Fibers and Polymers 2005, Vol.6, No.4, 332-335

New Definition of the Fibrogram and Its Application to Cotton Blending

Boong Soo Jeon*

Department of Textile Engineering, Sungkyunkwan University, Suwon 440-746, Korea
(Received July 26, 2005; Revised September 19, 2005; Accepted September 26, 2005)

Abstract: The fibrogram theory is newly derived from the superposition principle of the conventional staple diagram, in which the left-hand ends of the fibers have to share a common starting point in order for the fiber length distribution to be measured, and the right-hand ends of the fibers form points. It is shown that the fibrogram is the staple diagram of the fiber sample having different random starting points, as well as the double cumulative distribution function of the frequency length function in the length biased sample. Also, the various means, viz. the numerical mean length, numerical mean length in median, length biased mean length, and length biased mean length in median, and the various upper half means, viz. the numerical upper half mean length, numerical upper half mean length in median, length biased upper half mean length, and length biased upper half mean length in median, are discussed in relation to the cotton blending process.

Keywords: Fibrogram, Fiber length distribution, Staple diagram, Beard diagram, Fiber blending

Introduction

The fiber length is considered to be one of the most important physical properties of cotton in terms of its processing performance and the quality of the final product. The judgment made by a cotton classer, who determines the fiber length based on a simple visual inspection of the sample beard, is made in a very unscientific manner and is subject to numerous sources of error, so that a device that is able to measure the fiber length more accurately is very much needed. Moreover, since it is very difficult and time consuming to measure the length of each individual fiber, most of the existing test methods and instruments that have been developed for fiber length analysis measure the length and mass or weight of each group of fibers. The fiber length characteristics are then obtained from the lengths and masses of the fiber groups. High volume fiber testing methods use this principle, and the fibrogram is one of these high volume fiber testing methods.

The theory of the fibrogram was first introduced by Hertel [1]. He considered the abscissa as the length axis and the ordinate as total length axis of all the fibers by the number. Since Hertel introduced his theory, this ordinate has been variously referred to as the weight axis [2], the number or relative number axis [1,3], the probability axis [2,4], or the proportional mass or mass probability axis [5]. Differences in the interpretation of the methods and equipment required for preparing fiber beards may have given rise to these different theories. The particular interpretation depends on whether the sample selected from the cotton population to be measured is length biased or not. Hertel noted that "the simple selection of a single fiber at a time would give a length biased sample, whereas there is no length bias in selecting fibers by groups." However, under the assumption that the fibrosamplers used to prepare the fiber beards have an equal probability of selecting any given length fiber rather than the probability being proportional to the fiber length, it has been reported that the ordinate of the fibrogram experimentally represents the number of fibers [6].

Some techniques have been proposed for measuring the fiber length, such as the Suter-Webb Array, Almeter, AFIS (Advanced Fiber Information System), and HVI(High Volume Instrument) methods. In general, obtaining samples from the entire fiber population has been carried out in two different ways. One is the numerical sampling method, in which a numerical sample of length data is the set of all fiber lengths, and the number of fibers of each length in the sample is proportional to the corresponding number of fibers in the overall population. The other is the length or weight biased sampling method, in which the number of fibers of each length is proportional to the total length or weight of these fibers. Also, two different methods of measurement are used, viz. the numerical and weight biased methods. The Suter-Webb Array method uses a numerical sampling method and a weight biased measurement, while the Almeter and AFIS techniques are based on a numerical sampling method and a numerical measurement. It is well known that the HVI method is based on the length biased sampling method and a numerical measurement. Therefore, the fibrogram obtained from the HVI method represents the length biased distribution, which contains less short fibers and more long fibers than the original population, with the discrepancy being in direct proportion to the fiber length.

In this work, we newly derive the definition of the fibrogram from a frequency-density function of the fiber length and show that the fibrogram represents a length biased distribution of the fiber length. We also obtain various fiber mean lengths from the fibrogram, such as the numerical mean length, the numerical mean length in median, the length biased mean length, the length biased mean length in median, the numerical upper half mean length, the numerical upper half mean length in median, the length biased upper half mean length, and the length biased upper half mean length in median. We

*Corresponding author: jeonbs@skku.edu

also discuss the application of these mean lengths and their ratios to the process of cotton blending.

Derivation of the Fibrogram

The sliver is clamped at various random positions along its length, and the fibers, which are not clamped, are combed out. Then, the fibers that are clamped form the beard, and the fiber length is measured by using the shape of this beard. This is the principle of the fibrogram. Before the clamped sliver is combed out, the sample is a numerical sample and is similar to the sample picked up by the fibrosampler used to prepare the fiber beards from the cotton lump. If the fibers are combed out of the beard, the probability of a fiber being clamped becomes proportional to its length. This kind of sample is a length biased sample. This beard is very similar to the final sample of the fibrosampler, in the case of fibers which are carded as they pass over a section of card wire. If the different lengths of fibers are uniformly distributed in the sliver along its every length, all the length distributions of the fibers held by the clamp at the different random positions should be identical.

