

# Optimum Weight in Spline for Surface Model

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## ABSTRACT

The digital surface model (DSM) is used for several purposes in photogrammetry, remote sensing and laser scanned data such as orthoimage production, contours derivation, extraction of height information. Creation of a surface model from point-clouds (3-D sparse points) that can be derived from stereo imagery and range data (e.g. laser scanned data) can be done with several mathematical interpolation models. In this paper, thin-plate-spline (TPS) is used for digital surface modeling. Determination of suitable weight is an important problem in thin-plate function for a surface. The Voronoi algorithm has been proposed as a method for determination of the weight in thin-plate-spline. In this paper, methods has been tested for different surfaces. The results show that thin-plate-spline can be independent of weight.

## Key words

weight, thin-plate-spline, point density, Voronoi algorithm, adjustment

## 1. Introduction

TPS is the abbreviation for the Thin Plate Spline, which was first introduced by Duchon(1976). It was shown that the radial based functions admits a solution and only one under the condition that the sample be counted at least on three non-aligned points (Meinguest, 1979). TPS has been found for so many uses, as an example, geophysical applications in aeromagnetic & gravimetric surveying (Billings *et al.*, 2002), modeling of fingerprints (Bazen and Gerez, 2002), medical researches (Pedersen, 2000), converting elevation contours to a grid (Gousie, 2000; Maillet, 2004). In other research it was proved that TPS is compatible to Advection-Diffusion equations and is method with the unique answer for these kinds of differential equations (Boztosun *et al.*, 2002).

## 2. Mathematical Model of the Thin-plate-splines

The thin-plate-spline is a physically based on 2-D interpolation which represents a thin metal sheet that is constrained not to move at the grid points, and is free from any external force relied upon control points, from this sight the bending energy in control points should have been minimized. This bending energy function is shown by nether formulation in 2-D space (Wolfram, 2005),

$$I[f(x, y)] = \text{dint} \int_R^2 f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2 \quad (1)$$

In Eq. 1,  $f$  represents a height in 3D models. This equation is invariant under translation, rotation, or scaling of either set of control points (Maillet, 2004), supports norm of the surface and when it is minima it means that the bending of the surface in control points have to become minimized and the slope variation reduced to minimum in the tangential plates.

From another point of view the thin-plate-spline as an approximation for the real surface of the control points will enable user to get to the control points as near as possible. Therefore a constraint of the distance between the control points and surface must also be minimized so L2 norm is used,

$$E_{TPS} = \sum_{i=1}^n \mu_i (f(x_i, y_i) - z_i)^2 + \text{dint} \int_R^2 f_{xx}^2 + 2f_{xy}^2 + f_{yy}^2 \quad (2)$$

Solving of the bending part of the Eq. 2 is just like inhomogeneous biharmonic equation,  $\nabla^4 f + \sum_{m=1}^M \lambda_m \delta(x - x_m) = 0$  (Jenkins, 2000), which has a unique answer in the polar coordinate system (Maillet, 2004). Duchon and Suber (1992) achieved the under gone answer for Eq. 1,

$$\begin{aligned} &\sum_i a_i \phi(r_i(x_j + y_j)) \\ &+ b_0 + b_1 x_j + b_2 y_j = z_j; (j=1, \dots, n) \\ &\sum_j a_i = 0; \sum_j a_i x_i = 0; \sum_j a_i y_i = 0; \end{aligned} \quad (3)$$

By considering  $\mu_i$  weights and adjustment Eq. 2, upper answer can be rewritten as follows:

$$\begin{aligned} &\frac{8\pi a_i}{\mu_j} + \sum_i a_i \phi(r_i(x_j + y_j)) \\ &+ b_0 + b_1 x_j + b_2 y_j = z_j; (j=1, \dots, n), \end{aligned} \quad (4)$$

$\phi$  components are radial base functions which can be in  $\phi(x, x_j) = \phi(r_j) = r_j^{2m} \log(r_j)$  form and have  $C^{2m-1}$  continuity (Boztosun *et al.*, 2002), selection of form of  $\phi$  (order, m) is out of the argument of this article, therefore based on usual shape of the radial base functions in TPS. By assuming m is 1, so  $\phi$  have  $C^1$  continuity and have  $\phi(x, x_i) = \phi(r_i) = r_i^2 \log(r_i)$  shape; in which  $r$  is Euclidean norm in 2D space  $(r_i(x_j, y_j) = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2})$ .

Duchon offered a straight solution for this equation series which they are linear system with  $n+3$  equations,

$$\begin{pmatrix} 8\pi/\mu_1 & c_{21} & \cdot & \cdot & \cdot & c_{n1} & 1 & x_1 & y_1 \\ c_{21} & 8\pi/\mu_2 & \cdot & \cdot & \cdot & c_{n2} & 1 & x_2 & y_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ c_{1n} & c_{2n} & \cdot & \cdot & \cdot & 8\pi/\mu_n & 1 & x_n & y_n \\ 1 & 1 & \cdot & \cdot & \cdot & 1 & 0 & 0 & 0 \\ x_1 & x_2 & \cdot & \cdot & \cdot & x_n & 0 & 0 & 0 \\ y_1 & y_2 & \cdot & \cdot & \cdot & y_n & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \cdot \\ \cdot \\ a_n \\ b_0 \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ \cdot \\ \cdot \\ z_n \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad (5)$$

