THE JOURNAL OF THE ACOUSTICAL SOCIETY OF KOREA VOL.24, NO.4E 2005. 12 pp. 152~157

Joint Phase and Frequency Offset Estimator for Short Burst MPSK Transmission with Preamble

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(Received November 17 2005; accepted December 21 2005)

Abstract

In this paper, a new data-aided joint phase and frequency estimator, which has very low computational complexity, is proposed and its variances of phase and frequency estimates are derived. To estimate the phase and frequency offset, first of all, the overall observation interval is divided into same length sub-intervals, and then phase estimates are independently computed based on symbols of the each sub-intervals. To be continue the sequence of computed phase estimates, proper integer multiples of 2π are added to (or subtracted from) the computed phase estimates, which is called linearized phase estimate. The phase offset of the proposed joint estimator is estimated by averaging the linearized phase estimates and the frequency offset by averaging the differences between consecutive linearized phase estimates. The variance of the proposed phase offset estimate is same to MCRB of phase if there is no frequency offset, but it is smaller than MCRB of phase if there is frequency offset. However, the variance of the proposed frequency offset estimate is bigger by at least 0.5 dB than MCRB of frequency with the same observation interval.

Keywords: *Frequency Offset Estimation, Phase Offset Estimation, Transmission, Burst, MPSK*

I. Introduction

For a long time, non-coherent transmission scheme, such as FSK, was commonly considered as a unique scheme for underwater acoustic transmission because it is severely hard to acquire phase synchronization and tracking in the underwater acoustic channel which can be characterized as a time-varying multipath channel with fast fading and Doppler effect[1]. After, however, Stojanovic and et. al. demonstrated the feasibility of phase coherent transmission technique in the underwater in [2], many researches are concentrates on the phase coherent transmission schemes, such as DPSK, PSK, QAM due to the bandwidth efficiency[1]. The state-of-art in the underwater acoustic coherent transmission is that the transmission data rate is from several bits per second (bps) to tens of kbps and the transmission range is from several meter to tens of kilometer[3]. The data rate is generally inversely proportional to the transmission range.

For the coherent communication, frequency and phase offsets should be correctly compensated for correct decision of transmitted data because the transmitting data information is contained in the phase of transmitting signal. To obtain phase and frequency offsets in the burst transmission, a short preamble is attached to the beginning of each burst and the feed-forward schemes are generally adopted to remove hang-up phenomena which occur in the feed-back schemes[4~9]. The phase and frequency offsets are estimated independently in a practical system, i.e., there are two dedicated hardwares for phase and frequency estimates. Typically, the phase offset is estimated using maximum likelihood (ML) phase estimator as in [4] and the frequency offset is estimated using data-aided frequency offset estimator as in [5~9]. Furthermore, the phase estimating procedure begins after the frequency offset compensation is

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completed in the conventional synchronization scheme. Therefore, the latency, the time interval from the instance at which the first received signal is enforced to demodulator to the instance at which the first demodulated data is decided, increases and the hardware complexity increases.

In this paper, a new joint phase and frequency estimator for coherent MPSK burst transmission is proposed and the variances of phase and frequency offsets of the proposed joint estimator are derived under high signal-to-noise ratio (SNR) assumption and compared with modified Cramer-Rao bounds (MCRB) of phase offset and frequency offset. The proposed joint estimator concurrently estimates the phase and frequency offsets and shares lots of operations required to obtain the estimates, so that the latency is minimized and the hardware complexity is reduced.

This paper is organized as follows; in Section II, we describe the ML phase offset estimator and derive its variance when there is a small frequency offset and the received SNR is high. In Section III, we explain the proposed joint frequency and phase offset estimate, derive the variances of frequency offset estimate and phase offset estimate, and derive the optimal length of sub-observation length to obtain the interim phase estimates. In Section IV, the simulations are performed to verify the derived estimate variance for the small frequency difference between the local oscillator of transmitter and receiver case. We, finally, conclude the paper in Section V.

