Mechatronic Control Model of the Wind Turbine with Transmission to Split Power

Tong Zhang, Wenyong Li*, and Yu Du

Abstract: In this paper, a wind turbine with power splitting transmission, which is realized through a novel three-shaft planetary, is presented. The input shaft of the transmission is driven by the rotor of the wind turbine, the output shaft is connected to the grid via the main generator (asynchronous generator), and the third shaft is driven by a control motor with variable speed. The dynamic models of the sub systems of this wind turbine, e.g. the rotor aerodynamics, the drive train dynamics and the power generation unit dynamics, were given and linearized at an operating point. These sub models were integrated in a multidisciplinary dynamic model, which is suitable for control syntheses to optimize the utilization of wind energy and to reduce the excessive dynamic loads. The important dynamic behaviours were investigated and a wind turbine with a soft main shaft was recommend.

Keywords: Mechatronic control model, power splitting transimission, renew energy, variable speed wind turbine.

1. INTRODUCTION

The power generation with wind turbines has grown during the past twenty years. The customers who are environmentally conscious have now been using this clean wind energy from power providers. The global market for the electrical power produced by wind turbine generator has been increasing steadily, which directly pushes the wind power technology into a more competitive area [1]. But in recent years many wind turbines were incapacitated due to damage caused by excessively high dynamic loads from the strong fluctuations of the wind speed. Several solutions have been presented to reduce the dynamic loads and to utilize the wind energy more effectively. To reduce the dynamic loads wind turbines, which use the asynchronous generator at constant rotor speed, was discussed in [2]. Later the wind turbines at variable speed are investigated by many researchers [3-5,7,9,12]. There are two important structures for the variable speed wind turbines. The first structure consists of a fixed ratio transmission and a variable speed generator. The generator is usually connected to the electrical supply system through a rectifier and inverter. The main disadvantages of these solutions are the high cost of electronic hardware, off-setting the main objectives of increased cost effectiveness, a loss of 5-10% in the energy output caused by the rectification and inversion of all generated power, and the harmonics generated by the electronic conversion induce electrical noise into the electrical grid [5]. In order to overcome these disadvantages, a new transmission technology was applied, which splits the power into the main induction generator and a controllable motor. By adjusting the torque of the control motor the speed of the main induction generator can be always run at a desired constant speed to maximize the capture of the wind energy, although wind speeds are strongly variable. This new structure was originally developed for vehicle transmission [6], and has been recently applied in wind turbine [5,7]. Because of the constant speed of the main generator, it is possible to connect the main generator directly to the electrical supply system without the expensive rectifier and inverter. The inverter between the control motor and the electrical supply system is relative simple, small and cheap. The dynamic load on components of the wind turbine can be adjusted with this special transmission [8], but its complete mathematical model, which is suitable for the control syntheses, is not clear.

In this paper the complete mathematical model of

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this new mechatronical structure was derived from combining the sub models, which are the aerodynamics of the rotor, the dynamics of the drive train with the planetary transmission and the dynamics of the power generation unit. All the sub models and the complete mathematical model of the wind turbine were described in Section 2. Some suggestions for optimising the utilization of wind energy and a suitable linearized model for control syntheses were given in Section 3. Some important dynamic behaviours of the wind turbine were investigated in Section 4.

2. MODEL OF THE WIND TURBINE

The schematic structure of the wind turbine with power splitting transmission is shown in Fig. 1. The rotor with the blades converts the wind energy to mechanical energy. The mechanical energy is divided by the planetary transmission into two power flows. One of them is converted into electrical energy through the main generator, which is directly connected to the grid. The other is connected to the grid via the control motor, the rectifier and the inverter. Whether the control motor absorbs energy from the grid or offers energy to the grid depends on the wind velocity. By means of controlling the motor torque an optimised absorption rate of wind energy can be realized.

2.1. Aerodynamic models

The aerodynamic torque T_R depends nonlinearly on the wind speed v_W, the tip-speed ratio λ , and the blade pitch angle β, such that we have [2]

$$T_R = \frac{\rho}{2}\pi R^3 v_W{}^2 c_M(\lambda,\beta) \quad \text{with} \quad \lambda = \frac{R\cdot\omega_R}{v_W}, \quad (1)$$

where R denotes the rotor radius, ρ is the density of air, v_W the wind speed and ω_R is the rotor speed

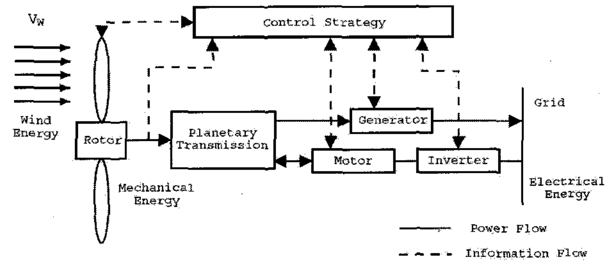


Fig. 1. Schematic structure of the wind turbine with power splitting transmission.

of the wind turbine. Furthermore c_M denotes the aerodynamic torque coefficient and c_P the aerodynamic power coefficient. The wind turbine has different stable operating points according to different wind speeds, at which the rotative accelerations of the system are zero, these operating points are also called balance operating points. The aerodynamic torque $T_R = f(\omega_R,\beta,v_W)$ can be linearized at a balance operating point as

$$\Delta T_R = \frac{\partial T_R}{\partial \omega_R}\Delta\omega_R + \frac{\partial T_R}{\partial \beta}\Delta\beta + \frac{\partial T_R}{\partial v_W}\Delta v_W. \quad (2)$$

In the following, it is assumed that all sub models would be linearized at the same operating point.