$c_{ij} = \phi(r_i(x_j, y_j))$

Which can be written as:

$$\begin{pmatrix} C & F \\ F^T & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} Z \\ 0 \end{pmatrix}.$$

Coefficient matrix  $\begin{pmatrix} C & F \\ F^T & 0 \end{pmatrix}$  is a positive symmetrical matrix. It was suggested that solving the system 5 by QR factorization as follows:

Where  $Q$  is an n by n orthogonal matrix and  $R$  is n by n by 3 matrix in the following form:

$$R = \begin{pmatrix} U \\ 0 \end{pmatrix}; \quad Dim[U] = 3 \times 3.$$

With  $U(3, 3)$  triangular superior matrix.  $F$  is of rank3, since there are at least three non-aligned points, therefore  $R$  is of rank 3, thus  $U$  is invertible,

$$\begin{aligned} \begin{pmatrix} C & F \\ F^T & 0 \end{pmatrix} &= \begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} Q^T C Q & R \\ R & 0 \end{pmatrix} \begin{pmatrix} Q^T & 0 \\ 0 & I \end{pmatrix}; \\ I &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned} \quad (7)$$

By replacing Eq. 7 in Eq. 5 we have:

$$\begin{aligned} \begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} Q^T C Q & R \\ R & 0 \end{pmatrix} \begin{pmatrix} Q^T & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} &= \begin{pmatrix} Z \\ 0 \end{pmatrix} \Rightarrow \\ \begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} Q^T C Q & R \\ R & 0 \end{pmatrix} \begin{pmatrix} Q^T a \\ b \end{pmatrix} &= \begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} Z \\ 0 \end{pmatrix} \quad (Q^{-1} = Q^T) \Rightarrow \\ \left\{ \begin{pmatrix} Q^T C Q & R \\ R & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \right\} \begin{pmatrix} Q^T Z \\ 0 \end{pmatrix} &= \begin{pmatrix} Q^T C Q & R \\ R & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = Qd \end{aligned} \quad (8)$$

And by supersede  $Q = (Q_{1 \times 3} Q_{2 \times (n-3)})$  and

$$d = \begin{pmatrix} d_{1 \times 1} \\ d_{3 \times 1} \\ d_{(n-3) \times 1} \end{pmatrix} \text{ one can write:}$$

$$\begin{pmatrix} Q_1^T C Q_1 & Q_1^T C Q_2 & U \\ Q_2^T C Q_1 & Q_2^T C Q_2 & 0 \\ U^T & 0 & 0 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} Q_1^T Z \\ Q_2^T Z \\ 0 \end{pmatrix}; \quad (9)$$

$$a = Q_1 d_1 + Q_2 d_2$$

One deduces  $U^T d_1 = 0$ , thus  $d_1 = 0$ , from equation  $Q_2^T C Q_2 d_2 = Q_2^T Z$ , one deduces  $d_2$ , from where  $a = Q_2 d_2$ ; from the equation  $Q_1^T C Q_2 d_2 + U b = Q_1^T Z$ , one can finally extract b.

It is necessary to notice that the  $C$  matrix is full. Therefore the matrix  $Q^T {}_1 C Q_2$  is also full. So, if we have more than 3 control points this method can never be singular, but weights  $\mu_i$  in Eq. 4 are one of the weaknesses of these methods and answers touched by them, we will discuss weights in TPS in the following section:

### 3. WEIGHT COMPARISON ON TPS

For simplicity of discussion, Eq. 2 can be rewritten as:

$$E_p = p \sum_{i=1}^n r_i (f(x_i, y_i) - z_i)^2 + I(f), \quad (10)$$

With  $P = \sum_{i=1}^n \mu_i$  and  $r_i = \frac{\mu_i}{P}$  that  $r_i$  is the relative weight of the point  $(x_i, y_i, z_i)$  and the  $P$  weight fixes the importance of the adjustment criterion,

$$e(f) = \sum_{i=1}^n r_i (f(x_i, y_i) - z_i)^2, \quad (11)$$

In relation to the curvature criterion  $I(f)$ .

#### 3.1 Examination of the Effect of the Weight P

To  $P > 0$ , corresponds a unique thin plate spline surface, denoted  $f_p$ , minimizing,  $E_p(f) = P \times e(f) + I(f)$ ; in the same way, to  $Q > 0$  corresponds the surface  $f_q$ , minimizing,  $E_q(f) = Q \times e(f) + I(f)$ .

Because of the minimal character of  $E_p(f_p)$  and of  $E_q(f_q)$ , one has,

$$\begin{aligned} E_q(f_q) &\leq E_q(f_p) \\ E_p(f_p) &\leq E_p(f_q) \end{aligned} \quad (12)$$

Or else:

$$P \times e(f_p) = I(f_p) \leq P \times e(f_q) + I(f_q) \quad (13)$$

$$Q \times e(f_p) + I(f_p) \leq Q \times e(f_p) + I(f_p) \quad (14)$$

If  $P < Q$  then  $I(f_p) \geq I(f_q)$  and  $e(f_p) \leq e(f_q)$ ; it means that the surface  $f_q$  is less bent than  $f_p$ , or "smoother". Form another point of view, if  $P$  becomes arbitrarily too large, and becomes near infinite,  $f_p$  must have been perfectly adjusted to the sample.

