

# Design of Reinforced Concrete Members for Serviceability Based on Utility Theory

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## Abstract

A methodology for design of reinforced concrete members for serviceability in general and deflection control in particular is presented based on application of utility theory. The approach is based on minimizing total cost including both initial construction and cost of failure considering variability in structural behavior and various forms of serviceability loss function. The method is demonstrated for the case of a simply supported slab for example.

*Keywords: Utility Theory, Serviceability, Deflection Control, Reinforced Concrete*

## 1. INTRODUCTION

Ultimate limit states are generally well defined by discrete limit states at which discontinuities in structural behavior occur. Serviceability limit states such as deflection and cracking on the other hand are generally not well-defined. Probability-based design requirements for strength are now well established and are embodied in modern building codes and design specifications in the form of load and resistance factors or partial factors. Deflection and crack control requirements are still handled by empirical rules based largely on previous experience.

In the design of concrete building structures, deflection control for floors and roofs is an important design consideration. While the current code procedures have provided adequate designs in the past, developments in design practice such as the use of higher strength materials and longer spans leading to more flexible structures, as well increasing expectations by owners for building performance suggest that a more rational approach to design for deflection control may be required in the future. Such an approach should consider the uncertainties inherent in predicting deflections of concrete members and structures as well as the difficulties associated with defining acceptable limits for deflection of members. This paper explores the application of utility theory to the problem. Since serviceability failure can occur in structures with adequate safety against collapse, the question becomes an economic issue. The utility theory approach balances the initial cost of construction against the potential costs of repair considering uncertainties associated with structural behavior at service load levels, and lack of a well-defined limit for deflection.

The formulation of the approach is based on the work of Reid and Turkstra (1980, 1981) and Turkstra and Reid (1981) at McGill University. A deterministic model is used to calculate deflections for a member with defined time-

dependent material properties and loading history. Monte Carlo simulation is then used to develop histograms of deflection with assumed statistical distributions for the input parameters. A loss function is then defined that specifies the onset of damage due to deflection and an upper limit at which the structure is assumed to be unusable. Both discrete (or step-wise) functions and continuous functions are considered in the applications. Utility theory is then applied to maximize utility for the member by minimizing the total cost considered as the sum of the initial cost and the probabilistically determined cost of failure. In this study two types of damage are considered explicitly; perception of deflection and damage to non-structural elements.

The paper presents the basic formulation of the problem based on utility theory and discusses the types of serviceability loss function considered. Costs of construction and costs of failure are then discussed. The application of the method is illustrated through a one-way slab example for which the optimum (maximum utility, minimum total cost) thickness of the slab is calculated. Suggestions for further research to fully develop the capability of the approach are presented.

## 2. UTILITY THEORY APPROACH

Following the formulation presented by Reid and Turkstra (1980, 1981), serviceability is considered as a specific type of structural utility,  $U$ , which can be expressed as:

$$U = B - C_I - \sum_i c_{F_i} \quad (1)$$

where,

$B$  = benefit derived from fully serviceable structure

$C_I$  = initial construction cost

$c_{F_i} = C_{F_i} \times F\left(\frac{\Delta_i}{L}\right)$ , cost due to failure in mode  $i$

$C_{F_i}$  = cost of failure due to being completely

unserviceable in mode  $i$

$\Delta_i$  = deflection in mode  $i$

$L$  = span length of a member

$F(\frac{\Delta_i}{L})$  = serviceability loss function as a function of deflection to span length ratio in mode  $i$ .

Mode  $i$  corresponds to the various structural effects such as elastic deflection, creep, and shrinkage. The cost due to

failure in mode  $i$  is calculated by multiplying  $F(\frac{\Delta_i}{L})$ ,

serviceability loss function by  $C_{F_i}$ , the cost of failure

due to being completely unserviceable. Forms of the serviceability loss function are discussed below.

Considering failure in a particular mode  $i$ , the utility function may be expressed as:

$$u(\frac{\Delta_i}{L}) = B - C_I - c_F(\frac{\Delta_i}{L}) \quad (2)$$

where,

$u(\frac{\Delta_i}{L})$  = utility function for failure mode  $i$

$c_F(\frac{\Delta_i}{L})$  = failure cost function for failure mode  $i$

Noting that  $\Delta_i$  is a random variable, a generalized

measure of structural utility with regard to failure mode  $i$  is

expected utility,  $E[U_i]$ . Expressing  $\frac{\Delta_i}{L}$  as  $x_i$  in Eq. (2),

the expected utility is defined as:

$$E[U_i] = \int_{-\infty}^{\infty} u(x_i) f(x_i) dx_i \quad (3a)$$

where,  $f(x_i)$  = probability density function of  $x_i$ .

