논문 2005-42TC-12-14

다중 안테나를 갖는 공간 다중화 시스템을 위한 maximum likelihood 검출기의 성능 분석

(Performance analysis of maximum likelihood detection for the spatial multiplexing system with multiple antennas)

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요 약

본 논문에서는 다중안테나를 갖는 공간 다중화 시스템에서 주어진 채널에 대한 maximum likelihood 검출기의 성능을 수식적으로 유도하였다. 벡터 심볼 오차율을 구하기 위해 nearest neighbor의 방향을 나타내는 에러벡터를 정의하였다. 전송 벡터 심볼이 랜덤한 채널에 의해 변환 될 경우 확정적으로 4개의 nearest neighbor를 가짐을 정의된 에러벡터의 특성을 이용해 입증하였다. 변형된 구 검출기로부터 획득 가능한 에러벡터와 최소거리 정보를 바탕으로 주어진 랜덤 채널 하에서 벡터 심볼 오차율을 수식적으로 도출하였다. 유도된 벡터 심볼 오차식을 검증하기 위해, 랜덤 채널을 unitary 채널, dense 채널, sparse 채널로 분류한 후 각 채널 상황에서 유도된 결과 식을 모의 실험 결과와 비교하였다. 모의실험 결과로부터 유도된 벡터 심볼 오차식이 다양한 랜덤 MIMO 채널에서 ML 검출기의 성능을 잘 근사하고 있음을 입증하였다.

Abstract

The performance of maximum likelihood(ML) detection for the given channel is analyzed in spatially multiplexed MIMO system. In order to obtain the vector symbol error rate, we define error vectors which represent the geometrical relation between lattice points. The properties of error vectors are analyzed to show that all lattice points in infinite lattice almost surely have four nearest neighbors after random channel transformation. Using this information and minimum distance obtained by the modified sphere decoding algorithm, we formulate the analytical performance of vector symbol error over the given channel. To verify the result, we simulate ML performance over various random channel which are classified into three categories: unitary channel, dense channel, and sparse channel. From the simulation results, it is verified that the derived analytical result gives a good approximation about the performance of ML detector over the all random MIMO channels.

Keywords : MIMO, maximum likelihood detection, error vector, channel density

I. Introduction

The multiple-antenna systems have a great attention, mainly due to its promising performance compared to conventional single-antenna systems. The previous studies in information theory have shown that a multi-input multi-output(MIMO) system is able to support enormous capacities, provided that the multipath scattering of a wireless channel is exploited with appropriate space-time signal processing techniques[1][2].

To achieve these advantages, various space-time modulation and detection techniques have been proposed[3]-[8]. Spatial multiplexing, which is one of the well-known space-time modulation techniques, can provide optimal capacity by multiplexing the incoming data into multiple substreams and transmitting each substream on a different antenna

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※ 본 논문은 한국전자통신연구원의 지원에 의한 연구 결과입니다.
접수일자: 2005년10월24일, 수정완료일: 2005년12월5일

[3] The substreams can be separated at the receiver by means of various detection techniques including zero-forcing(ZF), minimum mean square error (MMSE), vertical Bell laboratories layered space time(V-BLAST), and maximum likelihood(ML) detections[4]-[7]. Though ZF, MMSE and V-BLAST need reasonable complexity to implement, they can not enjoy the full diversity provided by multiple receivers[4]. Since the ML detector searches transmit vector symbol over the set of all candidates, it guarantees optimal performance in terms of minimizing the overall error probability and obtains full receive diversity. However, extremely high complexity required in implementing it, precludes its use in practical MIMO systems. To make its use possible, the ML detection algorithms with reduced complexity have been proposed, such as sphere decoding algorithm and K-best algorithm[6]-[8].

The performance of ML detector has been analyzed in the several literatures[9]-[12]. In [9], the union bound of ML performance for quadrature amplitude modulated(QAM) MIMO systems was derived. However, it gives a loose union bound. More tight bounds have been derived for ML detection in [10] and [11] with considering the effect of channel estimation error. For the performances in those literatures, they considered the averaged error probability with respect to Rayleigh MIMO channels. Thus these results are not of use for closed loop MIMO techniques, such as adaptive modulation[13], antenna subset selection[14], quantized beamforming [15], in which the transmitter requires the error probability for the current channel realizations. In [12], the analytical error probability for the given channel was derived. However, the results can not be readily expressed in closed form.