2.2. Drive train dynamics

The mechanical power, which is produced by aerodynamic energy, will be transferred via the main shaft and transmission to the generator and the control motor. The mechanical model consists of the shafts, the flexible couplings and the planetary transmission. In Fig. 2 a schematic interpretation of the mechanical model of the wind turbine with planetary transmission is represented.

The losses in the drive train, although small, are important, since the mechanical dynamic modes would otherwise be excessively resonant [9]. The losses are represented as viscous damping on the low-

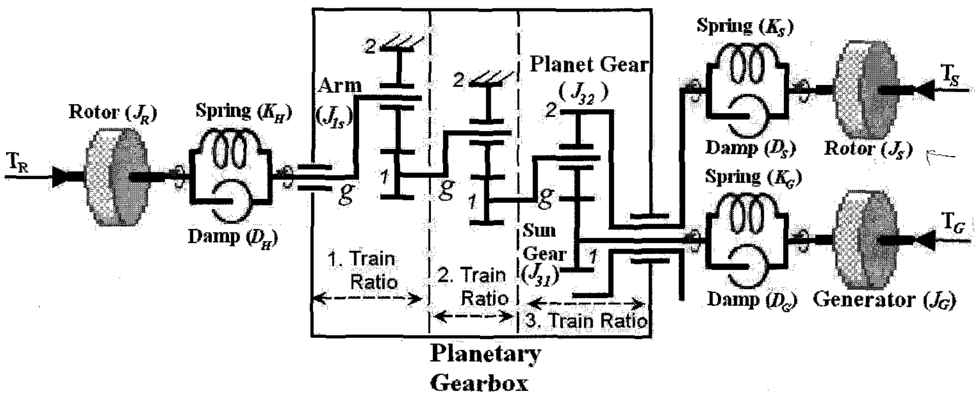


Fig. 2. Mechanical model of the wind turbine with planetary transmission.

speed and high-speed shafts. Most of the damping, e.g. losses in the gearbox, is related to the absolute rotation of the drive train, but part, e.g. internal losses from shafts twisting, is related to the relative rotation of components of the drive train [9].

The angular velocities of third train ratio in Fig. 2 have the following relation:

$$\omega_{31} - i_{312}\omega_{32} - (1 - i_{312})\omega_{3g} = 0$$

with the relations of the angular velocities in the first ratio and the second ratio

$$\omega_{3g} = \frac{1}{i_{2g1}}\omega_{2g} = \frac{1}{i_{1g1}i_{2g1}}\omega_{1g},$$

then we have the equation of angular velocities for the planetary gearbox,

$$\omega_{31} - i_{312}\omega_{32} - \frac{1 - i_{312}}{i_{2g1}i_{1g1}}\omega_{1g} = 0. \quad (3)$$

The torques of third train ratio in Fig. 2 have the following relations [13]:

$$\begin{cases} \dfrac{T_{32}}{T_{31}} = -i_{312}\eta_{321}^{\tau} \\ \dfrac{T_{3g}}{T_{31}} = i_{312}\eta_{321}^{\tau} - 1 \\ \dfrac{T_{3g}}{T_{32}} = \dfrac{1}{i_{312}\eta_{321}^{\tau}} - 1 \end{cases} \quad (4)$$

where $\tau := \frac{T_1(\omega_1 - \omega_g)}{|T_1(\omega_1 - \omega_g)|}$. The relations of the torques in the first ratio and the second ratio are

$$\frac{T_{1g}}{T_{3g}} = \frac{T_{1g}}{T_{11}}\frac{T_{11}}{T_{2g}}\frac{T_{2g}}{T_{21}}\frac{T_{21}}{T_{3g}} = \frac{1}{i_{1g1}i_{2g1}\eta_{1s1}\eta_{2g1}} \quad (5)$$

from (4) and (5) we have the equation of torques for the planetary gearbox,

$$\begin{cases} \dfrac{T_{32}}{T_{31}} = -i_{312}\eta_{321}^{\tau} \\ \dfrac{T_{1g}}{T_{31}} = (i_{312}\eta_{321}^{\tau} - 1)\dfrac{1}{i_{1g1}i_{2g1}\eta_{1g1}\eta_{2g1}} \\ \dfrac{T_{1g}}{T_{32}} = (\dfrac{1}{i_{312}\eta_{321}^{\tau}} - 1)\dfrac{1}{i_{1g1}i_{2g1}\eta_{1g1}\eta_{2g1}}. \end{cases} \quad (6)$$

