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Concurrent Mapping and Localization using Range Sonar in Small AUV, SNUUV I

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Abstract

Increased usage of AUVs has led to the development of alternative navigational methods that use the acoustic beacons and dead reckoning. This paper describes a concurrent mapping and localization (CML) scheme that uses range sonars mounted on SNUUV-I, which is a small test AUV developed by Seoul National University. The CML is one of such alternative navigation methods for measuring the environment that the vehicle is passing through. In addition, it is intended to provide relative position of AUV by processing the data from sonar measurements. A technique for CML algorithm which uses several ranging sonars is presented. This technique utilizes an extended Kalman filter to estimate the location of the AUV. In order for the algorithm to work efficiently, the nearest neighbor standard filter is introduced as the algorithm of data association in the CML for associating the stored targets the sonar returns at each time step. The proposed CML algorithm is tested by simulations under various conditions. Experiments in a towing tank for one dimensional navigation are conducted and the results are presented. The results of the simulation and experiment show that the proposed CML algorithm is capable of estimating the position of the vehicle and the object and demonstrates that the algorithm will perform well in the real environment.

Keywords: AUV, CML, mapping, localization, range sonar

1 Introduction

In the last decade, the use of AUV has been increasing in various areas of Ocean engineering due to the autonomous and long navigation capability substituting for the diver's working time. With the increase in the usage of AUV, it has become essential to have a technique to bound navigation error and to identify the position of AUV during navigation in order to ensure AUV's safe and fully autonomous navigation. In the past, AUV employed the dead reckoning and the Inertia Navigation System (INS). This method, however, has the demerit in that the error of navigation grows unbounded as the navigation times elapses, and thus requires the use of acoustic beacon or Global Position System (GPS) to prevent further occurrence of error so as to obtain the ground fixed relative positioning information for INS (Lee et al, 2003). The drawback of using the acoustic beacon is that it is difficult to deploy and recover the acoustic beacons. In addition, when the GPS is used, AUV must be placed at the surface due to the need to reset their navigation system many times during the operation for the initialization the navigational error which tends to increase whenever they move away from the surface. Thus, there are some missions for AUV to carry out where these method are undesirable or impossible (Smith et al, 1997).

Other alternative approaches have been suggested such as using measured data, for example range and angle of objects, obtained from the environment where the vehicle passes through, as the source of information for ground-fixed relative position (Smith *et al.* 1997). Since most AUVs are equipped, some way or the other, with sonar for measuring altitudes or for detecting obstacles, that sonar data could be used to provide ground fixed navigation in operating environments.

Concurrent Mapping and Localization (CML) is one of the techniques that use sonar data for navigation (Smith et al, 1990). CML is designed to simultaneously identify distinct features of an unknown environment where *a prior* map is not available. It, then, utilizes this information to map the movement of the vehicle (Carpenter, 1998) (Newman, 1999) (Harris and Carpenter, 2001). This technique utilizes EKF to estimate the position. With the Forward Looking Sonar (FLS), which is used by current CML technique, a large amount of information about the environment such as the distance and bearing of object located in front of AUV, etc. can be obtained. But, it has a drawback in that it is difficult to install it in small AUV because of its size and computing power. Therefore, the CML proposed in this paper uses the range sonar as a device offering information on distance to the obstacle and the AUV's position.

The discussion of this paper proceeds as follows. First, the small AUV developed by Seoul National University and the range sonar system is introduced briefly. Then, a decoupled EKF is developed for an idealized measurement environment, and gating to choose the one object close to the existing object and object deleting/creating techniques are discussed. The results of the simulation and experiment conducted to verify the proposed CML algorithm are presented.

2 SNUUV I and range sonar system

SNUUV Ⅰ is a small AUV that was designed and assembled by the Department of Naval Architecture and Ocean Engineering of Seoul National University. SNUUV Ⅰ was installed with a sonar system composed of 4 transducers and amplifier for detecting and avoiding objects that are potentially hazardous. The sonar system was designed to operate under certain constraints such as the range of the sonar must exceed 100 m. Thus, the transducer was selected based on the relationship between the frequency and the absorption of sound in underwater environment. From the result obtained by the relationship mentioned above, the transducer that was selected was TC3029 manufactured by Reson Inc. which can operate under an operating frequency of 500 kHz for over a detecting range of 100m. It is placed at the nose of the AUV, forming a cross pattern to detect the front, starboard, port and downward direction of SNUUV Ⅰ. The detailed features of SNUUV Ⅰ are summarized at Table 1 and a picture of SNUUV Ⅰ is presented in Figure 1.