#### 3.2 Considering the Weight $r_i$ , under the supposing of fixed P weights

The simplest choice is to give the same weight  $r_i = \frac{1}{n}$  (supposing  $r_i = 1$ ) but because of irregularity of points and differences in local density of control points one deduces using of Voronoi polygon areas for introducing weights (Maillet, 2004; Goncalves *et al.*, 2002). Thus as we discuss this method briefly, we would emphasize on the point, which this paper is based on.

Voronoi algorithm is a way to partition  $R^2$  space. Those boundary condition of these sets are bisector lines between neighbor control points.  $R^2$  Space has a dominion from  $+\infty$  till  $-\infty$  thus control points can never achieve the supremum or infimum of this space, therefore area of edge polygons must be separately computed (Fig.1).

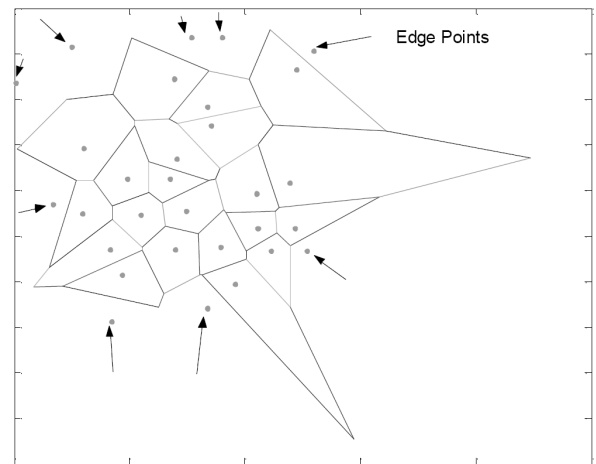


Fig. 1 Voronoi polygons.

#### a) Normalized Voronoi with fixed P equal to one, $\mu_i$ in edge polygons are assumed to be zero.

In this method as you can see from its name,  $P$  is equal to one,  $r_i$  is a normalized area of Voronoi polygons and in edge polygons,

$$\mu_i = \begin{cases} r_i = \frac{A_i}{\sum_{j=1}^n A_j}; & \text{if } (i = 1, 2, 3, \dots, n \& A_i \neq \infty) \\ 0; & \text{if } (A_i = \infty) \end{cases} \quad (15)$$

from which  $A$  is Voronoi polygons area.

**b) Normalized Voronoi with fixed  $P$  equal to one,  $\mu_i$  in edge polygons area assumed to be one.**

Weights obeys the undergone relations:

$$\mu_i = \begin{cases} r_i = \frac{A_i}{\sum_{j=1}^n A_j}; & \text{if } (i = 1, 2, 3, \dots, n \& A_i \neq \infty) \\ 1; & \text{if } (A_i = \infty) \end{cases} \quad (16)$$

**c) Normalized Voronoi with fixed weight  $P=100$ .**

The surface constructed by normalized Voronoi,  $P=1$  is a surface with softness more than contiguity of control points so based on what we said on the start of this section,  $P$  fixed brings itself forth. It is necessary to say  $P$  is a digit between  $10^3$  to  $10^5$ . So, the four following equations analyse the versatility of  $P$ .

$$\mu_i = 100 \times \frac{A_i}{\sum_{j=1}^n A_j}; i = 1, 2, 3, \dots, n. \quad (17)$$

**d) Normalized Voronoi with fixed weight  $P=1000$ ,**

$$\mu_i = 1000 \times \frac{A_i}{\sum_{j=1}^n A_j}; i = 1, 2, 3, \dots, n, \quad (18)$$

**e) Normalized Voronoi with fixed weight  $p=10000$ ,**

$$\mu_i = 10000 \times \frac{A_i}{\sum_{j=1}^n A_j}; i = 1, 2, 3, \dots, n, \quad (19)$$

**f) Normalized Voronoi with fixed weight  $P=100000$ ,**

$$\mu_i = 100000 \times \frac{A_i}{\sum_{j=1}^n A_j}; i = 1, 2, 3, \dots, n, \quad (20)$$

**g) Voronoi,**

This method is the comparison between usual and normalized methods,

$$\mu_i = A_i; i = 1, 2, 3, \dots, n. \quad (21)$$

**h) Fixed weight equal to one,**

$$\mu_i = 1; i = 1, 2, 3, \dots, n \quad (22)$$

it means that  $P$  is the total number of control points, and  $r_i$  are equal to one.

#### 4. Eeperimental Results

The elevation data of a location with area of  $910 \times 804 \text{ m}^2$  includes 7,330 points. Selection of points by their preliminary acquisitions order and not to regularize them, for this purpose one needs a start point number and a count of points must be used. Secondly, regularizing data and selection of a window of it based on construction of  $m \times m$  network. For example first column of Table 1 which named by "Data Location" represents these two methods of point selections. The differing on the last row, represent the first method, selecting points with point number 1 as start point and 1500 points data selected from original dataset. Or first row of this table is a selection of the window on the first row and the first column of the  $5 \times 5$  network of dataset.