II. ML Phase Offset Estimation with Frequency Offset

The followings are assumed: the symbol timing is ideal; the normalized frequency offset is much smaller than 1 ($f_dT \ll 1$); and the received symbol sequence is a known sequence (preamble). The k-th received signal is represented as

$$r(kT) = c_k e^{j(2\pi k f_d T + \theta_0)} + n(kT) \tag{1}$$

where c_k is the known MPSK signal at the k-th symbol interval and $|c_k|^2 = 1$, θ_0 is a constant phase over observation interval, T is the symbol period, f_d is the frequency offset which is much less than $1/T$, and $n(kT)$ is the additive white complex Gaussian noise whose real and imaginary components have zero mean and variance of $N_0/2E_s$, respectively, and independent each other. It is well known that, for the observation length L, the ML phase offset estimator is given as[4]

$$\widehat{\theta_{0,ML}} = \arg\left\{ \sum_{k=0}^{L-1} c_k^* r(kT) \right\}. \tag{2}$$

To reveal the significant effect of frequency offset in the phase estimate, substituting (1) into (2) obtains

$$\widehat{\theta_{0,ML}} = \arg\left[\{1 + V_L\} \rho_L(f_{dT}) e^{j(\pi(L-1)f_dT + \theta_0)} \right] \tag{3}$$

where

$$\rho_L(f_dT) \equiv \frac{\sin(\pi L f_d T)}{L \sin(\pi f_d T)}, \tag{4}$$

$$V_L \equiv \frac{e^{-j\pi(L-1)f_dT + \theta_0)}}{\rho_L(f_dT)} \cdot \frac{1}{L} \sum_{k=0}^{L-1} c_k^* n(kT), \tag{5}$$

V_L is a complex Gaussian random variable with zero mean and the variance of $N_0/(2L\rho_L^2(f_dT)E_s)$. Assuming high SNR, (3) is approximated as

$$\widehat{\theta_{0,ML}} \approx [\theta_0 + \pi f_d T(L-1) + Im\{V_L\}] \mod 2\pi \tag{6}$$

where $Im\{V_L\}$ is the imaginary part of V_L and $(x \mod 2\pi)$ means that $x$is folded into $[-\pi, \pi)$. From (6), it is clear that the mean of the phase offset estimate coincides with the phase at the center of the observation interval which corresponds $t = (L-1)T/2$, that is, the mean of the estimated phase offset is biased by the amount of $\pi f_d T(L-1)$, which gives a clue of the proposed frequency offset estimator: the mean of the difference between two adjacent phase offset estimates includes a term of a function of f_d and L. Also, under high SNR assumption, the variance of ML phase offset estimate is well known as[4]

$$var(\widehat{\theta_{0,ML}} - \theta_0) = \frac{1}{2L\rho_L^2(f_dT)E_s/N_0}. \tag{7}$$

Since $\rho_L(f_dT)$ has its maximum of 1 at $f_dT = 0$ and decreases as f_dT departs from 0, the variance of phase offset estimate is larger than MCRB of θ_0 except for $f_dT = 0$.

III. Joint Phase and Frequency Offset Estimator

Based on the discussion in Section II, we propose a new joint phase and frequency offset estimator, which estimates the frequency and phase offset from the sliced phase offset estimates as depicted in Figure 1.

The overall observation interval, LT ($L=NM$), is sliced into N sub-observation intervals spanning over MT, and then phase estimates are independently estimated on each sub-observation interval. Since the result of $\arg(\cdot)$ is confined within $[-\pi, \pi)$, if the phase estimate on the i-th sub-observation interval, $\theta_M(i)$, is close to $\pm\pi$, the difference between the i-th and the (i-1)-th phase estimates, $\Delta\theta_{M(i)}$, might be approximately $\pm 2\pi$, which introduces large phase offset estimation error. Subsequently, the estimated phases are linearized to keep the phase continuous over the border of $\pm\pi$ as follows:

$$\widehat{\theta_M}(i) = \widehat{\theta_M}(i-1) + \Delta\widehat{\theta_M}(i) \tag{8}$$

where

$$\Delta\widehat{\theta_M}(i) = \begin{cases} \Delta\theta_M(i) & , \text{ for } |\Delta\theta_M(i)| < \pi \\ \Delta\theta_M(i) + C & , \ otherwise \end{cases} \tag{9}$$

$$\Delta\theta_M(i) = \theta_M(i) - \theta_M(i-1), \tag{10}$$

$$C = -2\pi \cdot sgn(\Delta\theta_M(i)), \tag{11}$$

$\theta_M(0)=0$ and $sgn(x)$ is -1 when $x<0$ or 1 when $x \geq 0$. Under high SNR, the linearized phase estimate on the samples of the i-th sub-observation interval can be represented as

$$\widehat{\theta_M}(i) \approx \theta_0 + 2\pi f_d T\left\{iM + \frac{M-1}{2}\right\} + Im\{V_M(i)\}, \tag{12}$$

where $V_M(i)$ has identically independent distribution and the statistical characteristics of $V_M(i)$ are same to those of V_L in