In this paper, we derive the analytical performance of ML detection for the given channel. To analyze error performance of ML, we define error vector which represents the geometrical relation between the lattice points after channel transformation, and derive the properties of the error vector. By taking advantage of this information, we formulate the vector symbol error rate performance. In order to make out our results, we classify the random MIMO channels into three categories: unitary channel, dense channel, and sparse channel, and perform Monte-Carlo simulations over these channels. From the results in the sequel, we can declare that the analytical performance approximates well the performance of ML for the given channel.

Ⅱ. System overview

We consider a MIMO system with M_T transmit and M_R receive antennas, where the transmitted symbols are assumed to be independent in time as well as space, so that the transmit symbol vector written as $\boldsymbol{s}$ consists of M_T QAM symbols. In this paper, we focus on the case in which $M_T \le M_R$ to avoid rank deficient condition and assume that $E\{\boldsymbol{s}\boldsymbol{s}^H\} = \frac{1}{M_T}\boldsymbol{I}_{M_T}$ for unit transmit power constraint, where $E\{\cdot\}$ denotes expectation, $(\cdot)^H$ denotes conjugate transposition, and $\boldsymbol{I}_{M_T}$ is $M_T \times M_T$ identity matrix. Suppose that the bandwidth is much smaller than the coherent bandwidth of channel so that the discrete-time equivalent channel can be modeled as an $M_R \times M_T$ matrix $\boldsymbol{H}$. Assume that the channel is spatially uncorrelated, and the element of channel matrix $\boldsymbol{H}$ is drawn from complex Gaussian random variable with $CN(0,1)$. The received vector added with spatially uncorrelated noise can be written as $\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}$, where $\boldsymbol{n}$ is the complex Gaussian noise vector whose entry is distributed according to $CN(0, N_o)$. We will define signal to noise ratio(SNR) as transmit power to noise power ratio, that is $\sigma = 1/N_o$.

To obtain an equivalent real lattice representation of this MIMO system, we transform the complex equation into real matrix equation $\boldsymbol{r} = \boldsymbol{M}\boldsymbol{x} + \boldsymbol{n}$, where $\boldsymbol{r} = [Re\{\boldsymbol{y}^T\}\ Im\{\boldsymbol{y}^T\}]^T$, $\boldsymbol{x} = [Re\{\boldsymbol{s}^T\}\ Im\{\boldsymbol{s}^T\}]^T$, $\boldsymbol{w} = [Re\{\boldsymbol{n}^T\}\ Im\{\boldsymbol{n}^T\}]^T$, and $\boldsymbol{M} = \begin{bmatrix} Re\{\boldsymbol{H}\} & -Im\{\boldsymbol{H}\} \\ Im\{\boldsymbol{H}\} & Re\{\boldsymbol{H}\} \end{bmatrix}$.

The real transmit vector $\boldsymbol{x}$ has pulse amplitude modulation(PAM) signal for the lth element;

$$x_l \in \boldsymbol{U} = \{u | u = 2k-1-K,\ k = 1, 2, \cdots, K\} \tag{1}$$

where $K=2^{b/2}$ and $b/2$ means the number of bits to be loaded at the real or imaginary component of QAM symbol. Since the entries of $\boldsymbol{M}$ and $\boldsymbol{w}$ are the real and imaginary elements of $\boldsymbol{H}$ and $\boldsymbol{n}$ they are distributed according to $N(0,1/2)$ and $N(0,N_0/2)$, respectively.

As the channel is assumed to be estimated accurately using training symbols, we assume in the sequel that the channel is known at the receiver. The receiver determines the transmitted vector $\boldsymbol{x}$ with the knowledge of the channel matrix $\boldsymbol{M}$ and received vector $\boldsymbol{r}$ by ML detection criterion, which can be written as $\hat{\boldsymbol{x}}=\arg\min_{\boldsymbol{x}\in\chi}\|\boldsymbol{Mx}-\boldsymbol{r}\|^2$, where $\|\cdot\|$ denotes vector norm, arg min returns global minimizer, and $\chi=U^{2M_T}$ is the whole set of possible transmit vectors. Though the ML detection provides optimal solution, the complexity of it is prohibitively extreme. To relieve this burden in real application, we will use the sphere decoding algorithm for ML [6]. This algorithm can be also used to obtain a minimum distance between the lattice points transformed by channel and the error vector, which indicates the direction of nearest neighbors. The squared $d_{\min}$ is defined by

$$d_{\min}^2=\min_{\boldsymbol{x}_a,\,\boldsymbol{x}_b\in\chi,\ \boldsymbol{x}_a\neq\boldsymbol{x}_b}\|\boldsymbol{M}(\boldsymbol{x}_a-\boldsymbol{x}_b)\|^2 \tag{2}$$

and the error vector corresponding to $d_{\min}$ is defined by $\boldsymbol{e}=\boldsymbol{x}_a-\boldsymbol{x}_b$ where $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$ are the nearest neighbors in χ after channel transformation.