According to the principle of conservation of angular momentum the drive train has the following relations

$$\begin{aligned} T_R - T_{KH} - T_{DH} &= \frac{d\omega_R}{dt}J_R, \\ T_{KH} + T_{DH} - T_{1g} &= \frac{d\omega_{1g}}{dt}J_{1g}, \\ T_G - T_{KG} - T_{DG} &= \frac{d\omega_G}{dt}J_G, \\ T_{KG} + T_{DG} - T_{31} &= \frac{d\omega_{31}}{dt}J_{31}, \\ T_S - T_{KS} - T_{DS} &= \frac{d\omega_s}{dt}J_S, \\ T_{KS} + T_{DS} - T_{32} &= \frac{d\omega_{32}}{dt}J_{32}, \end{aligned} \quad (7)$$

where

$$\begin{cases} T_{KH} := K_H\Delta\varphi_H = K_H(\varphi_R - \varphi_{1g}) \\ T_{DH} := D_H\Delta\omega_H = D_H(\omega_R - \omega_{1g}) \\ T_{KG} := K_G\Delta\varphi_G = K_G(\varphi_G - \varphi_{31}) \\ T_{DG} := D_G\Delta\omega_G = D_G(\omega_G - \omega_{31}) \\ T_{KS} := K_S\Delta\varphi_S = K_S(\varphi_S - \varphi_{32}) \\ T_{DS} := D_S\Delta\omega_S = D_S(\omega_S - \omega_{32}). \end{cases}$$

The dynamic model of the drive train relating to the aerodynamic torque T_R, the generator reaction torque T_G and the torque of the controlled actuator T_S can be derivate from the (3), (6), (7) as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\begin{bmatrix} T_R \\ T_G \\ T_S \end{bmatrix}, \quad \mathbf{A} \in \Re^{9\times 9}, \mathbf{B} \in \Re^{9\times 3}, \quad (8)$$

where the system matrix is

$$\mathbf{A} = \begin{bmatrix} \mathbf{E}_3\mathbf{D}_J & -\mathbf{E}_3\mathbf{D}_J & \mathbf{E}_3\mathbf{K}_J \\ \mathbf{aD} & -\mathbf{aD} & \mathbf{aK} \\ \mathbf{E}_3 & -\mathbf{E}_3 & 0\cdot\mathbf{E}_3 \end{bmatrix} \quad (9)$$

with

$$\mathbf{D}_J := \left[-\frac{D_H}{J_R} \quad -\frac{D_G}{J_G} \quad -\frac{D_S}{J_S}\right]^T,$$

$$\mathbf{D} := [D_H \quad D_G \quad D_S]^T,$$

$$\mathbf{K_J} := \left[-\frac{K_H}{J_R} \quad -\frac{K_G}{J_G} \quad -\frac{K_S}{J_S}\right]^T,$$

$$\mathbf{K} := [K_H \quad K_G \quad K_S]^T,$$

$$\mathbf{a} := \begin{bmatrix} J_{1g} & \frac{1 - i_{312}}{i_{1g1}i_{2g1}}J_{31} & 0 \\ 0 & -i_{312}J_{31} & -J_{32} \\ \frac{1 - i_{312}}{i_{1g1}i_{2g1}}J_{31} & 1 & -i_{312} \end{bmatrix}^{-1} \cdots .$$

$$\cdots \begin{bmatrix} 1 & \frac{1-i_{312}}{i_{1g1}i_{2g1}} & 0 \\ 0 & -i_{312} & -1 \\ 0 & 0 & 0 \end{bmatrix},$$

and the input matrix is

$$\mathbf{B} = \begin{bmatrix} \mathbf{E}_3 & \mathbf{0} \end{bmatrix}^T \begin{bmatrix} \frac{1}{J_R} & \frac{1}{J_G} & \frac{1}{J_S} \end{bmatrix}^T. \tag{10}$$

The vector of the state variable $\mathbf{x}$ is:

$$\mathbf{x} = \begin{bmatrix} \omega_R & \omega_G & \omega_S & \omega_{1g} & \omega_{31} & \omega_{32} & \bar{\varphi}_H & \bar{\varphi}_G & \bar{\varphi}_S \end{bmatrix}^T$$

with

$$\bar{\varphi}_H := \varphi_R - \varphi_{1g},$$
$$\bar{\varphi}_G := \varphi_G - \varphi_{31},$$
$$\bar{\varphi}_S := \varphi_S - \varphi_{32}.$$

2.3. Power generation unit dynamics

The electrical systems in the wind turbine, which include all components for transformation of the mechanical power into electrical power, The main generator and the elements for adjustment of the system (e.g. the actuator for regulation of the blade pitch and control motor), are of crucial importance. The main generator connected to the sun gear of the third ratio of the gearbox directly feeds into the grid. The asynchronous machine with a short circuit rotor is usually used as the main generator in wind turbines due to economy and robustness in operation. In contrast to the synchronous generator it has better damping characteristics, and its flexibility of the rotor speed can contribute to the reduction of the dynamic loads produced by wind fluctuations. In this paper it is also used as the control motor in wind turbines to splitting power. However, the asynchronous generator must absorb reactive power from the grid. in this paper used as the main generator in wind turbines