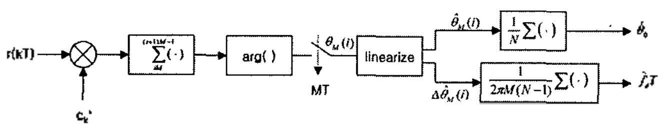

Figure 1. Block diagram of proposed joint phase and frequency offset estimator, based on linearizing phase estimates $\widehat{\theta_{M(i)}}$.

(5) except M replacing L. Note that (12) dose not include mod 2π function in (6).

By averaging the linearized phase estimate, the phase offset estimate can be computed as

$$\widehat{\theta}_0 = \frac{1}{N}\sum_{i=0}^{N-1}\widehat{\theta_M}(i) \tag{13}$$

and, under high SNR assumption, it is approximated as

$$\widehat{\theta}_0 \approx \theta_0 + \pi f_d T(N-1) + \frac{1}{N}\sum_{i=0}^{N-1} Im\{V_M(i)\}. \tag{14}$$

From (6) and (14), the proposed and ML phase offset estimators have the same bias. Under high SNR assumption, the variance of phase offset estimate is computed as

$$var(\widehat{\theta}_0 - \theta_0) = \frac{1}{2L\rho_M^2(f_d T)E_s/N_0}. \tag{15}$$

Note that the ratio of the variance of proposed phase offset estimate to that of ML one in (7) becomes $\rho_L^2(f_d T)/\rho_M^2(f_d T) \leq 1$. Therefore, the variance of proposed phase estimator has smaller variance when $f_d T \neq 0$.

The normalized frequency offset can be estimated using the phase difference between consecutive linearized phase estimates as

$$\widehat{f_d}T = \frac{1}{2\pi M(N-1)}\sum_{i=1}^{N-1}\Delta\widehat{\theta_M}(i) \tag{16}$$

Under high SNR assumption, $\Delta\widehat{\theta_M}(i)$ is approximated as

$$\Delta\widehat{\theta_M}(i) \approx 2\pi M f_d T + Im\{V_M(i)\} - Im\{V_M(i-1)\} \tag{17}$$

and the normalized frequency offset estimate is also approximated as

$$\widehat{f_d}T \approx f_d T + \frac{Im\{V_M(N-1)\} - Im\{V_M(0)\}}{2\pi M(N-1)}. \tag{18}$$

Since $\Delta\widehat{\theta_M}(i)$ is restricted to the interval $[-\pi, \pi)$, an operating range is limited to $|f_d T| < 1/2M$. Note that (18) includes two $V_M(i)$ terms only at the boundaries, i.e., $i=0$ and $i=N-1$, while summing up $\Delta\widehat{\theta_M}(i)$ from $i=1$ to $i=N-1$. Subsequently, according to the statistic characteristics

Table 1. The required number of multiplications and $\arg(\cdot)$ operations for estimating the frequency offset, when the overall observation length is L-symbol.

Method	number of multiplication	number of $\arg(\cdot)$
Kay	$9L-4$	$L-1$
Fitz	$\frac{L(3L+10)}{2}+1$	$L/2$
L&R	$\frac{L(3L+10)}{2}+1$	1
Proposed	$4L+1$	3

of $V_M(i)$, it is clear that frequency offset estimate is unbiased and, under high SNR, its variance is derived as

$$var(\hat{f}_d T - f_d T) = \frac{1}{4\pi^2 M^3 (L/M-1)^2 \rho_M^2 (f_d T)} \cdot \frac{1}{E_s/N_0}. \quad (19)$$

From (19), the error variance of the frequency offset estimate is a function of L, M, and E_s/N_0. If overall observation interval and SNR are once fixed, the variance of frequency offset estimate can be reduced by properly choosing sub-observation interval, M. The optimum M satisfies the following relation to minimize the variance as

$$L = \frac{M + 2M^2 \pi f_d T \tan^{-1}(M\pi f_d T)}{2M\pi f_d T \tan^{-1}(M\pi f_d T) - 1}. \quad (20)$$

After applying $\tan^{-1}(x) \approx x^{-1}$, the relation is approximately simplified into $M = L/3$. One should recognize that M should be a factor of L but $L/3$ might not be a factor of L. Therefore, we choose M as one of factors of L which is closest to $L/3$ to minimize the variance of frequency offset estimate. For example, when the observation interval L is 16, we choose M to 4. If the variance of the proposed frequency offset estimator in (19) with $M = L/3$ is compared with MCRB of the normalized frequency estimate in[4], the proposed frequency offset estimator has a larger variance about 0.5 dB, i.e., the proposed frequency offset estimator requires more than 0.5 dB symbol energy to get the same variance of MCRB on the same observation length.

