KYUNGPOOK Math. J. 45(2005), 539-547

Some Topologies Induced by b-open Sets

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Abstract. The class of b-open sets in the sense of Andrijević ([3]), was discussed by El-Atik ([9]) under the name of γ-open sets. This class is closed under arbitrary union. The aim of this paper is to use Λ-sets and $\vee$-sets due to Maki ([15]) some topologies are constructed with the concept of b-open sets. b-Λ-sets, b-$\vee$-sets are the basic concepts introduced and investigated. Moreover, several types of near continuous function based on b-Λ-sets, b-$\vee$-sets are constructed and studied.

1. Introduction

Andrijević ([3]) introduced a new class of generalized open sets in a topological space, so called b-open sets. This type of sets discussed by El-Atik ([9]) under the name of γ-open sets. The class of b-open sets is contained in the class of semi-preopen sets and contains all semi-open sets and preopen sets. The class of b-open sets generates the same topology as the class of preopen sets. In 1986, Maki ([15]) introduced the concept of generalized Λ-sets in a topological space and defined the associated closure operator by using the work of Levine ([14]) and Dunhem ([8]). Caldas and Dontchev ([7]) built on Maki's work by introducing Λ_s-sets, $\vee_s$-sets, $g\Lambda_s$-sets and $g\vee_s$-sets. Ganster and et. al. ([10]) introduced the notion of pre-Λ-sets and pre-$\vee$-sets and obtained new topologies defined by these families of sets. In [12] Khalimsky, Kopperman and Meyer proved that the digital line is a typical example of a $T_{1/2}$ space. Our aim is to introduce the notion of b-Λ-sets and b-$\vee$-sets to topological spaces and study some of its properties. Also, we prove that the topology generated by the class of b-open sets contains the topology generate by the class of preopen (resp. semi-open) sets by using the notions of Λ-sets and $\vee$-sets. Finally we introduce generalized b-Λ-sets, generalized b-$\vee$-sets, b-Λ-functions, b-$\vee$-functions and investigate some properties of the new concepts.

Definition 1.1. A subset A of a topological space is called:

(1) Semi-open [14] if $A \subseteq cl(int(A))$.

(2) Preopen [16] if $A \subseteq int(cl(A))$.

Received April 26, 2004.
2000 Mathematics Subject Classification: 54A10.
Key words and phrases: b-open sets, b-Λ-sets, b-$\vee$-sets, generalized b-Λ-sets, generalized b-$\vee$-sets, Λ_b-functions and $\vee_b$-functions.

(3) α-open [4] if $A \subseteq int(cl(int(A)))$.

(4) β-open [1] [Semi-preopen [5]] if $A \subseteq cl(int(cl(int(A))))$.

(5) b-open [3] if $A \subseteq int(cl(A)) \cup cl(int(A))$.

(6) Nowhere dense [2] if $int(cl(A)) = \phi$.

The class of all semi-open (resp. preopen, α-set, β-open and b-open) denoted by $SO(X, \tau)$ (resp. $PO(X, \tau)$, $\alpha O(X, \tau)$, $\beta O(X, \tau)$] and $BO(X, \tau)$.

The complement of these sets called semi-closed (resp. preclosed, α-closed, β-closed and b-closed) and the classes of all these sets will be denoted by $SC(X, \tau)$ (resp. $PC(X, \tau)$, $\alpha C(X, \tau)$, $\beta C(X, \tau)$ and $BC(X, \tau)$).

Definition 1.2. A topological space (X, τ) is said to be:

(1) Extremely Disconnected (abb. E. D) [2] if the closure of any open set is open.

(2) Submaximal [11] if all dense subset are open.

(3) Resolvable [11] if there is a subset D of X such that D and $(X - D)$ are both dense in X.

Definition 1.3. A subset A of a space (X, τ) is called:

(1) Λ-set (resp. -$\vee$-set) [15] if it is the intersection(resp, union) of open (resp. closed) sets.

(2) Λ_s-sets (resp. -$\vee_s$-sets) [7] if it is the intersection (resp. union) of semi-open (resp. semi-closed) sets.

(3) pre-Λ-sets (resp. pre-$\vee$-set) [10] if it is the intersection (resp. union) of preopen (resp. preclosed) sets.