Substituting Eq. (2) into Eq. (3a) gives;

$$E[U_i] = B - C_I - C_F \left[ \int_{-\infty}^{\infty} F(x_i) f_{x_i}(x_i) dx_i \right] \quad (3b)$$

Eq. (3b) can be converted into discrete type as follows:

$$E[U_i] = B - C_I - C_F \sum_{i=1}^n F(x_i) p(x_i) \quad (3c)$$

where,  $p(x_i)$  is probability mass function of  $x_i$  in a given member.

In order to evaluate the expected utility, first, the utility should be defined as a function of a variable and second, the probability mass function of that variable should be determined based on appropriate load-time history and structural response models. For a given member, if the

benefit associated with a fully serviceable structure is taken as a constant, the minimized total cost consisting of initial construction cost plus cost of serviceability failure can be obtained so as to maximize the value of utility. The initial construction cost and cost of failure can be assumed to vary with some structural parameter such as the depth of a member,  $h$ .

If the member depth is taken as the structural parameter, the initial construction cost will generally increase with increasing size of member while the cost of serviceability failure due to excessive deflection can be expected to decrease as the member depth increases. As illustrated in Fig. 1, the optimum member depth can be determined where the total cost function is a minimum. The initial cost function can be determined by designing a series of beams with varying depths, computing weights or volumes of resulting materials and using available unit costs for materials in place to determine the cost for each member design, while the serviceability failure costs can be estimated by assuming serviceability loss functions and applying these functions to histogram of deflection to span length ratio of each member design that can be obtained from probabilistic approaches such as Monte Carlo simulation. An expected value of utility with respect to a particular mode of failure is completely defined by relevant serviceability loss functions and histograms of the utility parameter. Particular characteristics of serviceability loss functions associated with a discontinuous serviceability loss function and a continuous serviceability loss function are discussed in the following section.

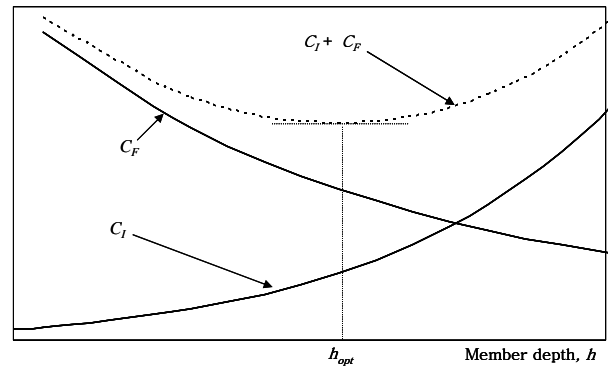


Figure 1. Function of member depth,  $h$ .

### Discontinuous serviceability loss function

A discontinuous serviceability loss function is a function that takes stepped values corresponding to the bounds at discrete failure points on its parameter such as deflection ratio to span length. For example, one-step discontinuous serviceability loss function,  $h(x_i)$  discontinuous at a failure point,  $x_{f_i}$  is expressed in Eq.(4).

$$u(x_i) = B - C_I - C_F \sum_{i=1}^n p(x_i) \cdot h(x_i) \quad (4)$$

$$\text{where, } h(x_i) = \begin{cases} 0 & \text{when } x_i < x_f \\ 1 & \text{when } x_i > x_f \end{cases}$$

Fig. 2 shows a deflection histogram and discontinuous serviceability loss function conceptually. The histogram developed by Monte Carlo simulation describes the variability of deflection for a particular member. The probability of obtaining a value associated with a particular cell is  $p(x)$ . The loss function  $h(x)$  is taken to be zero up to a specified threshold value and unity beyond that value. A complete serviceability failure is assumed to occur at the specified threshold value. The cost of failure increases as the overlap between the loss function and the histogram increases. In other words as the threshold value increases the cost of failure decreases.

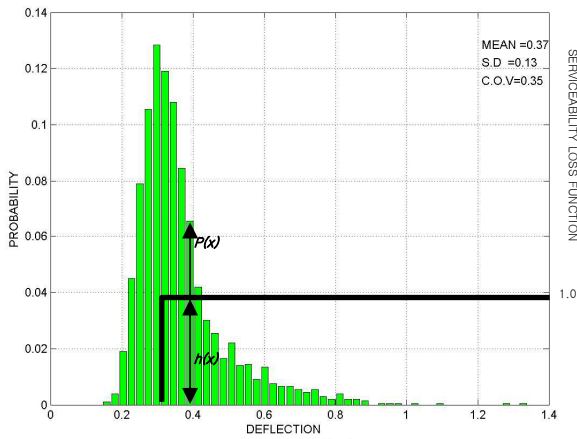


Figure 2. Probability histogram of deflection and discontinuous serviceability loss function.