- Mathematical model of the main Generator

It is assumed that the asynchronous machine satisfies the hypotheses: linear magnetic materials, symmetry of the rotor and the stator, nonlinear flux density distribution only arise from air-gap, and negligible magnetic hysteresis. The asynchronous machine can be simply modelled by the following equations written in the stator reference frame $S(\alpha, \beta)$ [10]:

$$\vec{U}_o^S = R_o \vec{I}_o^S + \frac{d\vec{\Psi}_o^S}{dt},$$
$$\vec{U}_\theta^S = R_\theta \vec{I}_\theta^S + \frac{d\vec{\Psi}_\theta^S}{dt} - j\Omega_L \vec{\Psi}_\theta^S \tag{11}$$

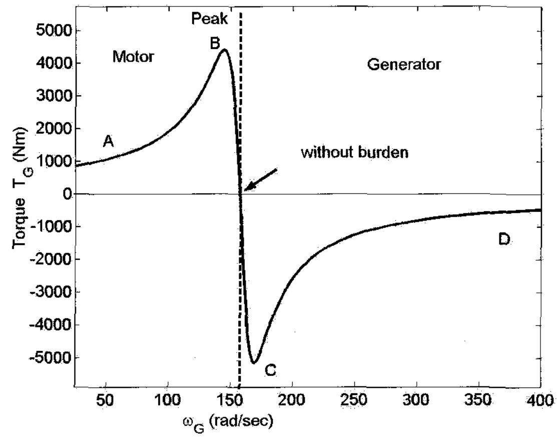


Fig. 3. Torque-speed curve of asynchronous motor.

with $\Omega_L = \Omega_m Z_p$. Die equations of flux linkage are

$$\vec{\Psi}_o^S = L_o \vec{I}_o^S + M \vec{I}_\theta^S,$$
$$\vec{\Psi}_\theta^S = M \vec{I}_o^S + L_\theta \vec{I}_\theta^S. \tag{12}$$

The torque of airgap is

$$T_M = \frac{3}{2} Z_p \frac{M}{L_o} \left(\Psi_{o\beta} I_{\theta\alpha} - \Psi_{o\alpha} I_{\theta\beta} \right). \tag{13}$$

Ψ_o, I_θ can be got from the (11) and (12). The dynamic mechanic equation can be written as

$$J_G \frac{d\Omega_m}{dt} = T_M - T_W. \tag{14}$$

Indices o, θ denote respectively stator and rotor.

- Steady state motor performance

The complex interactions in the electrical machines can be modelled by relatively simple equivalent circuits if the following assumptions are adopted. sinusoidal time variation of applied voltage, sinusoidal space variation of air gap m.m.f. and completely balanced operation will be assumed in the modelling of polyphase a.c. machines [11], the performance of the electrical machine could be represented on a per-phase basis. The nonlinear torque-speed curve of the asynchronous machine according the Kloss'schen equation is shown in Fig. 3. The curve includes three segment: AB , BC and CD . AB is the start process, and the machine operates at the segment BC. The torque-speed of the asynchronous machine could be linearized as

$$\Delta T_M = \frac{\partial T_M}{\partial \Omega_m} \Delta \Omega_m.$$

In the following Model, the airgap torque T_M will be replaced by the generator torque T_G and the mechanical angular velocity Ω_m will be replaced by the rotor speed of the generator ω_G.

$$\Delta T_G = \frac{\partial T_G}{\partial \Omega_G} \Delta \Omega_G. \tag{15}$$

2.4. Combined models

In the above sections, the mathematical models of the components of the wind turbine with planetary transmission for splitting the power were developed. The linearized aerodynamic model, the model of the mechanical system with planetary transmission and the linearized model of the induction generator can be combined in a mathematical model for the whole system. From (8) a linearized model of the mechanical system with planetary transmission can be written with help of the Taylor expansion:

$$\Delta \dot{\mathbf{x}} = \dot{\mathbf{x}} - \dot{\mathbf{x}}_0 = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \Delta \mathbf{x} + \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \Delta \mathbf{u}, \mathbf{f} := \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \mathbf{u} := \begin{bmatrix} T_R \\ T_G \\ T_S \end{bmatrix}.$$

Because the system matrix $\mathbf{A}$, the input matrix $\mathbf{B}$ are independent on $\mathbf{x},\mathbf{u}$, A linearized model of the mechanical system with planetary transmission can be given as

$$\Delta \dot{\mathbf{x}} = \mathbf{A}\Delta \mathbf{x} + \mathbf{B} \begin{bmatrix} \Delta T_R \\ \Delta T_G \\ \Delta T_S \end{bmatrix}. \tag{16}$$

the state variable:

$$\Delta \mathbf{x} = \begin{bmatrix} \Delta\omega_R & \Delta\omega_G & \Delta\omega_S & \Delta\omega_{1g} & \Delta\omega_{31} & \Delta\omega_{32} & \Delta\bar{\varphi}_H & \Delta\bar{\varphi}_G & \Delta\varphi_S \end{bmatrix}^T$$