For comparison of weight defining methods mentioned in section 3, 10 data with different dispersal was selected from original data. After applying of methods mentioned on this article on every set of data for finding precision of every set, check points with ratio of 2 to 1 for control points selected have been chosen. Standard deviation and average of residuals for every weight definition methods computed on check points which you can find results on Table 1 and 2 also Fig. 4.

The notable point is the smoothness behavior of TPS in void areas is lake existence naught extrapolation in it (Fig. 3) (Bazen *et al.*, 2002).

As it was expected, methods using normalized Voronoi polygon areas with  $P$  weight equal to one, because of greater bending effect than L2 norm discussed in Eq. 2 part 1 in section 3 in contiguity to control points relative to other methods showing a weakness and only in 16 point series that have a better answer. And it is because of lack of data and probably existence of unusual point in the set of data in comparison of these two methods (a, b). In fact participating edge points (a) have been a better response than not using them at all. Therefore in other methods we use weights equal to one for edge polygons.

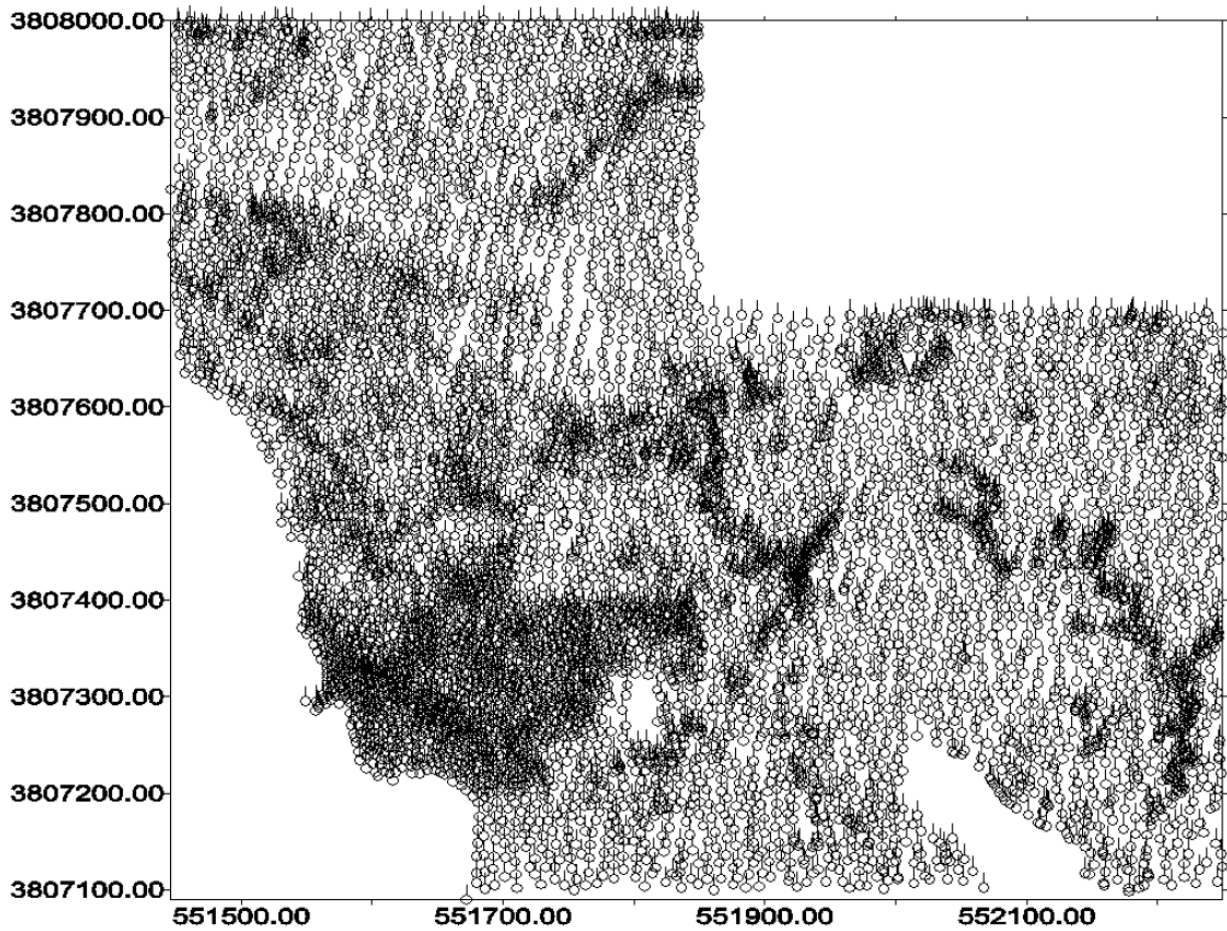


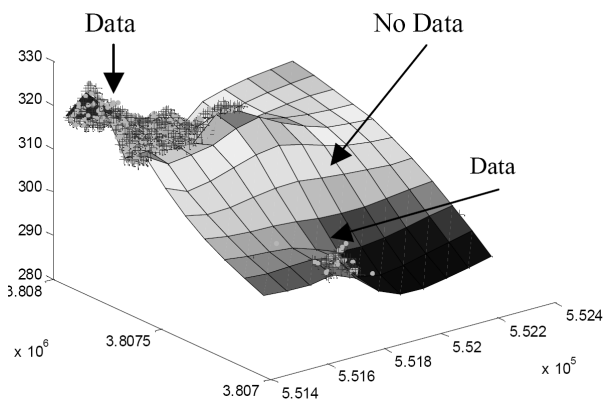
Fig. 2 Point-clouds.