The process for calculating the range from AUV to the objects that are located around the surrounding environment is based on the threshold method. The detail of the process is as follows. After the transducer transmits a signal, the reflected signal amplitude is compared with the threshold level. If the signal amplitude is greater than the threshold level, the first arrival signal which exceeds the threshold level is used to calculate the range from AUV to the object. Figure 2 shows a sample real data for calculating this range. As can be seen, there is ambient noise and thus it is necessary to filter the low frequency noise as it may affect the calculation of the range. Thus, in this study, a sonar system was equipped with the band-pass analogue filter with a center frequency of 550 kHz.

Table 1: Specification of the range sonar of SNUUV I

Operating Frequency	520 -550 kHz
Power	5 W
Input voltage	35V
Sampling time	1 Hz
Filter	Band-pass with center frequency 550 kHz

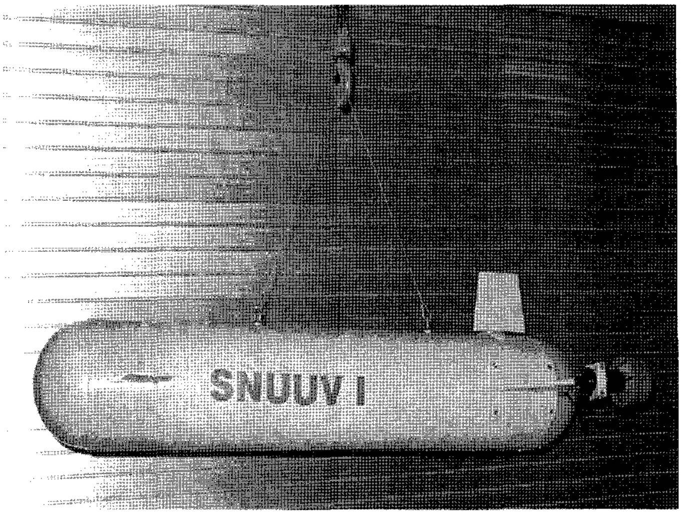


Figure 1: Picture of SNUUV I

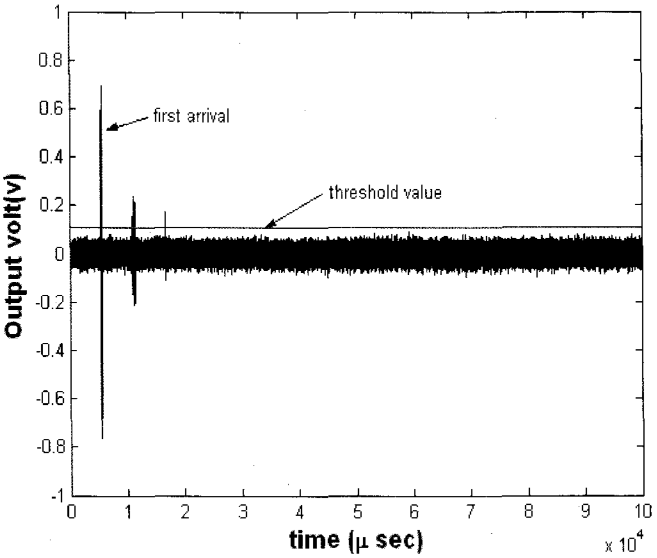


Figure 2: Detecting the first arrival signal in a single ping

3 Algorithm of CML

Let us begin our evaluation on the CML algorithm by considering an AUV navigating in an environment with a single object in the environment. If AUV is assumed to travel in a stationary2-dimensional environment, the state of the vehicle can be written as

$$\mathbf{x}_v = \begin{bmatrix} x & y & V & \psi \end{bmatrix}^T \tag{1}$$

where x, y, V and ψ denote the positions, total velocity and heading components respectively.