From the (2), (9), (10), and (15) one can educe the linearized state equation of the system:

$$\Delta \dot{\mathbf{x}} = \tilde{\mathbf{A}}\Delta \mathbf{x} + \tilde{\mathbf{B}} \begin{bmatrix} \Delta\beta \\ \Delta T_S \end{bmatrix} + \tilde{\mathbf{S}}\,[\Delta v_W], \tag{17}$$

the system matrix:

$$\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{E}_3\tilde{\mathbf{D}}_J & -\mathbf{E}_3\mathbf{D}_J & \mathbf{E}_3\mathbf{K}_J \\ \mathbf{aD} & -\mathbf{aD} & \mathbf{aK} \\ \mathbf{E}_3 & -\mathbf{E}_3 & 0\cdot\mathbf{E}_3 \end{bmatrix} \tag{18}$$

with

$$\tilde{\mathbf{D}}_J := \begin{bmatrix} -\frac{D_H}{J_R} + \frac{1}{J_R}\frac{\partial T_R}{\partial \omega_R} & -\frac{D_G}{J_G} + \frac{1}{J_G}\frac{\partial T_G}{\partial \omega_G} & -\frac{D_S}{J_S} \end{bmatrix}^T,$$

the input matrix:

$$\tilde{\mathbf{B}} = \begin{bmatrix} \tilde{\mathbf{B}}_\beta & \tilde{\mathbf{B}}_T \end{bmatrix}$$

with

$$\tilde{\mathbf{B}}_\beta = \begin{bmatrix} \frac{1}{J_R}\frac{\partial T_R}{\partial \beta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\tilde{\mathbf{B}}_T = \begin{bmatrix} 0 & \frac{1}{J_S} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

and disturbance matrix

$$\tilde{\mathbf{S}} := \begin{bmatrix} \frac{1}{J_R}\frac{\partial T_R}{\partial v_W} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T.$$

In this equation the change of the wind velocity could be seen as disturbance of the system, the blade pitch angle $\Delta\beta$ and the torque ΔT_S of the control motor are regarded as input, which should be controlled. This aerodynamic model islinearized at the operating point, however the derivatives $\frac{\partial T_R}{\partial \omega_R}, \frac{\partial T_R}{\partial v_W}$, and $\frac{\partial T_R}{\partial \beta}$ are time-varying, hence the system matrix, the input matrix and the disturbance matrix are time-varying too.

3. ENERGY OPTIMUM

In the below rated power of the wind turbine, the wind is weak. The task of the control system is to make an optimal utilization of the wind energy. The wind power, which can be taken up by a wind turbine, depends on the wind velocity v_W and the aerody-namic power coefficient $c_P(\lambda,\beta)$, which is a function of the blade pitch angle β and the tip-speed ratio λ. If the wind turbine runs in below rated power, the power conversion becomes larger with a smaller blade pitch angle. This means that the blade pitch angle should be equal to zero. In order to obtain optimal wind energy, a frequently considered strategy in below rated wind speed is to track the optimal curve of the drive torque T_{Rmax} by setting the drive train torque directly to $k_1\omega_R^2$ [12].

$$T_{R_{max}} = T_W = k_1\omega_R^2, \tag{19}$$

where k_1 is a constant. According to the relationship between the torques of the input and the output shafts of the planetary gearbox, it is assumed that the efficiency of the transmission is equal to 100%, the torque of the control motor can be given as

$$T_S = k_1 \frac{i_{312}}{\left(\frac{i_{312}-1}{i_{1g1}i_{2g1}}\right)^3}(\omega_G - i_{312}\omega_S)^2. \tag{20}$$

It is clear that the rotor speed of the main generator ω_G and the angular velocity of the control motor ω_S can be control by the torque of the control motor, by which the optimal process can be realized.
The output torque of the control motor is adjustable with the novel power splitting transmission. In below rated power, to get the optimal yield of the wind energy the torque of the control motor should be under control, which can be divided into two components: $\Delta T_{S\,d}$ and $\Delta T_{S\,opt}$. An optimal yield of the wind energy can be reached by controlling the $\Delta T_{S\,opt}$, the $\Delta T_{S\,d}$ (changing at the operating point) can be used to introduce a so-called active damping into the system to reduce the excessively high dynamic load. Through the division of the control torque (17) becomes

$$\Delta\dot{\mathbf{x}} = \tilde{\mathbf{A}}\Delta\mathbf{x} + \tilde{\mathbf{B}}_T[\Delta T_{S\,d}] + \tilde{\mathbf{B}}_T[\Delta T_{S\,opt}] + \tilde{\mathbf{B}}_\beta[\Delta\beta] + \tilde{\mathbf{S}}[\Delta v_W]. \tag{21}$$

4. DYNAMIC BEHAVIOURS

The complete mechatronical mathematical model of the wind turbine with special power splitting transmission was derived from combining the sub models. Now it is possible to investigate its dynamic behaviours under variation of the structure parameters, the change of blade pitch angle, the control torque of the control motor, as well as disturbances from the wind.