### Continuous serviceability loss function

In most cases serviceability failure does not occur suddenly at a particular threshold value. In general a deflection value can be defined below which no damage or failure is detected. As the deflection increases beyond this value the extent of damage also increases until a stage is reached at which the structure can be considered to be completely unusable. Fig. 3 shows a histogram for deflection of a given beam. Superimposed on the figure is a continuous serviceability loss function which varies from zero at a particular deflection limit to unity at a higher deflection limit. The lower limit can be considered as the deflection limit at which no damage is detected in which case there is no failure cost. The upper limit represents the onset of a completely unserviceable structure with an associated cost to remedy. Such a representation reflects the fact that there is usually not a crisp delineation between a serviceable and unserviceable structure. The costs of failure include not only construction costs of repairs but also costs of loss of production to the owner during repairs. One could also consider the cost of loss of reputation to the structural engineer as a result of the failure. As was the

case for the discontinuous loss function, the greater the overlap between the deflection density function and the serviceability loss function, the higher will be the expected cost of failure.

Based on the results of surveys of buildings estimates of deflection corresponding to the onset of damage or failure due to deflection can be obtained. Hossain and Stewart (2001) reviewed survey data and proposed probabilistic models of damaging deflections for floor elements. They classified the models into two kinds: partition wall damage and perception damage. Perception damage is defined as deflection sagging in a structural element that is noticeable to occupants and disturbing to people causing problems such as excessive curvature in floor with visual sagging, slanting furniture and floor finishing damage. Partition wall damage model is defined as damage to non-structural partition walls due to floor deflection. According to their study, the minimum value of the deflection to span length ratio for which perception damage was reported is 0.003. An earlier study by Mayer and Rusch (1967) also concluded that a deflection to span length ratio of up to 1/300 is not found to be visually disturbing. Table 1 shows the statistics obtained by Hossain and Stewart based on their review of deflection survey data. These data can be used to specify the upper and lower limits of the serviceability loss function and the cumulative density function can be used to define the loss function between the two limits.

Table 1. Statistical parameters of damaging deflections ( $\Delta/L$ ) (Hossain and Stewart 2001).

| Parameter     | Perception damage   | Partition wall damage |
|---------------|---------------------|-----------------------|
| Sample size   | 60                  | 51                    |
| Minimum value | 0.0030              | 0.0006                |
| Maximum value | 0.0171              | 0.0135                |
| Mean          | 0.0077              | 0.0054                |
| C.O.V         | 0.42                | 0.57                  |
| Distribution  | Truncated lognormal | Gamma                 |

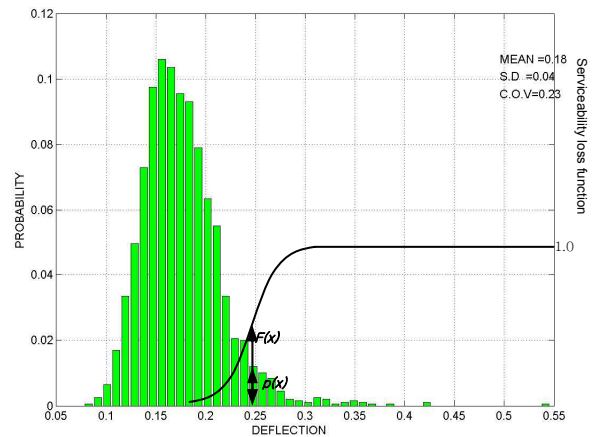


Figure 3. Probability histogram of deflection and continuous serviceability loss function.

### 3. COST INFORMATION

Data for the cost of initial construction and cost of failure are needed for implementation of the utility theory approach described earlier. For this study, RS Means building construction cost data (2002) is used for calculating cost of initial construction and RS Means repair and remodeling cost data (2002) for cost of failure. The cost due to loss of production is determined based on the national compensation survey data published by Bureau of Labor Statistics, U.S Department of Labor (2003).

#### Cost of initial construction

The cost of initial construction cost consists of formwork, reinforcement, concrete, finishing, curing, installation of reinforcement, pouring the concrete and shoring. Each unit cost includes the overhead and the profit of contractors. Table 2 shows the unit prices of each item based on Year 2002 data. To simplify the problem, only the cost of the floor itself was considered. For a complete analysis, the effect of varying the floor depth on column and foundation costs should also be considered.