Table 1 Standard deviation of dataset shown in Fig. 4.

Method	→ GCP	Norm. Voronoi (1 at edge ploy)	Norm. Voronoi (0 at edge poly)	P=100	P=1000	P=10000	P=100000	Voronoi	Weight=1
data Location	no.								
5*5section11	16	0.122	0.122	0.139	0.138	0.138	0.138	0.138	0.122
5*5section12	174	1.414	1.415	0.861	0.740	0.779	0.779	0.783	0.710
5*5section13	190	0.724	0.736	0.542	0.568	0.614	0.614	0.622	0.564
5*5section21	250	0.872	0.992	0.505	0.454	0.458	0.458	0.457	0.483
5*5section22	552	1.257	1.258	0.748	0.683	0.718	0.718	0.723	0.663
5*5section23	306	1.152	1.152	0.547	0.437	0.458	0.458	0.460	0.432
5*5section33	314	1.301	1.403	0.682	0.710	0.747	0.747	0.749	0.682
5*5section43	242	0.692		0.373	0.417	0.444	0.444	0.447	0.398
5*5section44	96	1.693		1.429	1.416	1.420	1.420	1.420	1.404
From Pnt. 1to1500	1000	1.401		0.867	0.536	0.507	0.507	0.510	0.504

**Table 2** Average of dataset shown in Fig. 4.

Method	→ GCP	Norm. Voronoi	Norm. Voronoi	P=100	P=1000	P=10000	P=100000	Voronoi	Weight=1
data Location	no.	(1 at edge plo)	(0 at edge poly)						
5*5section11	16	-0.032	-0.031	-0.045	-0.048	-0.049	-0.049	-0.049	-0.043
5*5section12	174	0.411	0.410	0.140	0.033	-0.034	-0.051	-0.051	-0.020
5*5section13	190	0.385	0.385	0.092	0.032	-0.009	-0.023	-0.025	0.001
5*5section21	250	0.123	0.124	0.056	0.040	0.040	0.041	0.041	0.035
5*5section22	552	0.140	0.142	0.088	0.042	0.021	0.008	0.006	0.016
5*5section23	306	-0.069	-0.068	0.003	0.005	0.017	0.019	0.019	0.012
5*5section33	314	0.177	0.239	-0.009	-0.054	-0.082	-0.093	-0.094	0.010
5*5section43	242	0.040		-0.022	-0.044	-0.068	-0.079	-0.080	-0.061
5*5section44	96	-0.390		0.112	0.133	0.139	0.143	0.143	0.117
From Pnt. 1to1500	1000	0.156		0.049	0.029	0.005	-0.010	-0.010	-0.013



**Fig. 3** Thin plate spline(TPS) surface.

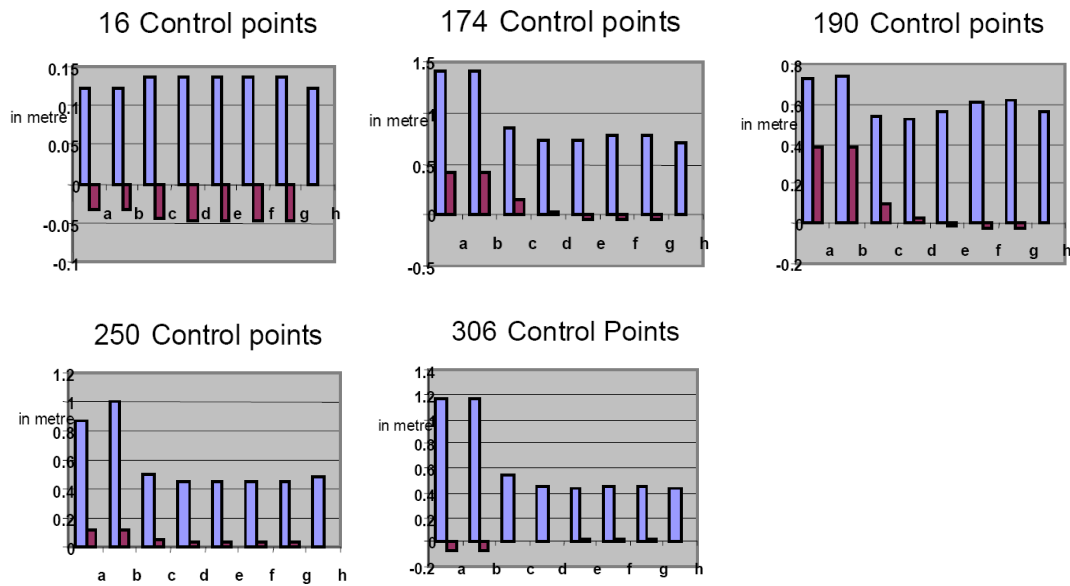
Voronoi methods with weights different from 1 are the best, but in Fig. 4 for the 242 point condition, 'c' have a less standard deviation than 'e', 'f' and 'd' Therefore for every set of data the best P must be optimized (Fig. 4, methods f, e, d and c). That is in the contrary with ideal best fitting described in ending of section 3.1 which P is leading to infinite. Method 'g' have a better answer compared with 'b' and 'a', but it is not so much worse than 'c', 'd', 'e' and 'f', so normal methods in TPS haven't any significant preferences. Based on 10 series as not expected, if  $\mu_i$  considered fixed and equal to one (method h) there is no significant difference between its result and the best results of

Voronoi methods.