(4) Generalized Λ-set (resp. generalized $\vee$-set) [15] if $\Lambda(A) \subseteq F$ whenever $A \subseteq F$ and $F \in C(X, \tau)$(resp. $U \subseteq \vee(A)$ whenever $U \subseteq A$ and $U \in O(X, \tau)$.

(5) Generalized semi-Λ-set (resp. generalized semi-$\vee$-set) [7] if $\Lambda_s(A) \subseteq F$ whenever $A \subseteq F$ and $F \in SC(X, \tau)$ (resp. $U \subseteq \vee_s(A)$ whenever $U \subseteq A$ and $U \in SO(X, \tau)$.

(6) Generalized pre-Λ-set (resp. generalized pre-$\vee$-set) [10] if $\Lambda_p(A) \subseteq F$ whenever $A \subseteq F$ and $F \in PC(X, \tau)$ (resp. $U \subseteq \vee_p(A)$ whenever $U \subseteq A$ and $U \in PO(X, \tau)$.

Lemma 1.1 ([9]). *For a space* (X, τ). *If* $U \subseteq Y \subseteq X$, $U \in BO(X, \tau)$ *and* $Y \in BO(X, \tau)$, *then*:

(1) $U \in \beta O(X, \tau)$

(2) $U \in BO(X, \tau)$ *if* $U \in \beta O(X, \tau) - BO(X, \tau)$.

2. Topological spaces via Λ_b-($\vee_b$-)sets

In this section we introduced the concept of b-Λ- (resp. b-$\vee$-) sets via the concept of b-open sets. We define the class of τ^{Λ_b} ($\tau^{\vee_b}$) and prove that they form topologies.

Definition 2.1. We define $\Lambda_b(A)$ and $\vee_b(A)$ for a subset A of a space (X,τ) as

$$\begin{aligned}\Lambda_b(A) &= \bigcap\{G : A \subseteq G,\ G \in BO(X,\tau)\} \text{ and}\\ \vee_b(A) &= \bigcup\{F : F \subset A,\ F \in BC(X,\tau)\}.\end{aligned}$$

Example 2.1. Let $X = \{a,b,c,d\}$, $\tau = \{X,\phi,\{a,b\},\{b\},\{b,c,d\}\}$. Then $BO(X,\tau) = \{X,\phi,\{b\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c,\},\{a,b,d\},\{b,c,d\}\}$ and $\Lambda_b(\{a\}) = \{a,b\}$, $\Lambda_b(\{c,d\}) = \{b,c.d\}$, $\Lambda_b(\{b\}) = \{b\}$, $\vee_b(\{a,b\}) = \{a,b\}$, $\vee_b(\{b,c,d\}) = \{c,d\}$, $\vee_b(\{a\}) = \{a\}$.

Lemma 2.1. *For subsets A, B and A_i, $i \in I$, of a space (X,τ) the following properties hold*:

(1) $A \subseteq \Lambda_b(A)$,

(2) *If* $A \subseteq B$, *then* $\Lambda_b(A) \subseteq \Lambda_b(B)$,

(3) $\Lambda_b(\Lambda_b(A)) = \Lambda_b(A)$,

(4) *If* $A \in BO(X,\tau)$, *then* $A = \Lambda_b(A)$,

(5) $\Lambda_b(\bigcup\{A_i : i \in I\}) = \bigcup\{\Lambda_b(A_i) : i \in I\}$,

(6) $\Lambda_b(\bigcap\{A_i : i \in I\}) \subseteq \bigcap\{\Lambda_b(A_i) : i \in I\}$,

(7) $\Lambda_b(X\backslash A) = X/V_b(A)$.

Proof. (1), (2), (4), (6) and (7) are immediate consequences of Definition 2.1. To prove (3), firstly from the Definition 2.1, $\Lambda_b(A) \subseteq \Lambda_b(\Lambda_b(A))$. For the converse inclusion, let $x \notin \Lambda_b(A)$. Then there exists $G \in BO(X,\tau)$ such that $A \subseteq G$ and $x \notin G$. Since $\Lambda_b(\Lambda_b(A)) = \{G : \Lambda_b(A) \subseteq G,\ G \in BO(X,\tau)\}$, so we have $x \notin \Lambda_b(\Lambda_b(A))$. Thus $\Lambda_b(\Lambda_b(A)) = \Lambda_b(A)$.