The state of the object acquired by the sonar is given by its position,

$$\mathbf{x}_o = \begin{bmatrix} x_o & y_o \end{bmatrix}^T \tag{2}$$

As CML is based on the stochastic mapping (Smith R. et al, 1990), it is able to combine $\mathbf{x}_v$ and $\mathbf{x}_o$ to form the system state vector

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_v^T & \mathbf{x}_o^T \end{bmatrix}^T \tag{3}$$

In the proposed approach, the vehicle motion is modeled as the second kinematics model, which is characterized by the constant velocity with white noise acceleration (Bar-Shalom and Li 1993). The object position is assumed to be stationary and modeled as random walk to account for apparent position change caused by the change in the distance between object and AUV. The discrete time state equation with sampling time T, is described as

$$\mathbf{x}(k+1) = \mathbf{F}\mathbf{x}(k) + \mathbf{q}(k) \tag{4}$$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & \cos\psi T & -V\sin\psi T & 0 & 0 \\ 0 & 1 & \sin\psi T & V\cos\psi T & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$\mathbf{Q}(k) = E[\mathbf{q}_k \mathbf{q}_k^T] = \begin{bmatrix} \cos^2\psi \frac{T^3}{3}\sigma^2_{q_x} & 0 & \cos\psi \frac{T^2}{2}\sigma^2_{q_x} & 0 & 0 & 0 \\ 0 & \sin^2\psi \frac{T^3}{3}\sigma^2_{q_x} & \sin\psi \frac{T^2}{2}\sigma^2_{q_x} & 0 & 0 & 0 \\ \cos\psi \frac{T^2}{2}\sigma^2_{q_x} & \sin\psi \frac{T^2}{2}\sigma^2_{q_x} & T\sigma^2_{q_x} & 0 & 0 & 0 \\ 0 & 0 & 0 & T\sigma^2_{q_\psi} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{q_o} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{q_o} \end{bmatrix} \tag{6}$$

Here $\sigma^2_{q_x}, \sigma^2_{q_y}$ are the process noises power governing the motion of the vehicle in the x, y directions respectively, and $\sigma^2_{q_o}$ is the process noise power governing the position of the object.

At sample time *k*, the sonar of SNUUV I provides a measurement of range and bearing object from AUV.

$$\mathbf{z}(k) = \begin{bmatrix} r(k) \\ \theta(k) \end{bmatrix} + \mathbf{w}(k) \tag{7}$$

where $r(k)$ and $\theta(k)$ represent the true range and bearing to the objects respectively.

$$\mathbf{w}(k) = \begin{bmatrix} w_r(k) \\ w_\theta(k) \end{bmatrix} \tag{8}$$

represents the measurement noises of sonar which are assumed to be zero mean, white Gaussian noise and uncorrelated with each other. The covariance matrix of $\mathbf{w}$ is

$$\mathbf{R}(k) = E[\mathbf{w}(k)\mathbf{w}^T(k)] = \begin{bmatrix} \sigma_r^2(k) & 0 \\ 0 & \sigma_\theta^2(k) \end{bmatrix} \tag{9}$$

The measurement in (7) can be described as a nonlinear function of the state variables, $\mathbf{z}(k) = \mathbf{h}(\mathbf{x}(k)) + \mathbf{w}(k)$, where

$$\mathbf{h}(\mathbf{x}(k)) = \begin{bmatrix} \sqrt{(x(k) - x_o(k))^2 + (y(k) - y_o(k))^2} \\ \arctan \dfrac{y(k) - y_o(k)}{x(k) - x_o(k)} \end{bmatrix} \tag{10}$$

Although the information of heading angle plays an important role in the navigation of AUV in the real environment, it is impossible to use a magnetic compass, which is normally used to measure the heading angle in most AUV, in a towing tank experiment because of the strong magnetic interference from the structure of the towing tank. In this case, the range sonar data is used to calculate the heading angle using the simple geometric relationship (Hwang et al, 2004). A simple illustration of the calculation process is given in Figure 3.

The estimation process for system state vector for the AUV in motion is based on the EKF. The EKF process can be summarized as follows.