Let us assume that a sliver contains a total of N fibers in the cross-sectioned areas, and that the frequency-density function of the fibers held by the clamp having a fiber length ℓ is $N(\ell)$. The left-ends of the fibers have to share a common starting point, in order for us to be able to measure the fiber length distribution, and the right-hand ends of the fibers would form points on the staple diagram, which is referred to as $Q(\ell)$. In $Q(\ell)$, the fibers lie horizontally and are arranged in order of decreasing length, with the left-hand ends located on the ordinate. The whole surface is uniformly covered by fibers. If all the fibers have uniform fineness, then the ordinate of $Q(\ell)$ represents the number of fibers in the beard having a length equal to ℓ. It is well known that $Q(\ell)$ is the first integral of the frequency-density function, $N(\ell)$, which can be expressed as follows.

$$Q(\ell) = \int_{\ell}^{\ell_m} N(\ell)d\ell \tag{1}$$

However, it is clear that the left-hand ends of the fibers in the sliver cannot share a common starting point, when the sliver is clamped at different random positions along its length. Therefore, the left-hand ends of the fibers clamped in the sliver need to have a common starting point, in order for us to be able to obtain the $Q(\ell)$ curve.

Among the fibers clamped at the random position, i, of the sliver, those having the same starting point are selected and arranged in order of decreasing length. The right-hand ends of the fibers form points on the diagram, $q_i(\ell)$. Let the number of fibers clamped be n_i. Similarly, those fibers clamped at the position, ΔL, apart from those clamped at the position, i, in the left direction are assumed to have corresponding values of $q_{i-1}(\ell)$ and n_{i-1}. By repeating this sampling method up to the largest fiber length, ℓ_m, in the left direction, we obtain $q_{i-2}(\ell)$, $q_{i-3}(\ell)$, ... $q_{i-p}(\ell)$, and n_{i-2}, n_{i-3}, ... , n_{i-p}, where the sampling number $p = \ell_m/\Delta L$. Since we assumed that the fibers were uniformly distributed in the sliver along its every length, all the length distributions and the number of fibers held by the clamp at the different positions satisfy equation (2).

$$q_i(\ell) = q_{i-1}(\ell) = q_{i-2}(\ell) = \dots = q_{i-p}(\ell) = q(\ell)$$
$$n_i = n_{i-1} = n_{i-2} = \dots = n_{i-p} = n \tag{2}$$

The $Q(\ell)$ curve is the staple diagram of all the fibers clamped at position i, and becomes the sum of the $q(\ell)$ curves, and $N = n_i = n_{i-1} = n_{i-2} = \dots = n_{i-p}$ is also satisfied. Therefore, the $q(\ell)$ and n of each sample satisfy $q(\ell) = Q(\ell)/(\ell_m/\Delta L)$ and $n = N/(\ell_m/\Delta L)$, respectively.

When the sliver is clamped at a random position, the number of fibers having the length $\ell \sim \ell + d\ell$ will be $q(\ell)d\ell$ in the $q_i(\ell)$ curve, and at that clamping position, the fibers that are clamped also include the fibers of length $(\ell + \Delta L) \sim (\ell + \Delta L + d\ell)$ in $q_{i-1}(\ell)$, $(\ell + 2\Delta L) \sim (\ell + 2\Delta L + d\ell)$ in $q_{i-2}(\ell)$, ..., and $(\ell + p\Delta L) \sim (\ell + p\Delta L + d\ell)$ in $q_{i-p}(\ell)$, and the corresponding numbers of fibers are $n_{i-1} = q_{i-1}(\ell + \Delta L)d\ell$, $n_{i-2} = q_{i-2}(\ell + 2\Delta L)d\ell$, ..., and $n_{i-p} = q_{i-p}(\ell + p\Delta L)d\ell$, respectively.

Then, the number of fibers having length ℓ or longer, N_1, can be described using equation (3), which is obtained from equation (2).

$$N_1 = [q(\ell) + q(\ell + \Delta L) + q(\ell + 2\Delta L) + \dots + q(\ell + p\Delta L)]d\ell \tag{3}$$

Substituting $Q(\ell) = p \times q(\ell)$ and $N = p \times n$ into equation (3), the number of fibers having length ℓ or longer, N_2, in the sample clamped at a random position can be expressed in the $Q(\ell)$ curve in the form of equation (4).

$$N_2 = [Q(\ell) + Q(\ell + \Delta L) + Q(\ell + 2\Delta L) + \dots + Q(\ell + \ell_m/\Delta L \times \Delta L)]d\ell \tag{4}$$

By dividing the interval, ΔL, into the infinitesimal interval, $d\ell$, we obtain the continuous function, $R(\ell)$ of equation (4), instead of N_2, as follows:

$$R(\ell) = \int_{\ell}^{\ell_m} Q(\ell)d\ell \tag{5}$$

The $R(\ell)$ curve is the so-called beard diagram, and its ordinate denotes the number of fibers with length ℓ or longer, when the fibers are held by the clamp at a random sliver position and those fibers that protrude from the clamp are arranged in order of decreasing length with their left-hand ends located on the ordinate.

