{-(\mu+\alpha)\chi_l}e^{-\lambda(kT-\chi_l)}\,d\lambda\,d\mu.$$

Also, the $(1-\gamma)100\%$ TBPI of μ, say (u_2,v_2), can be obtained by solving the following two equations with respect to u_2,v_2:

$$\frac{1}{I_0}P(a_1,b_1,a_2,u_2)=\gamma/2,\qquad \frac{1}{I_0}P(a_1,b_1,a_2,v_2)=1-\gamma/2,$$

where

$$P(a_1,b_1,a_2,w)=\int_{a_2}^{w}\int_{a_1}^{b_1}\lambda^{n}\left\{1-\frac{k\lambda}{\lambda-(\mu+\alpha)}\right\}^{n}e^{-k\lambda T}\,d\lambda\,d\mu$$

$$+\sum_{l=1}^{n}(-1)^{l}\sum_{1\le i_1<\cdots<i_l\le n}\int_{a_2}^{w}\int_{a_1}^{b_1}\lambda^{n}\left\{1-\frac{k\lambda}{\lambda-(\mu+\alpha)}\right\}^{n}e^{-(\mu+\alpha)\chi_l}e^{-\lambda(kT-\chi_l)}\,d\lambda\,d\mu.$$

The above systems of equations have no closed form solutions in (u_1,v_1) and (u_2,v_2), respectively. So, we have to use numerical method techniques to calculate the $(1-\gamma)100\%$ TBPI for λ and μ.

5. NUMERICAL RESULTS AND CONCLUSION

In this section we present two examples based on large simulation studies. In the first example, we generate a random sample from the underlying model. Then the sample generated is used to compute both MLE and Bayes estimate of the unknown parameters of interest. Furthermore, the asymptotic confidence and two sided Bayesian probability intervals for each parameter are calculated. The two types of estimates are compared based on the percentage error and the length of the interval estimates for each parameters. Also, the marginal posterior pdf and marginal posterior cumulative distribution function (CDF) of each parameter are plotted. The second is presented to study the following: (1) the influence of the sample size on the estimate of each parameter, (2) compare the two procedures used to derive each estimate. The criterions for the comparisons are the mean square errors associated with each estimate.

Example 5.1 The Monte Carlo method is used to generate a random sample from the underlying model when the exact values of the unknown parameters to be estimated are $\lambda=0.40$, $\mu=0.42$ and $\alpha=0.05$, $k=1$ and $m=2$. The sample with size 10 generated is 1.174, 1.704, 4.266, 3.393, 5.029, 1.322, 5.723, 0.561, 5.019 and 5.841. The total time on test $T=\sum_{i=1}^{10}x_i=34.032$ and $\eta=201.467$. The MLE of the parameters are $\hat{\lambda} = 0.588$, $\hat{\mu} = 0.538$ and the associated percentage errors are 46.92% and 28.024%, respectively. The Fisher information matrix is:

$$\mathbf{I} = 10^3 \times \begin{bmatrix} 2.036 & -2.022 \\ -2.022 & 2.036 \end{bmatrix}.$$

The 95% TCI of λ and μ are [0.223, 0.953] and [0.173, 0.903], respectively. For Bayes procedure we assumed the following three different choices of the prior intervals for λ and μ:

(1) choice I: $[a_1, b_1] = [0.01, 0.79]$ and $[a_2, b_2] = [0.02, 0.78]$,
(2) choice II: $[a_1, b_1] = [0.01, 1.00]$ and $[a_2, b_2] = [0.02, 1.00]$,
(3) choice III: $[a_1, b_1] = [0.01, 1.50]$ and $[a_2, b_2] = [0.02, 1.70]$.

Table 1 shows the Bayes estimates, the associated percentage error and the 95% TBPI of each parameter. The percentage error associated with the estimate $\hat{\theta}$ of the parameter θ can be computed by the following relation:

$$PE_{\hat{\theta}} = \frac{|\hat{\theta} - \text{exact value of } \theta|}{\text{exact value of } \theta} \times 100.$$

Table 1. The results obtained from Bayes procedure.

Choice	Bayes estimate		Percentage error		95% TBPI	
	$\tilde{\lambda}$	$\tilde{\mu}$	$\tilde{\lambda}$	$\tilde{\mu}$	λ	μ
I	0.559	0.533	39.826	26.878	[0.278, 0.775]	[0.235, 0.770]
II	0.619	0.594	54.756	41.365	[0.275, 0.965]	[0.231, 0.970]
III	0.695	0.741	73.782	76.446	[0.240, 1.425]	[0.212, 1.622]

Figure 1 shows the marginal posterior pdf of λ and μ for the three choices I, II and III. Figures 1.1, 1.2 and 1.3 are for λ considering the choices I, II and III, respectively, while figures 1.4, 1.5 and 1.6 are for μ considering the choices I, II and III, respectively. Figure 2 shows the corresponding marginal posterior CDF of λ and μ. Based on the results obtained above, one can see that:

(1) The percentage errors associated with the MLE of λ and μ are larger than those associated with the Bayes estimates of λ and μ, considering the choice III.
(2) The percentage errors associated with the MLE of λ and μ are smaller than those associated with the Bayes estimates of λ and μ, considering both choices I

and II.