4.1. Influence of torsional stiffnesses and dampings

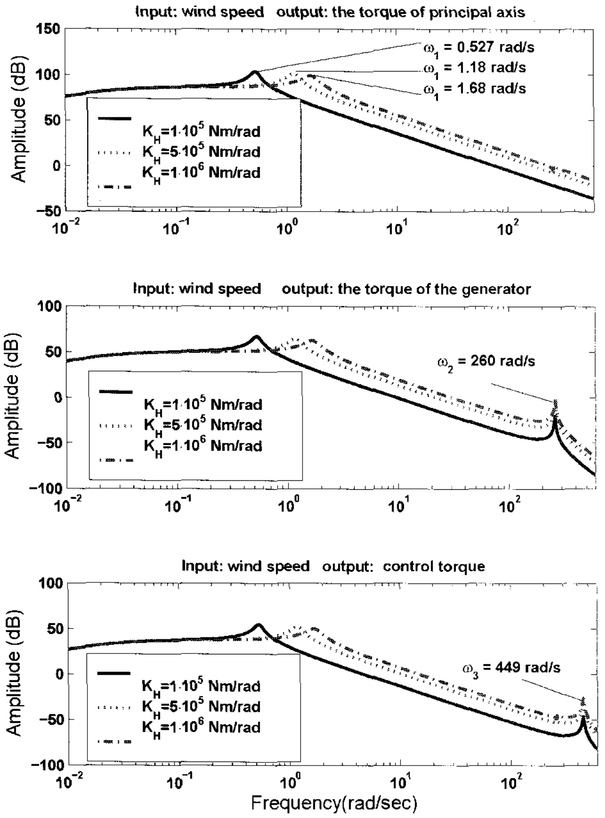
The structural parameters of the system, for example the torsional stiffnesses K_H, K_G, K_S and the damping D_H, D_G, D_S of the main shaft, the generator shaft and the shaft of the control motor, affect the dynamic characteristics of the system. The damping of the system is actually small, nevertheless very important, because otherwise the resonance arises within the system, particularly within the high frequency range. Here the influence of the torsional stiffness of the main shaft on the dynamic behavior of the system is discussed.
The eigenvalues of the system, which are affected by the torsional stiffness K_H, are shown in Table 1. From this table itcan be seen that there are three conjugated complex pairs of poles $s_{1,2}$ $s_{3,4}$ and $s_{5,6}$ in the system model. Therefore, oscillations arise in the system. The imaginary parts of the conjugated complex pairs of the poles are the resonant frequencies of the systems. The three conjugated complex pairs of poles correspond to the three oscillating frequencies of thesystem ω_1, ω_2 and ω_3. From Table 1, one can read out that frequencies corresponding to the eigenvalues $s_{3,4}$ and $s_{5,6}$ are $\omega_2 \approx 260\,\text{rad/s}$ and $\omega_3 \approx 449\,\text{rad}/xs$. The larger the real parts are, the faster the oscillations of the system weaken themselves. The eigenvalues $s_{1,2}$ depend on the torsional stiffness K_H. With rising rigidity K_H, the imaginary parts of the eigenvalues $s_{1,2}$ increase, however the other eigenvalues of the system remain always constant. This means that the change of the stiffness K_H only affects the imaginary parts of the eigenvalues $s_{1,2}$. If the stiffness K_H is changed from $1\cdot10^5\,\text{N m}/rad$ to $1\cdot10^6\,\text{N m/rad}$, the first natural frequency ω_1 varies from $0.53\,\text{rad}/s$ to $1.68\,\text{rad}/s$ accordingly.

Table 1. The eigenvalues of the system with the different torsional stiffness of the main shaft.

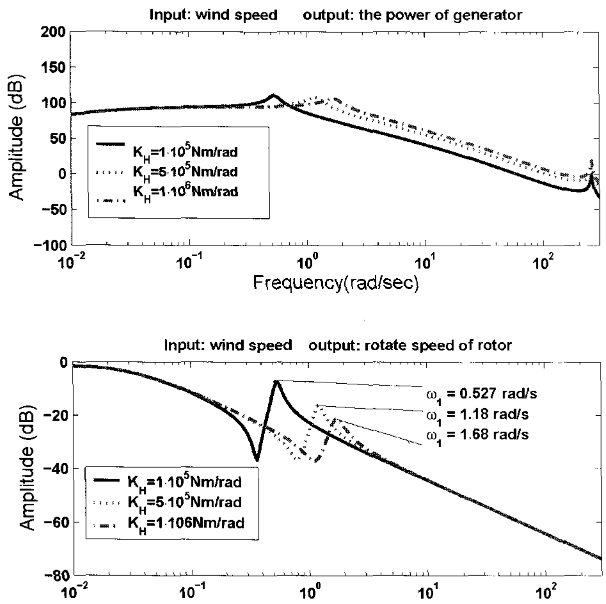
eigen-values	torsinal stiffness of the main shaft K_H (10^5 Nm/rad)				
	1.0	2.0	5.0	7.0	9.0
s_1	−0.04+0.53*i*	−0.05+0.75*i*	−0.11+1.18*i*	−0.14+1.40*i*	−0.18+1.59*i*
s_2	−0.04−0.53*i*	−0.05−0.75*i*	−0.11−1.18*i*	−0.14−1.40*i*	−0.18−1.59*i*
s_3	−5.05+449.4*i*	−5.05+449.42*i*	−5.05+449.46*i*	−5.05+449.49*i*	−5.05+449.52*i*
s_4	−5.05−449.4*i*	−5.05−449.42*i*	−5.05−449.46*i*	−5.05−449.49*i*	−5.05−449.52*i*
s_5	−1.73+259.2*i*	−1.73+259.34*i*	−1.73+259.76*i*	−1.73+260.04*i*	−1.73+260.32*i*
s_6	−1.73−259.2*i*	−1.73−259.34*i*	−1.73−259.76*i*	−1.73−260.04*i*	−1.73−260.32*i*
s_7	−15.03	−14.99	−14.88	−14.81	−14.74
s_8	−0.03	−0.03	−0.03	−0.03	−0.03
s_9	0	0	0	0	0