On the other hands, Voronoi algorithm add a new kind of problems which the fixed weight of one doesn't. those problems are:

- The probability of existence of more than 3 control points on one circle and creation of embedded polygons(n points). Because of the existence of more than  $C^2_n$  equitable perpendiculars, ambiguity in defining neighbor points come to existence.
- The problem of determination of these polygons created for these points is another problem of computer coding which is complex and time consuming
- Computing area of these irregular polygons is another problem.

Therefore revision necessities of indicated reasons for methods using Voronoi algorithm show their importance and make the writer to design another test for it. The algorithm of Voronoi has purposed because of this reason: this method is sensate of the density of control points, then in continuations ; has researched this significance in specific conditions of TPS and defined an experiment in this form that two adjusted areas of original data chosen, the first (I) has 454 points and second dataset (II) has 473 points. Control and check points have separated by 2:1 ratio till the created surface error by control points, to be determined by



**Fig. 4** Standard deviation and average for 8 chosen series of data that, “a” is the Voronoi normalized areas method with fixed weight P equals to 1 (P=1) and  $\mu$  in edge polygons equals to one, “c” is the Voronoi normalized areas method with fixed weight P equals to 100 (P=100), “d” is the Voronoi normalized areas method with fixed weight P equals to 1000 (P=1000), “e” is the Voronoi normalized areas method with fixed weight P equals to 10000 (P=10000), “f” is the Voronoi normalized areas method with fixed weight P equals to 100000 (P=100000), “g” is the method of Voronoi areas and “h” is the fixed weight (= 1) method.

check points.

Density of control points in I, is 10.51 points per square kilometers and in data II is 10.37 points per km<sup>2</sup>. Errors were computed for combination of two series, and results have been mentioned in Tables 3 and 4.

In the next stage, whereas the data of zone I not touched, zone II decreased to half and became 236 points. Again control and check points are separated in 2:1 ratio and the density of control points in II are decreased to 5.7 points per km<sup>2</sup> (i.e. less than half of II) results have been shown in row

$I + \frac{1}{2} II$  of Tables 3 and 4.

In resumption while zone I still unchanged, dataset of  $\frac{1}{2} II$  became half again and reached 119 points that is corresponds 2.58 points per km<sup>2</sup> density, in other word less than  $\frac{1}{4}$  zone II. The results are shown in row  $I + \frac{1}{4} II$  of Tables 3 and 4.

Now for last time data of zone  $\frac{1}{4} II$  becomes half and reaches 60 points and density of 1.32 points per km<sup>2</sup> which is one eights of zone II data. Results have been displayed in  $I + \frac{1}{8} II$  row of Tables 3 and 4.

**Table 3** standard deviation calculated for analyzing density effects.

Method	Norm. Voronoi	p=100	p=1000	p=10000	p=100000	Voronoi	Weight=1
I+II	1.789	0.816	0.637	0.583	0.595	0.609	0.581
$I + \frac{1}{2} II$	1.308	0.685	0.623	0.634	0.634	0.635	0.614
$I + \frac{1}{4} II$	2.638	1.004	0.648	0.648	0.622	0.638	0.620
$I + \frac{1}{8} II$	1.690	0.578	0.578	0.534	0.545	0.554	0.531

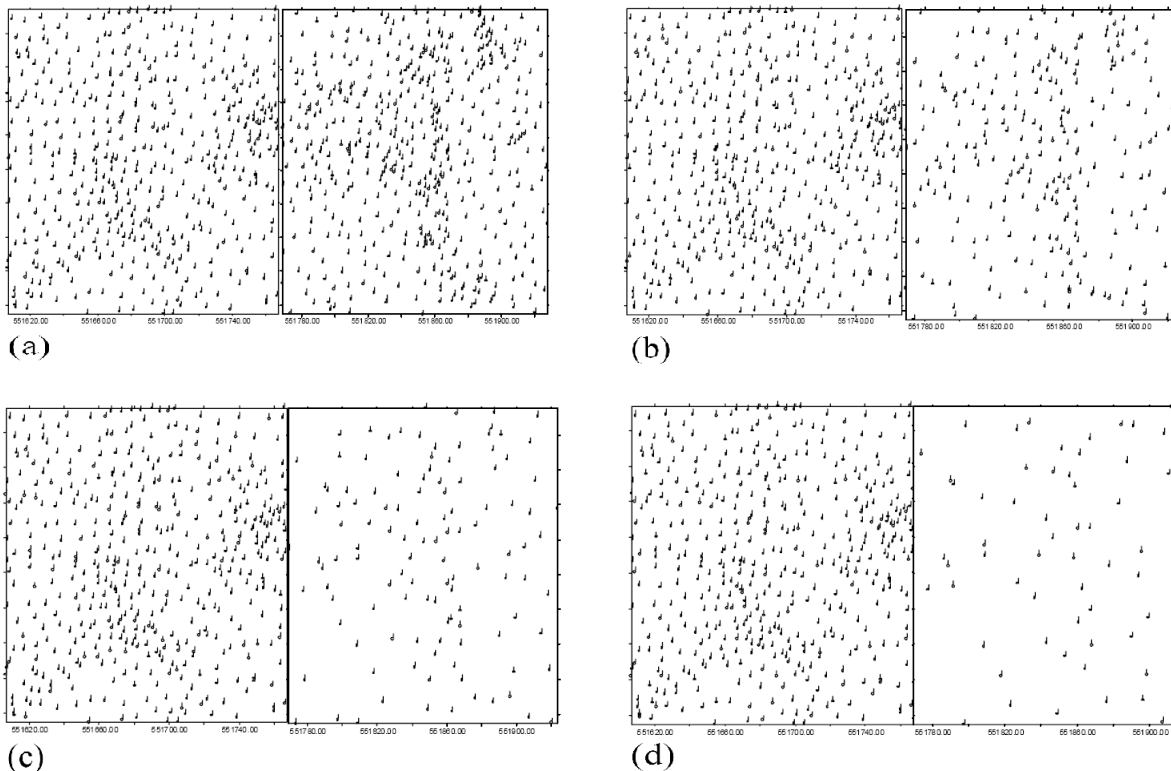
**Table 4** Average calculated for analyzing density effects.