To prove (5), let $A = \cup\{A_i : i \in I\}$. By (2) Since $A_i \subseteq A$, then $\Lambda_b(A_i) \subseteq \Lambda_b(A)$ and so $\bigcup\{\Lambda_b(A_i)\ : i \in I\} \subseteq \Lambda_b(A)$. To prove the converse inclusion, let $x \notin \bigcup\{\Lambda_b(A_i) : i \in I\}$, then for each $i \in I$ there exists $G_i \in BO(X,\tau)$ such that $A_i \subseteq G_i$ and $x \notin G_i$. If $G = \bigcup\{G_i : i \in I\}$ then $G \in BO(X,\tau)$ with $A \subseteq G$ and $x \notin G$. Hence $x \notin \Lambda_b(A)$ and so (5) holds. □

Lemma 2.2. *For subsets A, B and A_i, $i \in I$ of a space $BO(X,\tau)$ the following properties hold*:

(1) $\vee_b(A) \subseteq A$,

(2) *If* $A \subseteq B$ *then* $\vee_b(A) \subseteq \vee_b(B)$,

(3) $\vee_b(\vee_{B_P}(A)) = \vee_b(A)$,

(4) *If* $A \in BC(X,\tau)$ *then* $A = \vee_b(A)$,

(5) $\vee_b(\bigcap\{A_i : i \in I\}) = \bigcap\{\vee_b(A_i) : i \in I\}$,

(6) $\bigcup\{\vee_b(A_i) : i \in I\} \subseteq \vee_b(\bigcup\{A_i : i \in I\})$.

Proof. Obvious by Lemma 2.1 (7). □

Remark 2.1. We note, in general for any subsets A, B of $(X,\tau)\, \Lambda_b(A \cap B) \neq \Lambda_b(A) \cap \Lambda_b(B)$ and $\vee_b(A \cup B) \neq \vee_b(A) \cup \vee_b(B)$, as in the following example.

Example 2.2. In Example 2.1, if $A = \{a\}$, $B = \{b,c,d\}$ then, $\Lambda_b(\{a\} \cap \{b,c,d\}) = \Lambda_b(\phi) = \phi, \Lambda_b(\{a\}) \cap \Lambda_b(\{b,c,d\}) = \{b\}$. Also, $\vee_b(\{a\} \cup \{b,c,d\}) = \vee_b(X) = X$, $\vee_b\{a\} \cup \vee_b\{b,c,d\} = \{a\} \cup \{c,d\} = \{a,c,d\}$.

Definition 2.2. A subset A of a space (X,τ) is called b-Λ- (resp. b-$\vee$-) set if $A = \Lambda_b(A)$ (resp. $A = \vee_b(A)$). The class of all b-Λ-sets (resp. b-$\vee$-sets) will be denoted by τ^{Λ_b} (resp. $\tau^{\vee_b}$).

Example 2.3. Let (X,τ) be the same as in Example 2.1, the class of all b-Λ-sets is $\{X,\phi,\{b\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$ and b-$\vee$-sets is $\{X,\phi,\{a\},\{c\},\{d\},\{a,c\},\{a,d\},\{c,d\},\{a,c,d\}\}$.

Proposition 2.1. *In a space* (X,τ), *the class of all* b-Λ- (*resp.* b-$\vee$-) *sets is a topological space.*

Proof. It is obvious from Definition 2.1 that X, ϕ and the intersection of b-Λ-sets is b-Λ-sets. To prove the union of b-Λ-sets is b-Λ-sets, let $\{A_i : i \in I\}$ be a family of b-Λ-sets in (X,τ) and $A = \bigcup\{A_i : i \in I\}$. Then by Lemma 2.1 we have $A = \bigcup\{\Lambda_b(A_i) : i \in I\} = \Lambda_b(A)$ which complete the proof. □

Remark 2.2. Since τ^{Λ_b} (resp. $\tau^{\vee_b}$) is closed under arbitrary intersection, then such class is Alexandröff space. Then τ^{Λ_b} (resp. $\tau^{\vee_b}$) contains all b-open sets (resp b-closed sets). Also, τ^{Λ_b} (resp. $\tau^{\vee_b}$) containing τ^{Λ_s} ([7]) and τ^{Λ_p} ([10]) (resp. $\tau^{\vee_s}$ ([7]) and $\tau^{\vee_p}$ ([10])), but Andrijević ([3]) proved that the topology generated by b-open sets equal the topology generated by preopen sets.