4.2. Dynamical behavior under disturbances

For the investigation of the dynamical behaviors under disturbances, the turbulence of the wind velocity is regarded as input signal. In Figs. 4(a) and 4(b) the Bode diagrams of the transfer functions with different output are shown. Fig. 4(a) shows the Bode diagrams of the transfer function with the torque of the main shaft, the generator torque and the torque of the control motor as outputs. With a stiffness of $K_H = 5\cdot10^5\,\text{Nm}/rad$, the turbulence of the wind velocity with frequency $1\,\text{rad}/s$ and amplitude $1m/s$ causes an oscillation of the internal torque of the main shaft with an amplitude of approximately $6.5313\cdot10^4\,\text{N m}$. In lower frequency range, the change of the torque follows the fluctuation of the wind velocity. This means that the control system adjusts its resistive torque to suit the variation of the wind speed, in order to satisfy the need for optimizing the absorption rate of the wind energy. The disturbances within higher frequency range affect the torque of the system very slightly. It should be noticed



(a)



(b)

Fig. 4. Dynamic behaviors of the wind turbine under the turbulence.

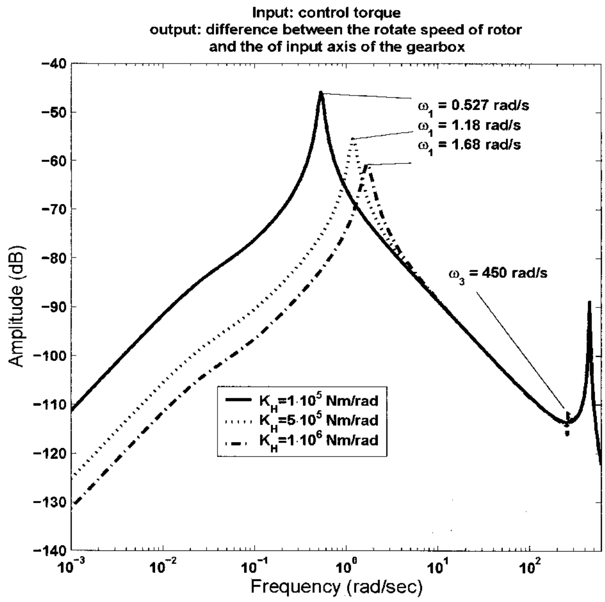
too, that the variation of the stiffness K_H does not affect the Bode diagrams in the low frequency range. The curves of the torques have different peaks at the frequencies of $0.527\text{rad}/s$ to $1.68\text{rad}/s$ accordingly, by which the resonances are caused in the system. The resonant frequency ω_1 increases with rising stiffness K_H. This corresponds to the eigenvalues s_1, s_2, shown in Table 1. In the high frequency range the turbulence of the wind exerts smaller influence on the torques in the case of small stiffness K_H than of large stiffness. This means that aflexible coupling between the rotor and the transmission can isolate high frequency disturbances. The resonant frequency $\omega_2 \approx 260\,rad/s$, which corresponds to the eigen-values s_5, s_6 (see Table 1), appears only in the Bode diagram of the generator torque. This means that the resonant frequency is caused by the flexible coupling and shaft between the generator and the gearbox. Accordingly, the peak, which appears at the frequency $\omega_2 \approx 449\,rad/s$ only in the Bode diagram of the torque of the control motor, corresponds to the eigenvalues s_3, s_4 (see Table 1), and is caused by the flexible motor shaft and coupling.

Fig. 4(b) shows the Bode diagrams with the generator power and the rotor speed as outputs. It can be seen that in the case of a stiffness of $K_H = 5\cdot 10^5\,\text{rad}/s$ a fluctuation of the wind speed around $0.1\,m/s$ with a frequency of $1.18\text{rad}/s$ causes an oscillation of the generator power with an amplitude over approxi-mately $28.1\,kW$. The rotor speed $\Delta\omega_r$ with different stiffnesses K_H have a similar amplitude-frequency response within the low frequency range. But they are strongly changed near the resonant frequency.