- First, predict the system state vector $\hat{\mathbf{x}}(k+1 \mid k)$, the system covariance matrix $\mathbf{P}(k+1 \mid k)$ and the measurement $\hat{\mathbf{z}}(k+1 \mid k)$ for time *k+1* and compute the Jacobian $\mathbf{H}(k+1)$.

$$\hat{\mathbf{x}}(k+1 \mid k) = \mathbf{F}\hat{\mathbf{x}}(k \mid k) \tag{11}$$

$$\mathbf{P}(k+1 \mid k) = \mathbf{F}\mathbf{P}(k \mid k)\mathrm{F}^T + \mathbf{Q} \tag{12}$$

$$\hat{\mathbf{z}}(k+1 \mid k) = \mathbf{h}(\hat{\mathbf{x}}(k+1 \mid k)) \tag{13}$$

$$\mathbf{H}(k+1) = \left.\frac{\partial \mathbf{h}(k+1)}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(k+1 \mid k)} \tag{14}$$

- Make a measurement at time *k+1* and compute the measurement residual $\Gamma(k+1)$, the residual covariance matrix $\mathrm{S}(k+1)$ and the Kalman gain $\mathbf{K}(k+1)$.

$$\mathbf{z}(k+1)=\begin{bmatrix} r(k+1) \\ \theta(k+1) \end{bmatrix}+\mathbf{w}(k+1) \tag{15}$$

$$\mathbf{\Gamma}(k+1)=\mathbf{z}(k+1)-\hat{\mathbf{z}}(k+1\mid k) \tag{16}$$

$$\mathbf{S}(k+1)=\mathbf{H}(k+1)\mathbf{P}(k+1\mid k)^{T}\mathbf{H}(k+1)+\mathbf{R}(k+1) \tag{17}$$

$$\mathbf{K}(k+1)=\mathbf{P}(k+1\mid k)\mathbf{H}^{T}(k+1)\mathbf{S}^{-1}(k+1) \tag{18}$$

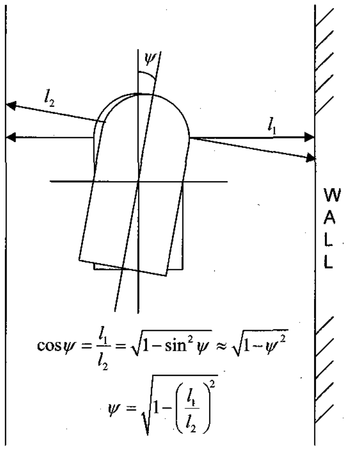


Figure 3: Calculation of heading angle from distance measurements in a towing tank

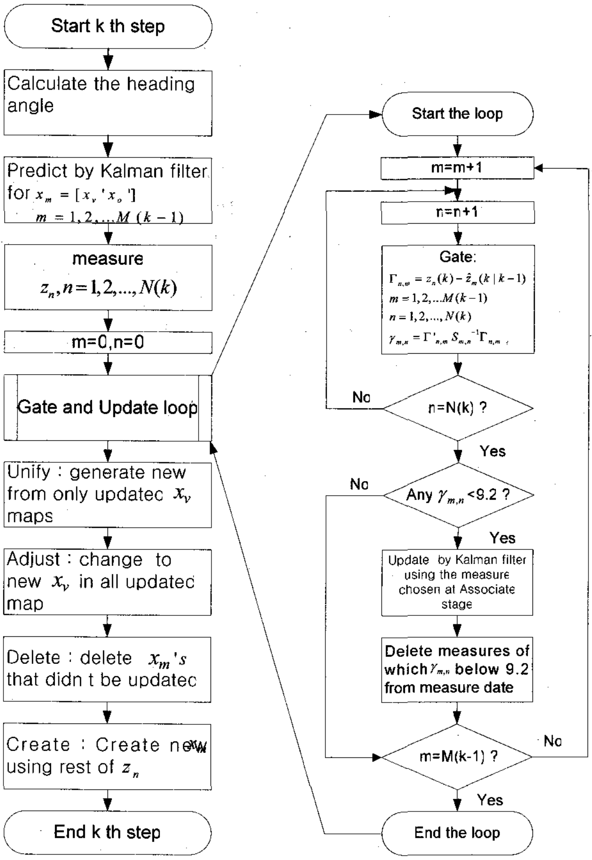


Figure 4: Flowchart overview of the CML Algorithm

- Update the system state vector $\hat{\mathbf{x}}(k+1\,|\,k+1)$ and the system covariance matrix $\mathbf{P}(k+1\,|\,k+1)$.