Now, we investigate some properties in the class of τ^{Λ_b} (resp. $\tau^{\vee_b}$).

Proposition 2.2. *In a space* (X,τ^{Λ_b}) *the following statements are verified*:

(1) *If every* $A \subset X$ *is nowhere dense then* $\tau^{\Lambda_b} = \tau^{\Lambda_s}$.

(2) *If* (X,τ) *is an indiscrete space, then each* b-Λ-*set is pre*-Λ-*set but not semi*-Λ-*set.*

Proof. (1) Since every A is nowhere dense, then $SO(X,\tau) = BO(X,\tau)$. So $\tau^{\Lambda_b} = \tau^{\Lambda_s}$.

(2) Obvious, since each b-open set in indiscrete space is preopen but not semi-open. □

Theorem 2.1. *If τ^{Λ_b} is quasi-discrete and for any $A \in \tau^{\Lambda_b}$ then $A \in \tau^{\Lambda_s}$ and $\tau^{\Lambda_b} = \tau^{\Lambda_s}$.*

Proof. Since $A \in \tau^{\Lambda_b}$ and $X - A \in \tau$, then A is Λ_b-open set and closed. Therefore $A \subseteq \bigcap\{B : B \subseteq A$ and B is b-open$\} \subseteq \bigcap\{cl(int(B)) \cup int(cl(B))\} = \bigcap\{cl(int(B)) \cup int(B)\} = \bigcap\{cl(int(B)) : B \subseteq A\}$. So $A \in \tau^{\Lambda_b}$. □

Theorem 2.2. *In a space (X, τ), any b-Λ-set A is pre-Λ-set if one of the following condition hold*:

(1) *(X, τ) is E.D.*

(2) *$(X - A)$ is dense in X.*

Proof. (1) Since A is b-Λ-set, then A is b-open set or the intersection of b-open sets. If A is b-open set and (X, τ) is E.D then $A \subseteq cl(int(A)) \cup int(cl(A) \subseteq int(cl(int(A))) \cup int(cl(A)) = int\,(cl\,(A))$. Therefore A is preopen set. So A is pre-Λ-set . When A the intersection of b-open sets and every b-open set in E.D space is preopen. Then A is the intersection of preopen set. So A is pre-Λ-set.

(2) Since A is b-Λ-set and $(X - A)$ dense in X, then $int(A) = \phi$ and $A \subseteq cl(int(A)) \cup int(cl(A)) = int\,(ci\,(A))$. Hence A is preopen and pre-Λ-set. □

Theorem 2.3. *In a space (X, τ), the following statements are verified*:

(1) *Every β-open and closed is b-Λ-set.*

(2) *If (X, τ) is Submaximal and E.D, then every α-open set is b-Λ-set.*

(3) *If $U \in \tau^{\alpha}$ and A is b-open then $A \cap U$ is b-Λ-set in τ_U.*

(4) *For $U \subset Y \subset X$ and U is b-open set in τ_Y and Y is b-open set then U is b-Λ-set if $U \notin [\beta O\,(X, \tau) - BO\,(X, \tau)]$.*

Proof. (1) Since if every $A \subset X$ is β-open and closed then it is b-open. Therefore A is b-Λ-set.

(2) If (X, τ) is Submaximal and E.D, then $\tau^{\alpha} = b(X, \tau)$. So α-open set is b-Λ-set.