To compare the computational complexity of the proposed scheme with that of the conventional frequency offset estimate schemes, such as, Kay[5], Fitz[6], and L&R method[7], the number of real multiplications and the $\arg(\cdot)$ operations for each schemes are counted when the overall observation length for frequency offset estimation is L.

Because multiplication and $\arg(\cdot)$ operation are much more complex operation than addition and/or subtraction. The counting results for each methods are listed in Table 1. In counting the required number of multiplication, one complex multiplication is counted as the four real multiplication. For Kay method[5], the smoothing function coefficients are computed in advance. For Fitz method[6] and L&R method[7], the computational complexity and the achievable minimum variance is directly related to the number of autocorrelations used for estimating the frequency offset in each methods. Since both methods have the minimum variance of frequency offset when the number of autocorrelations is $L/2$. In counting the required operations for both methods, this case is assumed. For the proposed method, it is assumed that L is an integer multiple of 3. As shown in Table 1, the computational complexity of Kay and the proposed methods are proportional to the observation length L and that of Fitz and L&R methods are proportional to $O(L^2)$. In the sequel, the proposed method requires the minimum computational complexity, that is, $4L$ multiplications for multiplying complex conjugate of preamble by the received signal, one multiplication for scaling, and 3 $\arg(\cdot)$ operations for computing the phase. Furthermore, if the ML phase offset is also estimated, one multiplication and one $\arg(\cdot)$ are required for conventional methods but only one multiplication for scaling is additionally required for proposed method. Therefore, the proposed method is the simplest frequency and phase offset estimating method.

IV. Simulation Results

The variances of phase and frequency offset estimator of the proposed joint scheme are investigated via computer simulation.

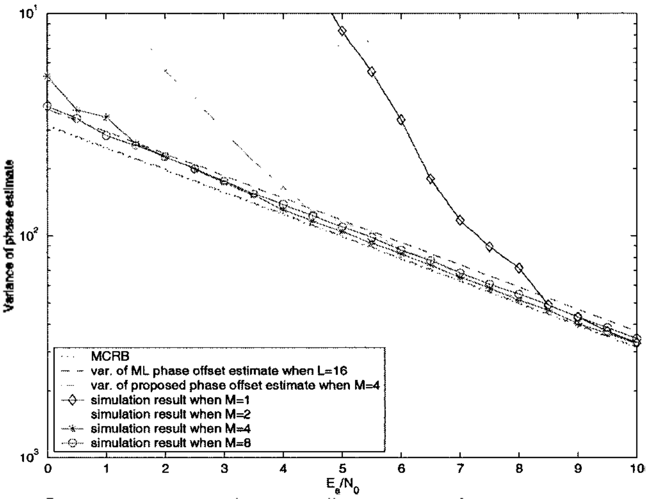

Figure 2. Variance of phase offset estimate for $L = 16$.

The transmitted frame is composed of two parts, that is, the preamble symbol, which is 16 symbol length CAZAC (constant amplitude and zero auto-correlation) symbol sequence[10], and the random QPSK symbol parts. We set the simulation parameter as the phase offset θ_0 is $\pi/2$, the normalized frequency offset $f_d T$ is 0.02, and the overall observation length L for estimations is 16. The variance of phase offset estimate and the variance of frequency offset estimate are investigate according to the divided observation length.