(3) The confidence limits of the λ and μ become wider when the prior limits of λ and μ become wider.

Thus, for the given sample, Bayes procedure provides better estimates than that obtained by using the maximum likelihood procedure, in the sense of having smaller percentage errors and narrow TBPI, when the exact values of parameters lie close to the middle of limits of the prior intervals of the parameters (choice I). When the prior limits of the parameters become wider, the maximum likelihood procedure provides better estimates than Bayes procedure. This conclusion can be confirmed from the plots shown in figure 1. Since the right tail of the marginal posterior pdf of λ and μ are proportional with the length of the prior limits of λ and μ, as it seems for the choices I and II. Therefore, the Bayes procedure is not recommended unless there is a good enough prior information about the unknown parameters. Otherwise, the maximum likelihood procedure is recommended.

Example 5.2 This example is presented to use Monte Carlo simulation method to compare the performance of the two procedures used based on the mean squared errors and the average of percentage errors associated with estimates obtained. The simulation is carried out according to the following scheme:

1. Specify the values of k, m, λ and μ.
2. Specify the sample size n.
3. Generate a random sample with size n from the lifetime of k-out-of-m cold standby system. This sample can be generated by using the following steps:
   3.1 Generating a random number on the unit interval (0,1), say u.
   3.2 Setting $R(x) = u$ in equation (2.2), we get the following equation:
   $$\frac{k\lambda}{\lambda-(\mu+\alpha)}y^{(k-1)\lambda+\mu+\alpha}-\frac{(k-1)\lambda+\mu+\alpha}{\lambda-(\mu+\alpha)}y^{k\lambda}-u=0\text{ , where } y=e^{-x}$$
   3.3 Solving the above equation with respect to y.
   3.4 The observed value of X is given by $x = -\ln(y)$.
   3.5 Repeat steps 3.1 to 3.4 n times, one gets the random sample $x_1, x_2, \cdots, x_n$.
4. Calculate both MLE and Bayes estimates of the parameters.
5. Repeat steps 3-4 1000 times.
6. Compute the mean squared error, say MSE, associated with each parameter.
7. Repeat steps 2-6 for n=5,10,...,100.

Figure 3 shows the MSE associated with MLE and Bayes estimates of the parameters λ and μ against n, when $\lambda = 0.40$, $\mu = 0.42, \alpha = 0.05$, $k = 1$ and $m = 2$. The prior intervals λ and μ in this example are assumed respectively to be $[a_1, b_1] = [0.01, 0.79]$ and $[a_2, b_2] = [0.02, 0.78]$.