(3) Since $U \cap A \subset int\,(cl\,(int\,(U))) \cap (int\,(cl\,(A)) \cup cl\,(int\,(A)))$

$$\begin{aligned}
&= (int\,(cl\,(int\,(U))) \cap int\,(cl\,(A))) \cup (int\,(cl\,(int\,(U))) \cap cl\,(int\,(A))) \\
&\subset int\ (int\,(cl\,(int\,(U))) \cap int\,(cl\,(A))) \cup cl\ \ (int\,(cl\,(int\,(U))) \cap int\,(A)) \\
&\subset int\ (cl\,(int\,(U)) \cap int\,(cl\,(A))) \cup cl\ \ (cl\,(int\,(U)) \cap int\,(A)) \\
&\subset int(cl\,(int\,(U)) \cap int\,(cl\,(A))) \cup cl\,(cl\,(int\,(U) \cap int\,(A))) \\
&\subset (U \cap int(cl\,(int\,(U)) \cap int\,(cl\,(A)))) \cup (U \cap cl\,(int\,(U) \cap int\,(A))) \\
&\subset int_U(cl\,(int\,(U)) \cap int\,(cl\,(A))) \cup cl_U\,(int\,(U) \cap int\,(A)) \\
&\subset int_U\,(cl\,(int\,(U) \cap int\,(cl\,(A))))) \cup cl_U\,(int\,(U) \cap int\,(A)) \\
&\subset int_U\,(cl\,(int\,(U) \cap cl\,(A)))) \cup cl_U\,(int\,(U) \cap int\,(A))
\end{aligned}$$

$$\subset \quad int_U\left(cl\left(cl\left(int\left(U\right)\cap A\right)\right)\right)\cup cl_U\left(int\left(U\right)\cap int\left(A\right)\right).$$

Then

$$\begin{aligned} & U\cap A\subset int_U U\cap\left(cl\left(int\left(U\right)\cap A\right)\right)\cup cl_U\left(int_U\left(int\left(U\right)\cap int\left(A\right)\right)\right) \\ \subset\quad & int_U\left(cl_U\left(int\left(U\right)\cap A\right)\right)\cup cl_U\left(int_U\left(U\cap A\right)\right) \\ \subset\quad & int_U\left(cl_U\left(U\cap A\right)\right)\cup cl_U\left(int_U\left(U\cap A\right)\right). \end{aligned}$$

Therefore $U\cap A$ is b-open set in τ_U and it is b-Λ-set in τ_U.

(4) Obvious from Lemma 1.2. □

Remark 2.3. The condition $U\notin\left[\beta O\left(X,\tau\right)-BO\left(X,\tau\right)\right]$ in Theorem 2.3.(4) is necessary as shown by the following example.

Example 2.4. Let $X=\{a,b,c,d\}$ with topology $\tau=\{X,\phi,\{b\},\{a,c\},\{a,b,c\}\}$ and $Y=\{b,c,d\}$ .The subset $A=\{c,d\}\in\tau^{\Lambda_b}$ while $A\notin BO(X,\tau)$.

Definition 2.3. A space (X,τ) is said to be:

(1) $B-T_{1/2}$ space if each singleton is either b-open or b-closed set.

(2) $B-T_1$ space if for each pair of distinct point x and y of X there exist two b-open sets U, V such that $x\in U$, $y\notin U$ and $y\in V$, $x\notin V$. Clearly a space (X,τ) is $B-T_1$ if each singleton is b-closed.

Lemma 2.3. *The space of b-open set is $b-T_{1/2}$ space.*

Proof. Since each singleton of each space is either preopen or preclosed ([10]) and each preopen set is b-open set. Then each singleton is either b-open or b-closed. This complete the proof. □

Theorem 2.4. *For a space (X,τ) the following are equivalent*:

(1) *(X,τ) is $B-T_1$.*

(2) *Every subset of X is b-Λ-set.*

(3) *Every subset of X is b-$\vee$-set.*

(4) *Every preopen and semi-open subsets of X is b-$\vee$-set*

Proof. Clearly (2) $\Leftrightarrow$ (3).

(1) $\Rightarrow$ (3) Let $A\subset X$. Since $A=\bigcup\{\{x\}:x\in A\}$, then A is a union of b-closed sets. Hence it is b-$\vee$-set.

(3) $\Rightarrow$ (4) Obvious.

(4) $\Rightarrow$ (1) Since each singleton is either preopen or preclosed. Let $x\in X$. If $\{x\}$ is preopen then, it is b-$\vee$-set and so is b-closed. If $\{x\}$ is preclosed then it is b-closed. For semi-openness,it is obvious from [7] Theorem 2.6(4). □

Proposition 2.3. *Let (X,τ) be a topological space. Then*

(1) (X, τ^{Λ_b}) *and* $(X, \tau^{\vee_b})$ *are always* $T_{1/2}$.