$$\hat{\mathbf{x}}(k+1\,|\,k+1)=\hat{\mathbf{x}}(k+1\,|\,k)+\mathbf{K}(k+1)\mathbf{\Gamma}(k+1) \tag{19}$$

$$\mathbf{P}(k+1\,|\,k+1)=\mathbf{P}(k+1\,|\,k)-\mathbf{K}(k+1)\mathbf{S}(k+1)\mathbf{K}^{T}(k+1) \tag{20}$$

The algorithm for single object is extended to the algorithm for multiple objects so that the algorithm can be implemented in the real environment. When there are M objects present, the system states the vector of each of the M objects concatenated to the vehicle state in the original CML technique. An EKF, as described above, is again used to estimate the system state vector $\mathbf{x}$, with a commensurate increase in computational complexity. However, in this paper, a method of decoupling the problem and forming an individual system state vector for each of the M objects is used to save computational power. Based on the specification of the sonar system of SNUUV I, the sonar can provide 4 measurements about the surrounding of AUV in terms of distance and angle. In CML, only 3 measurements that are obtained from the sensor located at the horizontal plane are used for estimating the system state vector. Due to the fact that only 3 measurements for object can be obtained from the sonar for the CML, the 3 system state vectors were formed as follows for the sample period *k+1*.

$$\mathbf{x}_m=\begin{bmatrix}\mathbf{x}_v^T & \mathbf{x}_{om}^T\end{bmatrix}^T,\ m=1,2,3 \tag{21}$$

Similarly to the single case, each of $\mathbf{x}_m$ is governed by the discrete time state equation

$$\mathbf{x}_m(k+1)=\mathbf{F}\mathbf{x}_m+\mathbf{q}_m(k) \tag{22}$$

where $\mathbf{F}$ and $\mathbf{q}_m$ are the transition matrix and covariance matrix of process noise, respectively, as given in (6). The separated EKF used 3 measurement from the SNUUV I sonar to provide 3 individual system state estimates $\hat{\mathbf{x}}_m$, $m=1,2,3$ in sample period *k+1*. The detailed process of algorithm is depicted as a flowchart in Figure 4. The procedure for estimating the system state vector for multiple objects is similar to that of a single case with only minor differences in two procedures related to gate, unify, adjust and delete/create.

In multiple cases, it is necessary to decide which measurement obtained by sonar will associate with the object existing at the old map in order to enhance efficiency of computation before the individual system state vector is updated. Gating is a method used to determine which measurement is closest to a particular object based on the Nearest Neighbor Standard Filter (NNSF) (Bar-Shalom and Forman, 1988). In NNSF, the innovation between the predicted measurement of m^{th} object and the measurement of n^{th} object,

$$\mathbf{\Gamma}_{m,n}(k+1)=\mathbf{z}_n(k+1)-\hat{\mathbf{z}}_m(k+1\,|\,k)\ ,\ m=1,2,3\ \ n=1,2,3\ \ k:\textit{time index} \tag{23}$$

is normalized by its residual covariance matrix $S_{m,n}(k+1)$ to produce the normalized innovation.

$$\gamma_{m,n}=\mathbf{\Gamma}^{T}_{m,n}S^{-1}_{m,n}\mathbf{\Gamma}_{m,n} \tag{24}$$

As $\gamma_{m,n}$ is χ^2 distribution with 2 degree of freedom under Gaussian assumption (Bar-Shalom and Forman, 1988), it is possible to fix the area of interest where measurements associated with the m^{th} object should be found. Only if $\gamma_{m,n}$ is less than the threshold, can the n^{th} object be considered to correspond with m^{th} object, used for updating the m^{th} object and be removed from gating process for other objects. Furthermore, only the system state vectors that include objects having the measurement obtained by gating can be updated by the EKF process.

After all system state vector is updated, the "unify step" resets the common vehicle state, which is to be used by sample mean of the vehicle state of individual system state vector. This ensures consistency of the map and the unified covariance matrix of vehicle is also derived by the same method. The "adjust step" replaces the existing vehicle state in individual system state vectors with the unified vehicle state. In "delete/create step", the individual system state vectors, which were not updated, are deleted in order to save memory space and power in order to conduct further computation. The unused measurements that have not yet passed any gates are than used to derive new individual system state vectors by appending new objects data to the unified vehicle state. The whole algorithm is depicted in Figure 4.