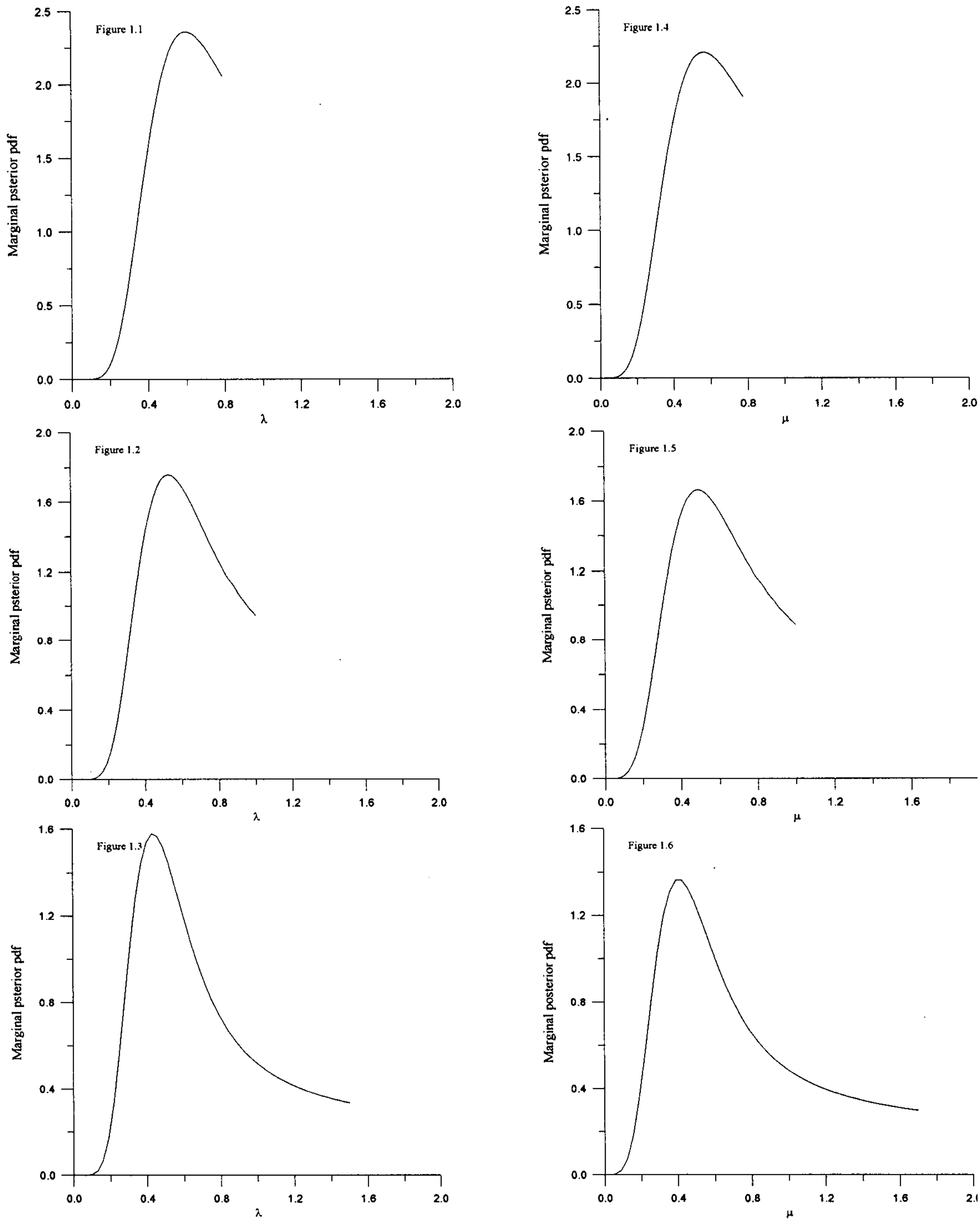

Figure 1. Marginal posterior pdf of λ and μ considering the three different prior intervals.