(2) *If* (X, τ) *is* $B - T_1$, *then* (X, τ^{Λ_b}) *and* $(X, \tau^{\vee_b})$ *are discrete space.*

Proof. Obvious from Theorem 2.4 and Lemma 2.3. □

Corollary 2.1. *If* (X, τ) *is resolvable, then* (X, τ^{Λ_b}) *and* $(X, \tau^{\vee_b})$ *are discrete.*

Proof. Since (X, τ) is resolvable, then (X, τ^{Λ_p}) and $(X, \tau^{\vee p})$ are discrete ([10]). Also, since $(X, \tau^{\Lambda p})$ and $(X, \tau^{\vee p})$ are subset of (X, τ^{Λ_b}) and $(X, \tau^{\vee_b})$ respectively, then (X, τ^{Λ_b}) and $(X, \tau^{\vee_b})$ are discrete. □

3. Generalized b-Λ- (b-$\vee$-) sets

In this article, we introduced a new concepts of generalized b-Λ- (resp. b-$\vee$-) sets and study its connection with b-Λ- (resp. b-$\vee$-) sets.

Definition 3.1. A subset A of a space (X, τ) is called:

(1) A generalized b-Λ-sets (abb. g-Λ_b-sets) if $\Lambda_b(A) \subseteq P$ whenever $A \subseteq P$ and $P \in BC(X, \tau)$.

(2) A generalized b-$\vee$-sets (abb. g-$\vee_b$-sets) if $V \subseteq \vee_b(A)$ whenever $V \subseteq A$ and $V \in BO(X, \tau)$.

Proposition 3.1. *Let A be a subset of a space* (X, τ) :

(1) *A is generalized Λ_b-set if and only if A is Λ_b-set.*

(2) *A is generalized $\vee_b$-set if and only if A is $\vee_b$-set.*

Proof. (1) Clearly, every Λ_b-set is generalized Λ_b-set. To prove the conversely, let A be a generalized Λ_b-set and there exists $x \in \{\Lambda_b(A) - A\}$. Observe that $\{x\}$ is b-open or b-closed and that $A \subseteq \{X - \{x\}\}$. If $\{x\}$ is b-open, then $\{X - \{x\}\}$ is b-closed . So $\Lambda_b(A) \subseteq \{X - \{x\}\}$,which gives a contradiction. If $\{x\}$ is b-closed, then $\{X - \{x\}\}$ is b-open and so $\Lambda_b(A) \subseteq \{X - \{x\}\}$, which also gives contradiction. Hence A is Λ_b-set.

(2) In the same manner of 1. □

Definition 3.2. Let Y be a subset of topological space (X, τ). A subset A of Y is a g-Λ_b-set (resp. g-$\vee_b$-set) relative to Y if A is a g-Λ_b-set (resp. g-$\vee_b$-set) of the subset space (Y, τ_Y) of (X, τ). For a subset A of Y. We define a subset $\Lambda_b(A_Y)$ by $\Lambda_b(A_Y) = \cap\{U \cap Y .: U \in BO(X, \tau), A \subseteq U \cap Y\}$. Then $\Lambda_b(A_Y) = \Lambda_b(A) \cap Y$.

Proposition 3.2. *Let Y be a b-Λ-set of a topological space (X, τ) and $A \subseteq Y$. Then A is g-Λ_b-set in (X, τ) if and only if A is g-Λ_b-set relative to Y.*

Proof. Obvious from Definition 3.2 and Proposition 3.1. □

4. Λ_b-functions and $\vee_b$-functions

Definition 4.1. A function $f : (X, \tau) \to (Y, \sigma)$ is called:

(1) Λ_b-function if $f(U) \in BC(Y, \sigma)$ for all $U \in \tau^{\Lambda_b}$.

(2) $\vee_b$-function if $f(G) \in BO(Y, \sigma)$ for all $G \in \tau^{\vee_b}$.

Remark 4.1. We note that Λ_b-functions and $\vee_b$-functions are independent as in the following example.