Table 2. Initial construction cost data.

| Item   | Unit price  | Remarks   |
|--|---|---|
| Formwork, \$/m <sup>2</sup> (\$/ft <sup>2</sup> )  | 60.00 (5.40)  | Plywood to 4.57 m (15') high, 3 use                                 |
| Reinforcement, \$/ton[metric] (\$/ton[short])      | 1444.44 (1,300.00)  | Grade 60, A615  |
| Concrete, \$/m <sup>3</sup> (\$/C.Y)               | $f'_c = 20.69$ MPa (3,000 psi) 100.00 (76.00)                                     | Concrete ready mix, Normal weight                                   |
|  | $f'_c = 27.58$ MPa (4,000 psi) 107.24 (81.50)                                     |   |
| Placing concrete, \$/m <sup>3</sup> (\$/C.Y)       | $h < 15.24$ cm (6") 32.90 (25.00)   | Pumped  |
|  | $15.24\text{cm} \leq h \leq 25.4\text{cm}$ (6" $\leq h \leq 10$ " ) 28.95 (22.00) |   |
|  | $h > 25.4$ cm (10" ) 25.40 (19.30)  |   |
| Finishing, \$/m <sup>2</sup> (\$/ft <sup>2</sup> ) | 25.40 (1.10)  | Integral topping and finish, using 1:1.2 mix, 0.48 cm (3/16") thick |
| Curing, \$/m <sup>2</sup> (\$/ft <sup>2</sup> )    | 6.11 (0.55)   | Curing blankets   |
| Shoring  | \$78.00/EA.   | 3538.02 kg (7800 #) capacity  |

#### Loss of production

In general, the disruption cost of serviceability failure comprises the loss of production during the time required for repair. An upper bound to this cost is assumed to correspond to roughly 4 weeks of lost production for the affected work area, which is twice the failed floor area. A monetary estimate of this loss is obtained by assuming that the monetary value of the productivity of an office worker is the associated payment received by the worker. According to the survey data conducted by Bureau of Labor Statistics, U.S. department of Labor (2002), annual average earning of full-time workers is approximately \$36,484. Assuming that an office worker occupies roughly 150 ft<sup>2</sup> of office floor area, a typical office production rate is roughly \$243/yr./ft<sup>2</sup> of serviceable floor area. This amount is converted to \$20.25/4 weeks/ft<sup>2</sup>. Assuming that the area disrupted for repair is twice the failed floor area (Reid and Turkstra, 1981) this unit cost should be multiplied by 2. If the deflection is greater than  $\frac{L}{240}$ , it is assumed that the

level below will also be affected by the repairs. Thus, a reasonable upper bound to the disruption cost of serviceability failure is estimated to be \$81/ft<sup>2</sup> of failed floor area. For the loss of production case a two-step discontinuous loss function is assumed as shown in Fig. 4.

The expected loss of production cost is expressed as:

$$E(c_{LF}) = C_{LF} \sum_{i=1}^n p\left(\frac{\Delta_i}{L}\right) \cdot h\left(\frac{\Delta_i}{L}\right) \quad (5)$$

where,

$$C_{LF} = \begin{cases} 0, & \Delta_i / L < 0.003 \\ \$40.5, & 0.003 < \Delta_i / L < (\lambda / 240) \\ \$81.0, & \Delta_i / L > (\lambda / 240) \end{cases}$$

= cost of loss of production

$n$  = number of simulations or classes

$p\left(\frac{\Delta_i}{L}\right)$  = probability of deflection ratio to span length

$$h\left(\frac{\Delta_i}{L}\right) = \begin{cases} 0, & \Delta_i / L < 0.003 \\ 1, & 0.003 < \Delta_i / L < (\lambda / 240) \\ 1, & \Delta_i / L > (\lambda / 240) \end{cases}$$

#### Cost of failure for repair

This part of cost of failure is a direct cost for repair, which consists of cut-out demolition cost and replacement cost including material cost, labor cost and contractor's overhead & profit. Detailed cost data based on RS Means (2002) are presented in Table 3. The expected cost of failure for repair can be obtained by using the continuous serviceability loss function. To calculate the expected cost of failure for repair  $E(c_{RF})$  with histogram of deflection, following equation is applied:

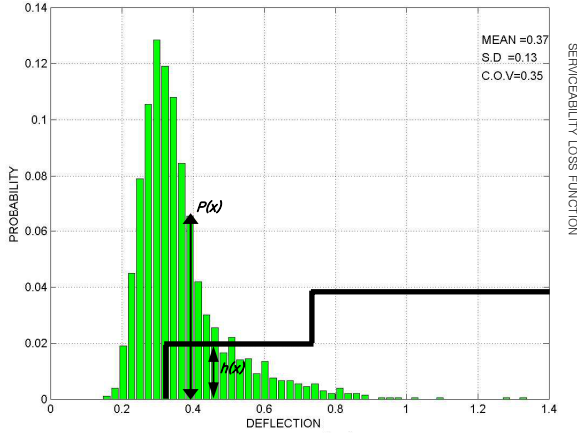


Figure 4. Two-step discontinuous serviceability loss function and histogram of deflection.

$$E(c_{RF}) = C_{RF} \sum_{i=1}^n p\left(\frac{\Delta_i}{L}\right) \cdot F\left(\frac{\Delta_i}{L}\right) \quad (6)$$

where,  $C_{RF}$  is cost of failure for repair due to fully unserviceable and  $F(x)$  is the continuous serviceability loss function as described earlier.

Table 3. Repair cost data.

| Type | Item   | Unit price                                 |                 |
|------|--|--|-----------------|
|      | Cutout demolition, \$/m <sup>3</sup> (\$/ft <sup>3</sup> ) | ≤ 0.17 m <sup>3</sup> (6 ft <sup>3</sup> ) | 1395.76 (39.50) |
|      |  | > 6 ft <sup>3</sup> (0.17 m <sup>3</sup> ) | 1254.42 (35.50) |
| Slab | Replacement, \$/m <sup>2</sup> (\$/ft <sup>2</sup> )       | 17.78 cm (7") thickness                    | 168.89 (15.20)  |
|      |  | 20.32 cm (8") thickness                    | 179.44 (16.15)  |
|      |  | 22.86 cm (9") thickness                    | 187.78 (16.90)  |
|      |  | 25.4 cm (10") thickness                    | 198.89 (17.90)  |

#### 4. EXAMPLE OF APPLICATION TO A ONE-WAY SLAB

To demonstrate the application of the method, the optimum thickness for a one-way slab is considered as an example. To obtain the histogram of computed deflections a deterministic model and a set of statistics for input parameters are needed to perform Monte Carlo simulation.

##### Deterministic deflection model

A layered beam finite element model developed by Choi et al. (2004) provides the basis for the deterministic model.

Concrete is assumed to be linear elastic in tension and compression under immediate loading. A linear post-peak decreasing branch in stress-strain diagram is considered to model the tension stiffening effect of concrete between cracks. If the tensile strength in a given layer is exceeded in each element, the analysis is repeated with a reduced modulus of elasticity. This process is repeated until the stress remains within the stress-strain envelope. The slope of the post-peak branch is defined by a tension stiffening parameter  $\beta = \epsilon_{tu} / \epsilon_{ti}$ .

Creep under sustained load is considered using the age-adjusted effective modulus proposed by Trost (1967):

$$E_{ca} = \frac{E_{ci}}{(1 + \chi \phi_t)} \quad (7)$$

where,  $E_{ci}$  is the instantaneous modulus of elasticity,  $\chi$  is the aging coefficient and  $\phi_t$  is the creep coefficient at time  $t$  defined as the ratio of creep strain at time  $t$  to initial elastic strain.

For the creep coefficient, a simple model presented by ACI Committee 209 (1992) based on Branson's work (1963) is used for standard conditions as follows:

$$\phi_t = \frac{t^{0.6}}{10 + t^{0.6}} \phi_u \quad (8)$$

where,  $\phi_u$  is the ultimate creep coefficient and  $t$  is the time after loading.

Also, for the effect of shrinkage, simple procedure suggested by ACI Committee 209 (1992) is adopted in this study. A typical two-dimensional beam element with three degrees of freedom at each end (one rotation and two displacements) was used to form the finite element model.

##### Parameter variability

Material properties and dimensions are assumed to be random variables with the statistical values listed in Table 4.

##### Loads

In order to compute the incremental deflection after installation of non-structural components the construction sequence and load history must be known. Fig. 5 shows a typical schematic load vs. time history for a slab system in a multi-story structure (Graham and Scanlon 1988). During construction, the load increases as floors above are supported temporarily on floors below. After construction, the load drops to the sustained load level. An increment in sustained load is added when non-structural components are installed. Live load is then applied intermittently during the service life of the structure. A simplified load vs. time history is shown in Fig. 6. A single instantaneous application of construction load,  $W_{co}$  is applied at time  $t_1$ . The load then drops to the sustained load level ( $W_s$ ) and remains constant thereafter. A single application of the non-sustained portion of live load,  $W_{lvar}$  is shown at time  $t_3$ .