Figure 2 shows the variances of phase estimate according to M. The variances of phase estimate are diverged from the theoretically expected variance over the lower than a some SNR due to the non-linear operation in computing the $\arg(\cdot)$ operation. As shown in Figuer 2, the diverging threshold SNR tend to smaller as M increases. For example, the diverging threshold SNR for $M=1$ case is about 9 dB SNR and that for $M=2$ case is about 4.5 dB SNR. For $f_d T \neq 0$, the proposed phase offset estimator shows smaller variance than ML one if the SNR is larger than the diverging threshold SNR for each M. Figure 3 shows the average of the normalized frequency offset estimates at 5 dB SNR for $L=16$ and $M=4$ or $M=8$ (solid curves) and the estimates without noise (dotted curves). As expected, the frequency offset estimate is unbiased for small normalized frequency offset ($|f_d T| < 1/2M$), and as M increases, the proposed algorithm has a narrower operating range. Figure 4 shows theoretic and simulated error variance of frequency offset estimator at $f_d T = 0.02$ where MCRB of $f_d T$ is also depicted as a reference. The theoretic error variances coincide with simulation results for practically interesting SNR region and, as expected, $M=4$ case has minimum variance with about 0.7 dB larger variance than MCRB. As M increases, theoretic and simulated error variance curves meet at lower SNR due to the non-linear operation in computing argument. Note that $M=1$ corresponds to Kay's method[5] except for the use of equal weighting. If a reader is interesting in very low SNR operating environments, such as lower than 0 dB SNR for QPSK modulation or lower than -3 dB SNR for BPSK modulation, one can use asymptotic CRB (ACRB) instead of MCRB as a lower bound, which is much closer to the true CRB at very low SNR[11].

V. Conclusions

In this paper, we proposed a new joint phase and frequency offset estimator and derived its error variances. For the phase offset estimate, ML-scheme and the proposed scheme have same bias but the proposed scheme has a smaller estimation variance than the ML one when the frequency offset is non-zero. For the frequency offset estimate, the proposed scheme is unbiased scheme for small normalized frequency offset ($|f_0 T| < 1/2M$) and it has a bigger variance by more than 0.5 dB than MCRB. However, the proposed frequency offset estimator requires much smaller computational complexity than the conventional ones. Furthermore, the proposed joint estimator shares lots of operations required to obtain estimates and can be simply implemented by slightly modifying the conventional ML phase estimator with simple additional arithmetic units. Therefore, the proposed joint estimator can be used to get the phase and the frequency offsets concurrently for coherent burst MPSK

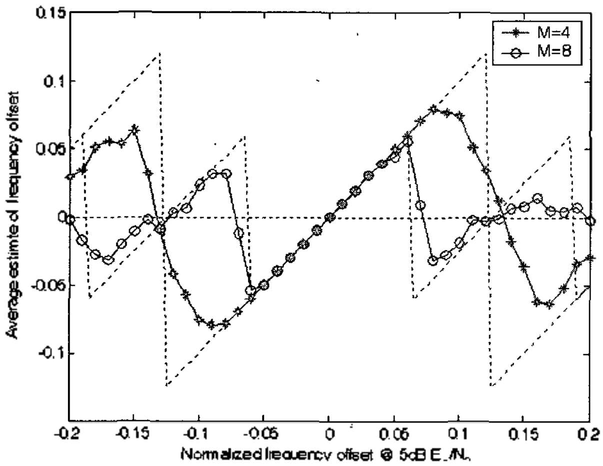

Figure 3. Average of normalized frequency offset estimate versus normalized frequency offset for $L=16$ at 5 dB SNR.

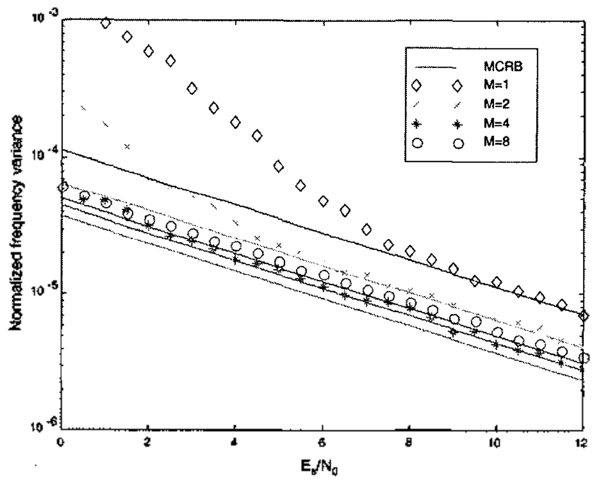

Figure 4. Variance of normalized frequency offset estimate versus SNR for $L=16$.

transmission with very low complexity.

Acknowledgements

This work is supported in part by "Development of Underwater 3-D Fusion Image Generation, Detection, and Transmission Techniques" of NRL program and "Development of Underwater Acoustic Wireless Communication Network" project of MOMAF.

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