Method	Norm. Voronoi	p=100	p=1000	p=10000	p=100000	Voronoi	Weight=1
I+II	0.342	0.046	0.021	0.017	0.010	0.003	0.011
I+ $\frac{1}{2}$ II	0.110	-0.014	-0.013	-0.001	-0.001	-0.001	-0.009
I+ $\frac{1}{4}$ II	0.412	-0.084	-0.090	-0.090	-0.081	-0.077	-0.082
I+ $\frac{1}{8}$ II	-0.045	-0.038	-0.038	-0.025	-0.017	-0.017	-0.026

In Fig. 5 a two dimensional diagram of the chain of density reduction described above have been illustrated, what was expected was because of the sensitivity of Voronoi methods compared with fixed weights methods, by reducing the density in a part of data, fixed weight methods must have shown weakness in final results.

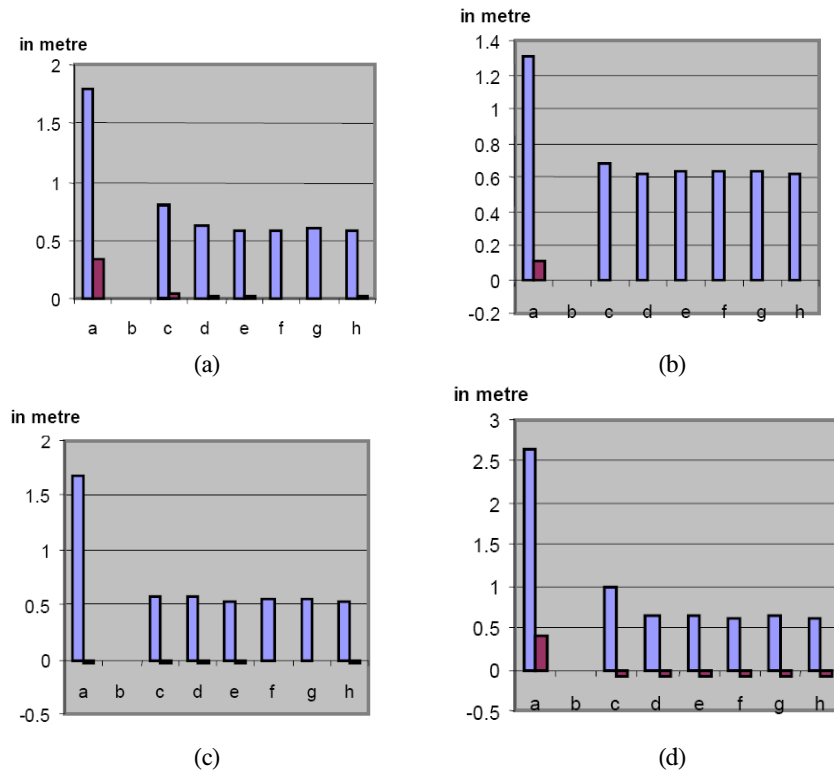
As you can see in Fig. 6 method 'h' even when the density reaches one eights in a part of original data, produces a comparable precision with Voronoi methods and will not weak-

en and will not adhere the expected pattern. In Fig. 7 behavior of every weight definition and will not adhere the expected pattern. In Fig. 7 behavior of every weight definition method except 'b'(because of lack of importance) is illustrated. This figure prove that with reduction of density even Voronoi methods will not follow a single pattern (p=100 till 100000, Fig. 7) and neither of results is strictly ascendan function nor strictly descendent function of density. Therefore one deduces by wonder that TPS with