Table 2: Basic Simulation Parameters

Sample time	1 Hz	Variance of object	$(1m)^2$
Operation time	50 sec	Variance of range	$(1m)^2$
Variance of velocity	$(0.1m/s^2)^2$	Variance of angel	$(5^\circ)^2$

Table 3: Conditions of straight line motion

Condition No.	Channels	Detection direction	Velocity	Range
1	1	Front	0.5 -2.0 m/s	25- 100 m
2	1	One side	1.0 m/s	4 m
3	2	Both sides	1.0 m/s	4 m
4	3	Front & Both sides	1.0 m/s	100 m,4 m

4 Simulation

For the purpose of verifying the proposed CML algorithm, several simulations were performed under various conditions. The vehicle state was recorded at coordinate fixed at initial position of the vehicle. Table 2 shows the basic setting for the simulation. The variance of velocity σ_{q_x} was designed based on the specification of SNUUV I. The variance of object σ_{q_o} and range σ_r and that of angle σ_θ were decided, taking into consideration time delay and the beam pattern of the transducer, respectively. The above-mentioned simulations were performed under these two conditions. The first condition was used for the straight run while the other for the zigzag motion. The conditions for the straight run are summarized in Table 3. Conditions 1, 2, 3 and 4 as shown in Table 3 are the conditions for straight runs with one or more object to evaluate the straight line performance respectively. Table 4 presents two conditions for non-straight run, i.e. the zigzag motion with multiple objects that will aid in verifying the functioning of CML in real AUV navigation. The results under these conditions were compared with the results of experiments in towing tank. Conditions 1 and 2 described in Table 3 were considered so as to verify the capacity of the proposed CML algorithm at various velocities and ranges.

And the purpose of condition 3 and 4 in Table 3 is the verification of the gating and association when various distances and bearings data are obtained from several sonars. The conditions in Table 4 were chosen for verifying the various functions of the proposed CML when the heading angle of AUV varies in a zigzag motion. Based on the length and width of real towing tank, all range in Table 3 and 4 was determined. The positive of heading is the counterclockwise and the negative is the clockwise. Figures 5 and 6 show the trajectory of vehicle's x direction in simulation when the condition is 1 as shown in Table 3 respectively. From Figures 5 and 6, it is clear that the proposed CML algorithm performed well in spite of the change in the distance for the object and velocity of AUV. Figure 7 shows the trajectory of vehicle in simulation performed under the conditions presented in Table 3 respectively. Figure 7-a) presents the results of simulation when

AUV used only one sonar to achieve the information of objects. From this result, it is clear that the proposed algorithm operates well when single information is available. Figure 7-b) shows

Table 4: Conditions of zigzag motion

Condition No.	Channels	Detection direction	Velocity	Range	Heading
1	3	Front & Both sides	1.0 m/s	100 m,4 m	0°~ ±5°
2	3	Front & Both sides	1.0 m/s	100 m,4 m	-5°~+ 5°

The results of simulation when AUV used multiple sonars obtained the 2 and 3 directions information of objects. From this result, it is clear that the gating and association function in the proposed CML algorithm does not confuse among the multiple information for associated multiple objects. Figure 8 shows the result when the AUV navigated in the tank with varying heading angle.

The left side of Figure 8 a) shows the result when the heading of AUV increases from 0 degree to 5 degree and then decreases back to 0 degree gradually. The right side of Figure8 b) shows the result when the heading varies from 0 degree to – 5 degree and returns to 0 degree. The left side of Figure 8 b) shows the result when the heading varies from 0 degree to 5 degree and from5 degree to – 5 degree and from -5 degree to 0 degree. From these figures, it can be seen that despite the existence of errors in the position of objects, the change in heading did not affect the mapping and localizing function of the proposed algorithm. From Figures 5, 6 and 7, it is verified that the states of vehicle and mapping objects were estimated appropriately under the