Here, we can conclude that $Q(\ell)$ is the staple diagram of the fiber sample having a common starting point, and $R(\ell)$ is the staple diagram of the fiber sample having different random starting points. $R(\ell)$ is also the double cumulative distribution function of the frequency length function in the length biased sample.

The value of $R(0)$ becomes the total length of the fibers in the sample, as follows:

$$R(0) = \int_0^{\ell_m} Q(\ell)d\ell = N\bar{\ell} \quad (6)$$

where $\bar{\ell}$ is the mean length of the length biased sample.

$Q(\ell)$ and $N(\ell)$ can be obtained by the differentiation of $R(\ell)$ and $Q(\ell)$, respectively, as follows:

$$Q(\ell) = -R'(\ell) \quad (7)$$

$$N(\ell) = -Q'(\ell) = R''(\ell) \quad (8)$$

Therefore, $R'(0) = -Q(0) = N$, or the tangent at the origin of the fibrogram is N and meets the axis of the abscissa at the point on the abscissa corresponding to the length biased mean length of the fibers.

Let the probability-density function of a numerical sample be $p(\ell)$. Then, that of a length biased sample is given by:

$$N(\ell) = N\frac{\ell p(\ell)}{\int_0^{\ell_m} \ell p(\ell)d\ell} \quad (9)$$

From equation (9), the frequency-density function of a numerical sample, $f_n(\ell)$, can be approximated as follows:

$$f_n(\ell) = N\frac{N(\ell)/\ell}{\int_0^{\ell_m} N(\ell)/\ell d\ell} \quad (10)$$

From equation (10), the staple diagram, $Q_n(\ell)$, and the beard diagram, $R_n(\ell)$, of a numerical sample can also be calculated by the integration of, as follows:

$$Q_n(\ell) = \int_{\ell}^{\ell_m} f_n(\ell)d\ell \quad (11)$$

$$R_n(\ell) = \int_{\ell}^{\ell_m} Q_n(\ell)d\ell \quad (12)$$

Therefore, $Q(\ell)$, $N(\ell)$, $f(\ell)$, Q_n, and $R_n(\ell)$ can be calculated by differentiation of the fibrogram, $R(\ell)$, and some additional mathematical operations. The numerical mean, L_m, and the length biased mean, L_{bm}, can also be calculated using $R(\ell)$ and $R_n(\ell)$. Since $Q(0.5)$ is the median, L_{be}, of the length biased sample and $Q_n(0.5)$ is the median, L_e, of the numerical sample, the four kinds of fiber mean lengths, viz. L_m, L_{bm}, L_e, and L_{be}, can be obtained from the fibrogram. The upper half mean length, UL_{bm}, of the length biased sample can easily be obtained, because the tangent at $R(L_{bm})$ of the fibrogram meets the axis of the abscissa at the point on the abscissa corresponding to the upper half mean length of the length biased sample. Similarly, the upper half mean length, UL_m, the upper half median length, UL_e, of the numerical sample, and the upper half median length, UL_{be}, of the length biased sample can be calculated by using the tangent lines at $R_n(L_m)$, $R_n(L_e)$, and $R_n(L_{be})$, respectively. Since it is also important to have a measure of the length variation, in order to investigate the length distribution of the fiber more precisely, it may be necessary to obtain some of the ratios of the mean lengths to the upper half mean lengths, instead of the means of the fiber lengths themselves.

When many different kinds of cotton are blended, the variables pertaining to the blended sample have to be controlled. Therefore, in the next section, the concept of the linear average will be discussed, in the case where the fiber length distributions follow the very simple model.

Application to Fiber Blending

In general, we selected the cotton bale so as to form a uniform property throughout the blended cotton lots, by measuring the mean length and uniformity index during the cotton blending process. The mean length and uniformity index in each bale are measured and averaged, in order that each blended lot has a uniform value. In this section, we will calculate the average value of each sample to be blended and compare it with the corresponding value of the blended sample.