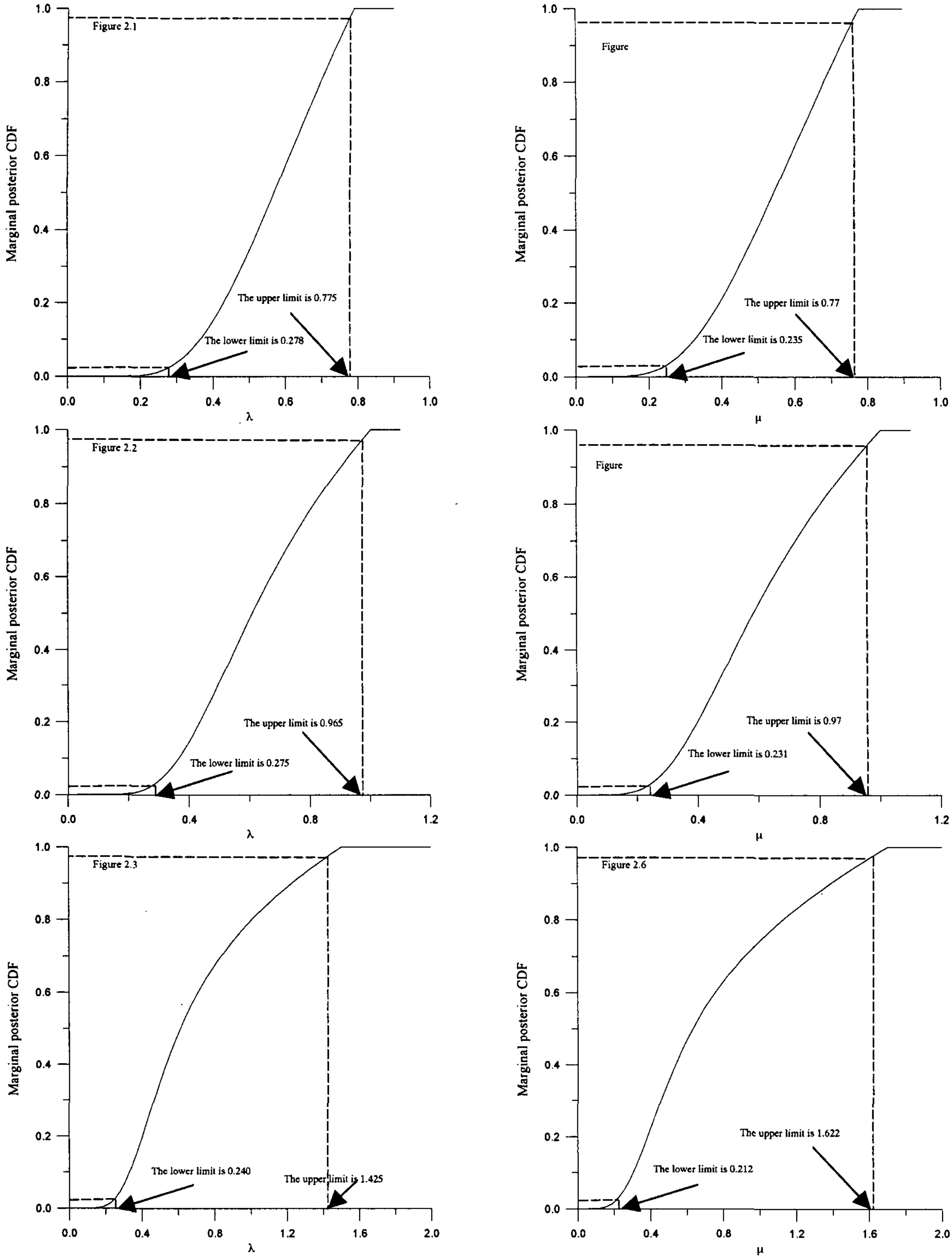

Figure 2. Marginal posterior CDF of λ and μ for different prior intervals.

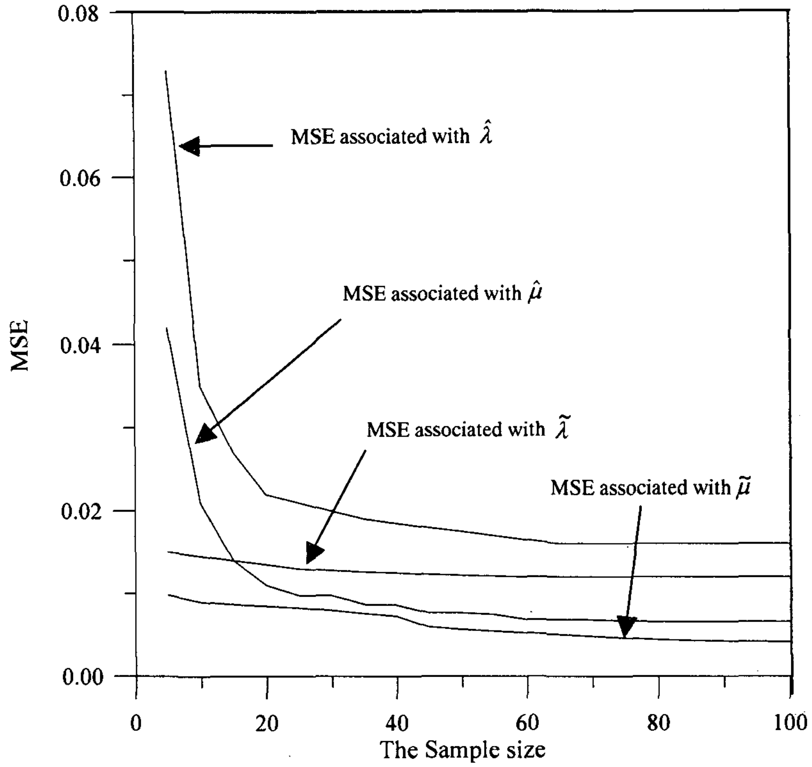

Figure 3. The MSE associated with Bayes estimates and MLE of λ and μ against *n*.

It seems from Figure 3 that

1. The MSE associated with the Bayes estimates of each parameter does not influence so much by increasing the sample size.
2. The MSE associated with the MLE of each parameter decreases dramatically with increasing the sample size when the sample size is smaller than 30. For the samples with sizes greater than or equal to 30 and smaller than or equal to 60, the MSE associated with MLE decreases slowly with increasing the sample size. For the sample size greater than 60 the MSE associated with MLE approximately constant.
3. The MSE associated with Bayes estimate of each parameters is smaller that associated with the MLE of such parameter for all sample sizes.
4. The difference between the MSE associated with Bayes estimate and MLE of each parameter decreases with increasing the sample size, especial when the sample size is small.

Therefore, one can conclude, as it was expected, that the Bayes estimate is better than the MLE of unknown parameter, in the sense of having smaller MSE, especially when the size of the available sample is small.

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