Table 4. Probability model of random variables

| Variable                              | Mean  | C.O.V   | S.D | Source                   |
|---------------------------------------|---|---|-----|--------------------------|
| $f'_c$ , MPa (psi)                    | $0.675f'_c + 7.58 \leq 1.15f'_c$<br>$(0.675f'_c + 1,100 \leq 1.15f'_c)$ | 0.176   | -   | Mirza et al. 1979        |
| Concrete (in-situ) $f'_r$ , MPa (psi) | $0.69\sqrt{f'_c}, (8.3\sqrt{f'_c})$                                     | 0.218   | -   | Mirza et al. 1979        |
| $E_c$ , MPa (psi)                     | $5015.21\sqrt{f'_c}, (60,400\sqrt{f'_c})$                               | 0.119   | -   | Mirza et al. 1979        |
| Reinforcement $A_s$                   | $0.99A_n$   | 0.024   | -   | Mirza and MacGregor 1979 |
| $E_s$ , MPa (ksi)                     | 201,326.91 (29,200)   | 0.024   | -   | Julian 1966              |
| Beam dimension $b$ , cm (in.)         | $b_n + 0.397, (b_n + \frac{5}{32})$                                     | 0.045   | -   | Naaman 1982              |
| $d_{st}, d_{sb}$ , cm (in.)           | $d_{sn} + 0.159, (d_{sn} + \frac{1}{16})$                               | $\frac{0.27}{h_n}, (0.68)$<br>$\frac{1}{h_n}$ | -   | Naaman 1982              |

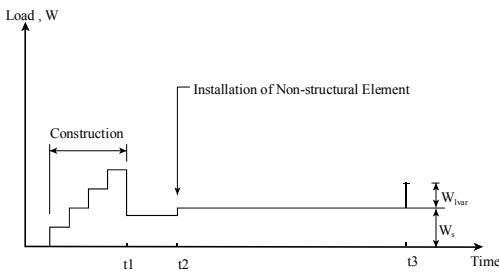
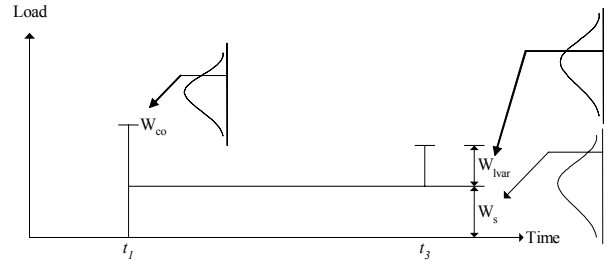
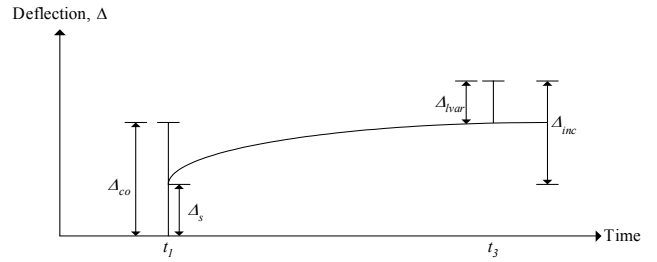


Figure 5. Schematic load-time history.

Installation of non-structural components is conservatively assumed to occur at time  $t_1$ . In addition, previous load surveys show that the average tenancy duration is 8 years and that most buildings will have eight different tenants (Ellingwood and Culver 1977). In the simplified load-time history model time  $t_3$  is assumed to be 8 years. Dead load, sustained live load and instantaneous additional live load are assumed to be the load components acting on the office floor during its service life. Also, sustained live load and instantaneous live load are subject to influence area of floor. Probabilistic models for these loads are given in Table 5.



(a) Simplified load-time history with probabilistic approach



(b) Simplified deflection-time history

Figure 6. Simplified load-time history with probabilistic approach and corresponding deflection-time history.