We first assume that the fiber length of the sample has a linear distribution, as shown in Figure 1, where M_O is the mode length. It is assumed that the largest fiber length of all four samples is 40 mm and their mode lengths are 22 mm, 26 mm, 30mm, and 34 mm, respectively. The numerical mean, L_m, the length biased mean, L_{bm}, the median, L_e, of the numerical sample, and the median, L_{be}, of the length biased sample are calculated and listed in Table 1. As is well known, the average numerical mean of the 4 fibers is the same as that of the blended fiber for the numerical sample. For the length biased sample, the average mean of the 4 fibers is 25.82, while the length biased mean for the blended fiber is 25.78. The resulting error is therefore 0.14. Similarly, errors of 0.89 % and 0.39 % are calculated for the numerical

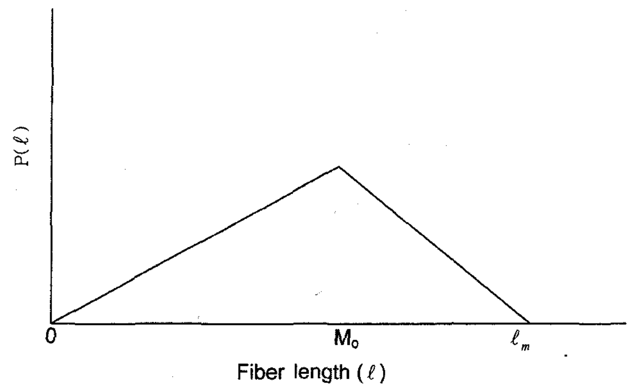

Figure 1. Linear distribution of fiber length.

Table 1. Mean lengths for the blended fibers

	L_m	L_e	L_{bm}	L_{be}
Fiber 1	20.67	20.98	23.90	23.89
Fiber 2	22.00	22.80	25.12	25.79
Fiber 3	23.33	24.49	26.43	27.59
Fiber 4	24.67	26.08	27.81	29.30
Average	22.67	23.59	25.82	26.64
Blended	22.67	23.40	25.78	26.75
%Error	0.00	0.80	0.14	−0.39

Table 2. Upper half mean lengths

	UL_m	UL_e	UL_{bm}	UL_{be}
Fiber 1	27.16	27.35	29.27	28.68
Fiber 2	28.38	28.80	30.11	30.53
Fiber 3	29.79	30.34	26.43	31.90
Fiber 4	31.32	31.95	32.74	33.45
Average	29.17	29.61	29.64	31.14
Blended	29.39	29.76	31.06	31.60
%Error	−0.76	−0.52	−4.58	−1.44

Table 3. Ratios of the mean length to the upper half mean length

	L_m/UL_m	L_e/UL_e	L_{bm}/UL_{bm}	L_{be}/UL_{be}
Fiber 1	0.76	0.77	0.82	0.83
Fiber 2	0.78	0.79	0.83	0.84
Fiber 3	0.78	0.81	1.00	0.86
Fiber 4	0.79	0.82	0.85	0.88
Average	0.78	0.80	0.88	0.85
Blended	0.77	0.79	0.83	0.85
%Error	0.7	1.2	5.4	1.0

median and the length biased median, respectively. These errors can be ignored if we assume that the length distribution is linear. Further investigation of this error would be necessary, however, if the fiber length has a distribution which is non-linear.

The upper half mean lengths of the 4 fibers are listed in Table 2. The discrepancies between the average of the 4 fibers and the upper half mean length of the blended fiber for the length biased sample are found to be larger than those for the numerical sample. In particular, the upper half mean length, UL_{bm}, of the length biased sample, shows a discrepancy of 4.6 %.

The ratios of the mean lengths to the upper half mean lengths are shown in Table 3. The largest error of 5.4 % occurs in the case of L_{bm}/UL_{bm}, which is the so called uniformity index. Here, we have to consider the possibility that some errors are likely to occur when the linear averages for the mean length and the uniformity index are used to control the cotton blending.

Conclusion

The fibrogram is newly derived from the superposition principle of the conventional staple diagram, resulting in the staple diagram of the fiber sample having different random starting points, and the double cumulative distribution function of the frequency length function in the length biased sample. Also, the various means and upper half means are investigated in the blended cotton sample. Some errors were found to occur, when the linear averages for the mean length and the uniformity index are employed to make the properties of the fiber length uniform throughout the cotton lots in the cotton blending process.

Acknowledgement

This paper was supported by Faculty Research Fund, Sungkyunkwan University, 2003.

References

1. K. L. Hertel, *Text. Res. J.*, **10**, 510 (1940).
2. C. B. Landstreet, *Textile Bull.*, **87**, 54-57, 74 (1961).
3. K. L. Hertel and R. Lawson, *Text. Res. J.*, **34**, 866 (1964).
4. H. W. Prier and P. E. Sasser, *Text. Res. J.*, **42**, 410 (1972).
5. R. S. Krowicki and K. E. Duckett, *Text. Res. J.*, **57**, 200 (1987).
6. Y. Chu and C. R. Riley, Jr., *Text. Res. J.*, **67**, 897 (1997).