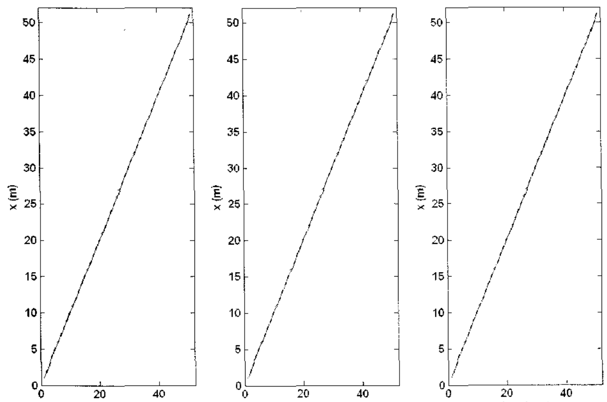


Figure 5: Trajectory of x for same velocity of 1 m/s for various ranges

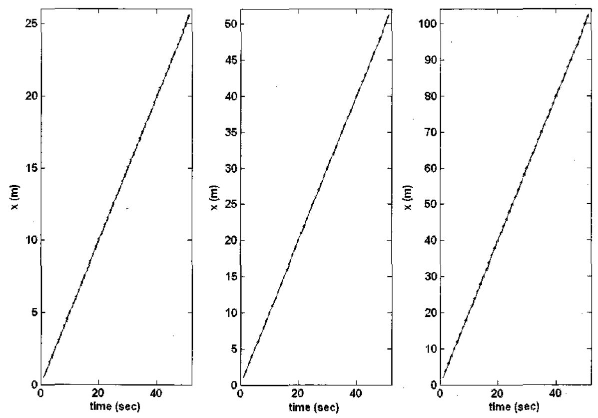
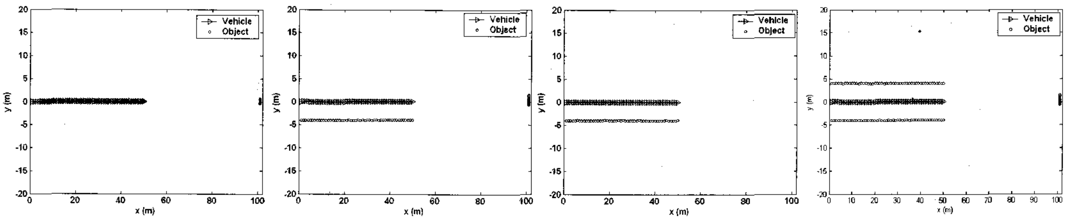
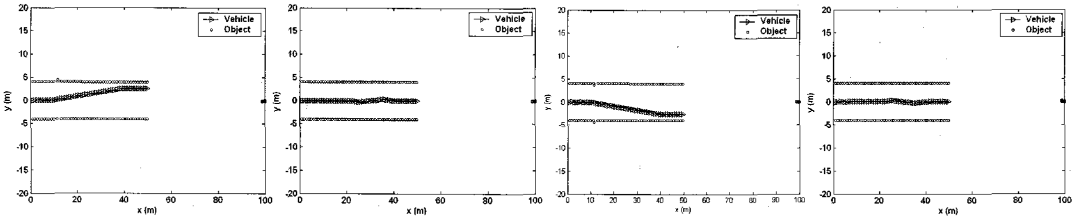


Figure 6: Trajectory of x for same range of 100m for various velocities



a) Single direction cases b) Multiple direction case

Figure 7: Results of CML from simulation of straight line motion



a) One direction heading changing case. b) Both directions heading changing cases.

Figure 8 : Results of CML from simulation of zigzag motion

condition that the velocity was constant. Then, Figure 8 shows that the change in the heading angle did not affect the function of CML. Therefore, based on these results of the simulation that was conducted under various conditions, it is expected that the proposed CML algorithm will perform well in a real towing tank environment

5 Experiment

For the purpose of verifying the capacity of the proposed CML algorithm in the real environment, the experiments were performed at a towing tank. As the sonar system has not yet been integrated into the main control system of SNUUV I , an instrument was used to fix the sonar in position within a towing carriage during the experiments. Therefore the experiment was set up in a straight line run. Basic configurations for variance, etc. were found to be the same as in the simulation. The conditions under which the