Table 5. Probability model of random variables.

| Load                             | Statistical parameters  | Distribution | Source                     |
|----------------------------------|---|--------------|----------------------------|
| Formwork load                    | mean=0.11 D <sub>n</sub><br>COV=0.10  | Normal       | El-Shahhat et al. 1993     |
| Sustained construction live load | mean=0.29 kPa (6.0 psf)<br>COV=1.10   | Gamma        | Ayoub and Karshenas 1994   |
| Stacking load                    | mean=0.974 kPa (20 psf),<br>COV=0.60  | Gamma        | Ayoub and Karshenas 1994   |
| Dead load                        | mean=1.05 D <sub>n</sub> , COV=0.10   | Normal       | Stewart 1996               |
| Sustained live load              | $\mu_{lsus} = 0.56 \text{ kPa}$<br>$(11.6 \text{ psf})$<br>$\sigma^2_{lsus} = 26.2 + \frac{6500}{A} \kappa$                       | Gamma        | Ellingwood and Culver 1977 |
| Extraordinary live load          | $\mu_E = \frac{\mu_Q \mu_R \lambda^2}{A}$<br>$\sigma^2_E = \frac{\lambda \kappa (\mu_Q^2 \mu_R^2 + \mu_R \sigma_Q^2 + \mu)}{A^2}$ | Gamma        | Ellingwood and Culver 1977 |

Note: D<sub>n</sub>= Nominal dead load;  $\kappa=2.76, (\mu_Q, \sigma_Q)=(7.305, 1.218)$  [kPa],

$(150, 25)$  [psf];  $(\mu_R, \sigma_R)=(4, 2), \lambda = \sqrt{\frac{A-155}{6.3}}$ ;  $A \geq 36 \text{ m}^2$  (400 ft<sup>2</sup>).

**Sensitivity analysis**

For the example, a simply supported one-way slab with a span length of 4.57 m (15 ft) and live load of 2.4 kPa (50 psf) was selected. Analyses were performed to assess the sensitivity of the results to assumed parameters. Specifically the lower bound on the sensitivity loss function was varied by plus or minus 30 %, the initial construction cost was varied by plus or minus 10 %, and the cost of failure was varied by plus or minus 10 %. The results, total costs

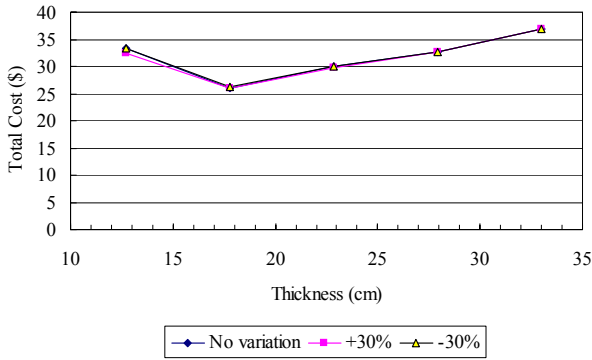


Figure 7. Effect of variation of lower bound of serviceability loss function of one-way slab.

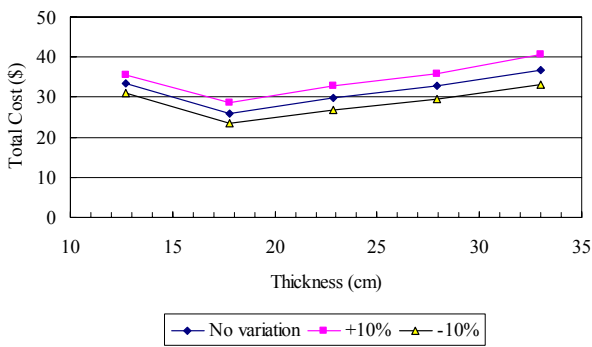


Figure 8. Effect of variation of cost of initial construction of one-way slab.

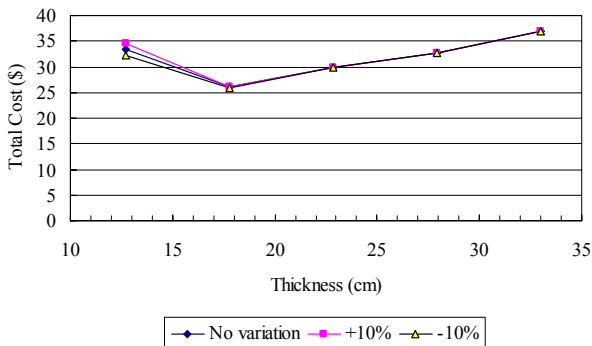


Figure 9. Effect of variation of cost of failure of one-way slab.

versus span length are illustrated in Fig. 7 to 9. As shown in the figures, the optimum thickness is calculated as 178 mm (7 in.) in each case.

As the application of the method, validity of ACI 318 minimum thickness requirement for both ends continuous one-way slabs are examined with the proposed utility theory. Table 6 shows geometric parameters of one-way slabs to examine the validity of ACI 318 minimum thickness requirement for both ends continuous condition. Live loads are assumed to be 2.4 kPa (50 psf) and 4.8 kPa (100 psf) at 0.95 of cumulative density function of the loading model respectively. After conducting Monte Carlo simulation, polynomial curve fitting is applied to a set of thicknesses in a given span length to estimate an optimized thickness.