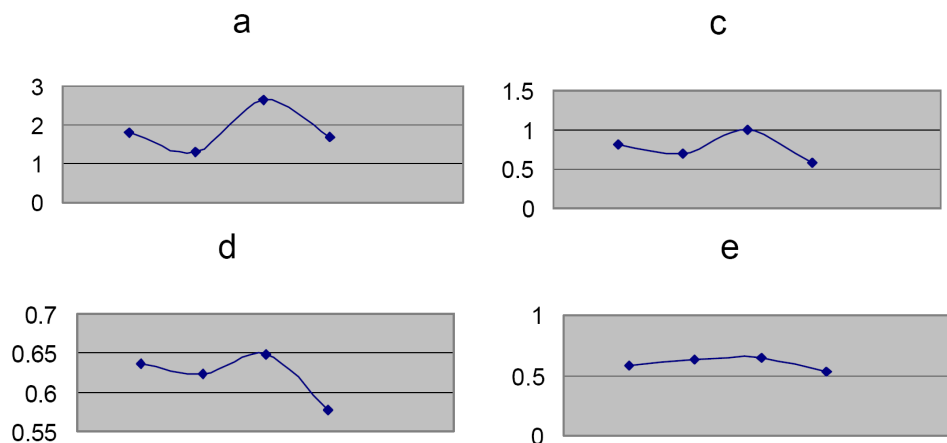


**Fig. 5** Sensitivity of TPS relative to control points density: a) I+II, b) I+ $\frac{1}{2}$ II, c) I+ $\frac{1}{4}$ II, d) I+ $\frac{1}{8}$ II.





**Fig. 6** Standard deviation and average for density analyzing dataset, “a” is the Voronoi normalized areas method with fixed weight P equals to 1 (P=1) and  $\mu$  in edge polygons equals to zero, “b” is the Voronoi normalized areas method with fixed weight P equals to 1 (P=1) and  $\mu$  in edge polygons equals to one, “c” is the Voronoi normalized areas method with fixed weight P equals to 100 (P=100), “d” is the Voronoi normalized areas method with fixed weight P equals to 1000 (P=1000), “e” is the Voronoi normalized areas method with fixed weight P equals to 10000 (P=10000), “f” is the Voronoi normalized areas method with fixed weight P equals to 100000 (P=100000), “g” is the method of Voronoi areas and “h” is the fixed weight (equal to 1) method.



**Fig. 7** Above 8 diagrams are for showing behavior of every weight definition methods relative to density variation, “a” is the Voronoi normalized areas method with fixed weight P equals to 1 (P=1) and  $\mu$  in edge polygons equals to zero, “c” is the Voronoi normalized areas method with fixed weight P equals to 100 (P=100), “d” is the Voronoi normalized areas method with fixed weight P equals to 1000 (P=1000), “e” is the Voronoi normalized areas method with fixed weight P equals to 10000 (P=10000), “f” is the Voronoi normalized areas method with fixed weight P equals to 100000 (P=100000), “g” is the method of Voronoi areas and “h” is the fixed weight (equal to 1) method.

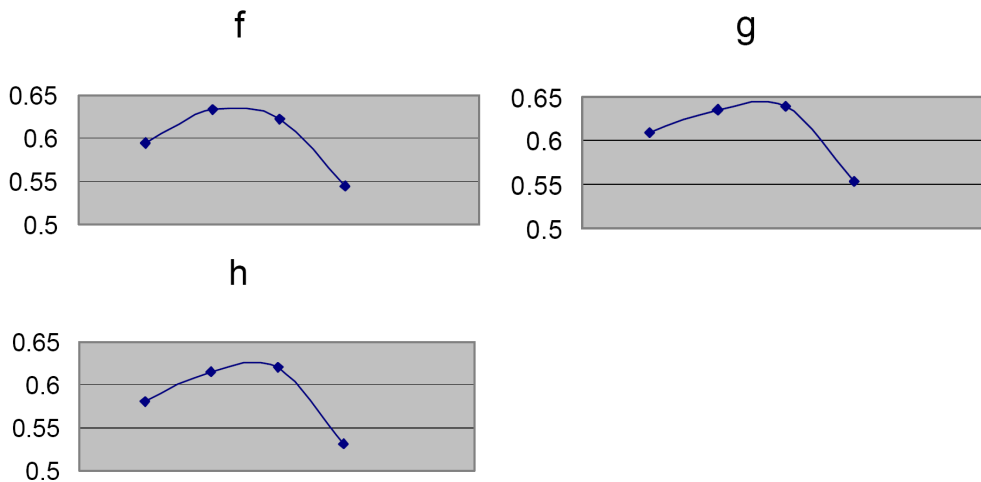


Fig. 7 Continued.

this solution is not sensitive to density of points.

What was discussed in this part illustrates that for this specific solution of thin plate Splines using of fixed weight is the best answer, need no complex calculations and make TPS as a unified method for miscellaneous datasets from which data density is not bothering.

## 5. Conclusions

TPS as a density independence method can be used for the construction of the surfaces in the new brand geomatics, because of its miscellaneous of data sources and differences in their densities, note that the weights are always of concern of the surveyors and differences in their densities, note that the weights are always of concern of the surveyors. On the other hand using TPS, without using the Voronoi algorithms, whereas it limits the number of control points and reduces the speed of operation, one can create a surface in contrary to other surface simulation methods such as BSplines all at once. Because of the density independence mathematical form of straight method, one can use it in CAD/CAM softwares.

It is certain that the independency of density must to be of specifications of the thin plate spline, therefore other researches on other solutions of TPS (Frank, 1982). must be considered. Because at least one can prove that, in the dense and the bare zones can be used with comparable error results. By the end result of this article, we recommend the

usage of the TPS method for mountainous-flat regions because variation of one zone doesn't affect other zones so much.

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