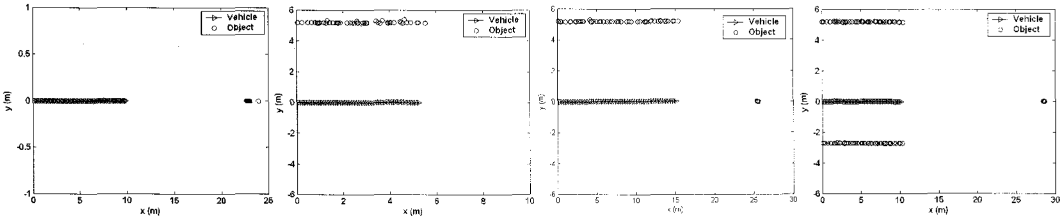
experiments were conducted are presented in Table 5 as follows: The velocity of experiment was designed to reflect the real cruising velocity of SNUUV I .Condition 1 is intended to verify the basic function of the proposed CML algorithm when the single information of sonar is available. Conditions 2 and 3 in Table 5 are intended to verify the functioning of gating and association when 2 and 3 sonars are used in the towing tank.
The experiment results under conditions 1 and 2 in Table 5 are depicted in Figure 9. From this result, it is verified that the change in the direction of sonar did not affect the functioning of the proposed CML algorithm in the real environment and the complexity of computation is small enough to implement the proposed algorithm into real operation of AUV in the real environment. It is also clear that even if the real AUV is not used, the proposed algorithm with multiple sonar information performs well in the towing tank when the motion is 1 dimensional without changing of heading angle.

6 Conclusion

In this paper, a CML algorithm which uses the range sonar applied to test-bed AUV, namely SNUUV I is presented. The algorithm uses a decoupled EKF to estimate the system state vector in multiple object condition and a data association to refresh the map for saving the computational power. The algorithm was tested through simulations under various conditions by varying the velocity, ranges, the number of objects, and the heading angle. The simulation results show that the algorithm is capable of estimating the vehicle state and mapping the objects. The experiments in the towing tank using a single and multiple sonar information under various velocities, ranges, and straight run also show that the algorithm will perform well in the real environment.

Table 5: Experimental Conditions

Condition No.	Channel	Detection direction	Velocity	Range
1	1	One side	0.1 – 0.5 m/s	20 – 50 m or 3 – 6 m
2	2	Front & One side	0.1 m/s	20 – 50 m & 3 – 6 m
3	3	Front & Both sides	0.1 m/s	20 – 50 m & 3 – 6 m



a) Front b) Side c) Front and one side d) Front and both sides

Figure 9: Result of CML from experiment at towing tank.

In the future, we plan to investigate the feasibility of integrating CML into the total navigation system of SNUUV I where it will function as a module for providing information about the environment and expanding it to vehicle state, including the heading angle, for controlling heading of the AUV.

Acknowledgments

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References

Bar-Shalom Y. and T.E. Forman. 1988. Tracking and Data Association, Academic Press.

Bar-Shalom Y. and X. Li. 1993. Estimation and Tracking: Principles, Technique, and Software. Artech House Inc.

Carpenter R.N. 1998. Concurrent mapping and localization with FLS, Proceedings of AUV 98, 133-148.

Harris J.D. and R.N. Carpenter. 2001. Results from the simulation of a decoupled concurrent mapping and localization technique, **4**, 2366 -2376.

Hwang A. and W. Seong. 2004. Concurrent mapping and localization for controlling the heading angle of SNNUV I in towing tank. Proc. the Annual Autumn Meeting, SNAK 431-437.

Lee C.M., P.M. Lee, S.M. Kim, S.W. Hong, J.W. Seo and W.J Seong. 2003. Rotating arm test for assessment of underwater hybrid navigation system for a semi-autonomous underwater vehicle. J. of KOCRE, **17**, **4**, 73-80.

Newman P.M. 1999. On the Structure and Solution for the Simultaneous Localisation and Map building Problem. PhD Thesis, Australian Centre for Field Robotics, The University of Sydney.

Smith C.M, J.J. Leonard, A.A. Bennett and C. Shaw. 1997. Feature-based concurrent mapping and localization for autonomous underwater vehicles. Oceans 1997, **2**, 896-901.

Smith R., M. Self and P. Cheeseman. 1990. Estimating Uncertain Spatial Relationships in Robotics, in I. Cox and G Wilfong, editors, Autonomous Robot Vehicles, Springer-Verlag.