Example 4.1. Let $X = \{a, b, c, d\}$ with the topology $\tau = \{X, \phi, \{a\}\}$ and $Y = \{u, v, w\}$ with the topology $\sigma = \{Y, \phi, \{u\}, \{w\}, \{u, w\}\}$. Then the class of b-open set are $BO(X, \tau) = \{X, \phi, \{a\}, \{a, b\}, \{a, c\}\{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$ and $BO(Y, \sigma) = \{Y, \phi, \{u\}, \{w\}, \{u, v\}, \{u, w\}, \{u, w\}\}$. Define f_1, f_2, f_3 from (X, τ) into (Y, σ) as follows $f_1\{a\} = f_1\{c\} = \{v\}$, $f_1\{b\} = f_1\{d\} = \{u\}$, $f_2\{a\} = f_2\{b\} = \{u\}, f_2\{c\} = f_2\{d\} = \{w\}$, $f_3\{a\} = \{v\}, f_3\{b\} = \{u\}, f_3\{c\} = f_3\{d\} = \{w\}$. Then f_1 is Λ_b-function but not $\vee_b$-function. Also, f_2 is $\vee_b$-function but not Λ_b-function. Also, f_3 are Λ_b-function and $\vee_b$-function.

Now, we state some results on such types of functions.

Theorem 4.1. *For a map $f : (X, \tau) \longrightarrow (Y, \sigma)$. The following are equivalent*:

(1) *f is Λ_b-map.*

(2) *For all $B \subseteq Y$, $F \in \tau^{\vee_b}$ with $f^{-1}(B) \subseteq F$, there exists $O \in BO(Y, \sigma)$ with $B \subseteq O$ and $f^{-1}(O) \subseteq F$.*

Proof. (1) $\Longrightarrow$ (2) Let $B \subseteq Y$, $F \in \tau^{\vee_b}$ with $f^{-1}(B) \subseteq F$ and put $O = [f\ (F^c)]^c$, since f is Λ_b-map, $f\ (F^c) \in BC(Y, \sigma)$. Hence $O \in BO(Y, \sigma)$. Since $f^{-1}(B) \subseteq F$, we have $f\ (F^c) \subseteq B^c$. Then $B \subseteq [f\ (F^c)]^c = O$. Moreover $f^{-1}(O) = f^{-1}[f\ (F^c)]^c \subseteq [F^c]^c = F$.

(2) $\Longrightarrow$ (1) Let $A \in \tau^{\Lambda_b}$, $y \in (f\ (A))^c$ and let $F = A^c$. Since $F \in \tau^{\vee_b}$ and by (2) when $B = \{y\}$, there exists $O_y \in BO(Y, \sigma)$ with $y \in O_y$ and $f^{-1}(O_y) \subseteq F$. So $y \in O_y$, $f^{-1}(O_y) \subseteq F$ and $y \in O_y \subseteq (f(A))^c$. Hence $(f(A))^c = \bigcup\{O_y : y \in (f(A))^c\} \in BO(y, \sigma)$. Thus $f(A) \in BC(y, \sigma)$. Therefore f is Λ_b-function. □

Theorem 4.2. *For a map $f : (X, \tau) \to (Y, \sigma)$ the following are equivalent.*

(1) *f is $\vee_b$-function.*

(2) *For all $B \subset Y$, $O \in \tau^{\wedge_b}$ with $f^{-1}(B) \subseteq O$, there exists $F \in BC(Y, \sigma)$ with $B \subseteq F$ and $f^{-1}(F) \subseteq O$.*

Proof. Similar as Theorem 4.1 and using Lemma 2.1 in [7]. □

Theorem 4.3. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \delta)$ be functions such that $g \circ f : (X, \tau) \to (Z, \delta)$ then*:

(1) *$g \circ f$ is Λ_b-function if f is Λ_b-function and g is $\vee_b$-function.*

(2) *$g \circ f$ is $\vee_b$-function if f is $\vee_b$-function and g is Λ_b-function.*

Proof. (1) Let $G \in \tau^{\Lambda b}$. Since f is Λ_b-function, then we have $f(G) \in BC(Y, \sigma)$. Consequently $f(G) \in \sigma^{\vee_b}$. Since g is $\vee_b$-function, then $g(f(G)) \in BO(Z, \delta)$.

(2) Similarly to part 1. □

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