Table 6. Geometric parameters of both ends continuous one-way slabs

| Length, mm (in.) | Depth, mm (in.)  |
|------------------|--|
| 4572 (180)       | 114.3 (4.5); 139.7 (5.5); 165.1 (6.5); 190.5 (7.5); 215.9 (8.5)      |
| 6096 (240)       | 114.3 (4.5); 165.1 (6.5); 215.9 (8.5); 266.7 (10.5); 317.5 (12.5)    |
| 7620 (300)       | 177.8 (7.0); 228.6 (9.0); 279.4 (11.0); 330.2 (13.0); 381 (15.0)     |
| 9144 (360)       | 228.6 (9.0); 279.4 (11.0); 330.2 (13.0); 381.0 (15.0); 431.8 (17.0)  |
| 10668 (420)      | 279.4 (11.0); 330.2 (13.0); 381.0 (15.0); 431.8 (17.0); 482.6 (19.0) |
| 12192 (480)      | 330.2 (13.0); 381.0 (15.0); 431.8 (17.0); 482.6 (19.0); 533.4 (21.0) |

Table 7. Comparison between optimized thickness obtained by utility analysis and ACI 318 minimum thickness limit for both ends continuous one-way slab.

| Span length, mm (inch) | Optimized thickness, mm, (inch) | ACI 318-02, mm (inch) | Difference     |
|------------------------|---------------------------------|-----------------------|----------------|
| 4572 (180)             | 114.3 (4.50)                    | 163.32 (6.43)         | -49.02 (-1.93) |
| 6096 (240)             | 193.04 (7.60)                   | 217.68 (8.57)         | -24.64 (-0.97) |
| 7620 (300)             | 284.48 (11.20)                  | 272.03 (10.71)        | 12.45 (0.49)   |
| 9144 (360)             | 381.0 (15.00)                   | 326.64 (12.86)        | 54.34 (2.14)   |
| 10668 (420)            | 464.82 (18.30)                  | 381.0 (15.00)         | 83.82 (3.30)   |
| 12192 (480)            | 533.4 (21.00)                   | 435.36 (17.14)        | 98.04 (3.86)   |

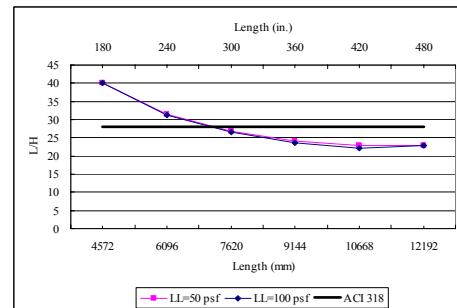


Figure 10. Comparison between optimized thickness of both ends continuous one-way slab and ACI 318 limit.

Table 7 and figure 10 show the result of the analysis. ACI 318 limit is conservative than the optimized thicknesses until around 300 inches span length, and it becomes minutely unconservative afterward. And, both live loads cases are almost consistent with each other.

### 5. SUMMARY AND CONCLUSIONS

A framework and methodology have been provided that can serve as a rational approach to design of concrete structures for deflection control in particular and serviceability design in particular. The use of utility theory provides a means of minimizing the total cost considering both initial construction cost and probabilistic cost of fail-

ure. This approach can be used to assess the reliability of code provisions for deflection control. Further research is needed to refine the serviceability loss functions for a range of serviceability requirements and to assemble cost data for various repair scenarios. For example in this analysis only repair of the slab it self was considered. Repair of non-structural partitions and other cases should also be considered.

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#### APPENDIX

Following parameters are used in Tables 4:

$A_s$  = area of reinforcement

$b$  = width of beam

COV = coefficient of variation

$d_{sb}$  = distance from top fiber to centroid of bottom steel

$d_{st}$  = distance from top fiber to centroid of top steel

$E_c$  = modulus of elasticity for concrete

$E_s$  = modulus of elasticity for steel

$\bar{f}'_c$  = concrete compressive strength

$f_r$  = modulus of rupture

Following parameters are used in Table 5:

$A$  = influence area

$D_n$  = nominal deal load

$\mu$  = mean

$\sigma$  = standard deviation

$\lambda$  = parameter for extraordinary live load

$\kappa$  = constant of sustained live load

Subscripts:

$L_{sus}$  = sustained live load

$E$  = extraordinary live load

$Q$  = weight of a single concentrated load in the cell

$R$  = number of loads per cell

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