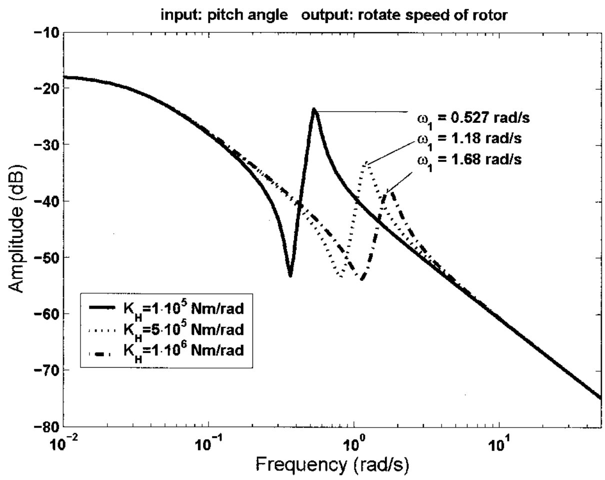
4.3. Dynamic behavior under controllable torque and blade pitch angle

The Bode diagram represented in Fig. 5(a) shows that the difference between the rotor speed and the rotating speed of the input shaft of the transmission ($\omega_R - \omega_{1g}$) is changed with the torque of the control motor, this means that the difference can be controlled by the control motor. When the difference ($\omega_R - \omega_{1g}$) is decreased, the dynamic loads on the main components of the wind turbine are reduced too. In this way an active damping can be realized by adjustment of the control torques. In addition, it should be noticed that the smaller the stiffness K_H is, the stronger the influence of the torque of the control motor on the difference ($\omega_R - \omega_{1g}$) is. Hence, the same control effect can be reached easier with a soft main shaft (small stiffness K_H) than a hard one.

The second controllable input of the system is the blade pitch angle β. The Bode diagram represented in Fig. 5(b) shows that the variation of the blade pitch angle leads to the change of the rotor speed ω_R. The slower the blade pitch angle changes, the stronger it



(a)



(b)

Fig. 5. Dynamic behavior of the Wind Turbine.

influences on the rotor speed. Hence, the wind turbine can always run without the over rated power through the slow change of the pitch angle. In the below rated power (weak wind) the blade pitch angle should be keep to zero, so that the maximum energy capture can be achieved. However the desired fast changes of the blade pitch angle are always not possible in real wind turbine. In the both cases the rotor speed of rotor will be very strongly changed by a slow adjustemnt of the blade pitch angle, its impact on the main Generator must be compensated by the control motor.

5. CONCLUSIONS

A wind turbine with power splitting planetary transmission is introduced and a mathematical model of this variable speed wind turbine is derived from combining three sub models, e.g. the rotor aerodynamics, the drive train dynamics and the power generation unit dynamics. The drive train dynamics consist of flexible elements and the planetary transmission. The complex mathematical model was linearized at an operating point, and a suitable model of the wind turbine was derived with help of the theory maximizing the wind energy capture. So that the multidisciplinary model can be not only directly used to control syntheses, but also to optimize the parameters of the structure by analyzing the dynamic loads on different components of the wind turbine. It shows that the stiffness of the main shaft is very important to the control syntheses. The wind turbine with a soft main shaft has the following advantages:

- reducing the influence of the undesired fluctuations of the wind speed,
- isolating the vibration transfer from rotor to the planetary gearbox and the main generator of the wind turbine,
- increasing the effect of an active damping supplied by the control motor.

The concrete control strategy to reduce the dynamic loads of the wind turbine will be investigated in future

APPENDIX A

Notation

D_i torsional damping
$\mathbf{E}_n$ $n \times n$ identity matrix
i_{jkl} transmission ratio
I current
J_i diametric moments of inertia
J_{jk} diametric moments of inertia of the components in gearbox
K_i torsional stiffness
L self-inductance
M mutual-inductance
R electrical resistance
T_i moment
T superscript denoting transpose
T_W load moment
U voltage
Z_p the pole pair number
α real part of a vector
β image part of a vector
φ angle
η_{jkl} efficiency
ω_i angular velocity
ω_{jk} angular velocity of the components in gearbox
Ω_m the mechanical angular velocity
Ω_L angular velocity of the rotor

Ψ flux linkage
i,j,k,l Indices
$i = R, G, H, S$
$R \rightarrow$ rotor with the blades
$H \rightarrow$ main shaft
$G \rightarrow$ generator shaft
$S \rightarrow$ shaft of the control motor
$j = 1,2,3$ (three train ratios)
$k,l = 1,2,g$ ($1 \rightarrow$ sun gear, $2 \rightarrow$ planet gear, $g \rightarrow$ arm)

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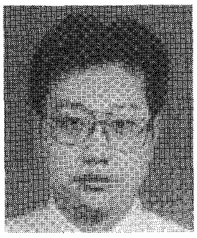

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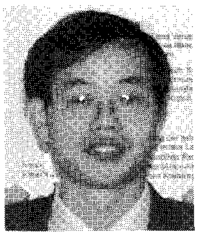

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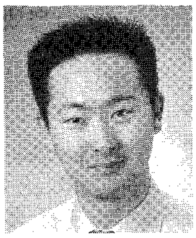

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