III. Vector symbol error rate for the given channel

To derive vector symbol error rate, we first consider pair wise error probability $P\{\boldsymbol{x}_i\rightarrow\boldsymbol{x}_j|\boldsymbol{M}\}$ which is the probability of confusing $\boldsymbol{x}_i$ with $\boldsymbol{x}_j$ when $\boldsymbol{x}_i$ is transmitted through the channel $\boldsymbol{M}$ and these are the only two hypotheses, can be written as [12]

$$P\{\boldsymbol{x}_i\rightarrow\boldsymbol{x}_j|\boldsymbol{M}\}=Q\left(\frac{\|\boldsymbol{M}(\boldsymbol{x}_i-\boldsymbol{x}_j)\|}{2\sqrt{N_0/2}}\right)=Q\left(\frac{\|d_{ij}\|}{2\sqrt{N_0/2}}\right) \tag{3}$$

where d_{ij} means the distance between $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ in the transformed domain.

The error probability $P_{e|\boldsymbol{x}=\boldsymbol{x}_i,\boldsymbol{M}}$ for the given $\boldsymbol{x}_i$ and $\boldsymbol{M}$ can be approximated by considering the union of error event to all the other vectors $\boldsymbol{x}_j,\forall j\neq i$. Further, in high SNR region, noise variance is so small that the error happens among the nearest neighbors more dominantly, so that we can obtain simple approximation of error rate, which becomes tight as SNR increases[12],

$$P_{e|\boldsymbol{x}=\boldsymbol{x}_i,\boldsymbol{M}}\approx Ne_iQ\left(\frac{d_{\min}}{2\sqrt{N_o/2}}\right) \tag{4}$$

where Ne_i is the number of nearest neighbors of $\boldsymbol{x}_i$. Define the nearest neighbor as a neighbor which is away from $\boldsymbol{x}_i$ by $d_{\min}$. Since $d_{\min}$ is determined in global set χ as given in (2), it does not depend on i.

The total vector symbol error probability can be obtained as follow by conditioning all possible symbol vectors, which are assumed to occur equally.

$$\begin{aligned}P_{e|\boldsymbol{M}}&=\sum_{i=1}^{N}P_{e|\boldsymbol{x}=\boldsymbol{x}_i,\boldsymbol{M}}P(\boldsymbol{x}=\boldsymbol{x}_i)\\&\approx\frac{1}{N}Q\left(\frac{d_{\min}}{2\sqrt{N_o/2}}\right)\left(\sum_{i=1}^{N}Ne_i\right)\end{aligned} \tag{5}$$

where N is the number of the total elements in χ.

As shown in (5), in order to obtain approximated error rate, $d_{\min}$ and $\sum_{i=1}^{N}Ne_i$ for the given channel $\boldsymbol{M}$ should be should be evaluated first. While $d_{\min}$ can be easily obtained by using the modified sphere decoding algorithm, the number of nearest neighbors for all vector symbols needs much effort because the neighbors of each vector symbol should be identified. Hence, we need a method to obtain the sum of nearest neighbor numbers efficiently. Fortunately, the error vector defined in the previous section provides a clue for it.

1. error vectors

Since, as defined in section II, the error vector indicates the direction of the nearest neighbor, we can enumerate the number of neighbor for the given error vector by investigating the properties of the error vector.

Lemma 1 : If the error vector $\boldsymbol{e}$ has minimum distance, then $\boldsymbol{e}$ has also minimum distance. The polarity of error vector dose not affect on the minimum distance.

Proof : $d^2_{\min} = \| \boldsymbol{Me} \|^2 = \| \boldsymbol{M}(-\boldsymbol{e}) \|^2$.

Lemma 2 : Let $\boldsymbol{e} = [\ \boldsymbol{e}_1{}^T\ \boldsymbol{e}_2{}^T]$, where $\boldsymbol{e}_1{}^T$ is the upper half vector of $\boldsymbol{e}$ and $\boldsymbol{e}_2{}^T$ is the lower half one. If the error vector $\boldsymbol{e}$ has minimum distance, then $\boldsymbol{e}_p = [-\ \boldsymbol{e}_2{}^T\ \boldsymbol{e}_1{}^T]$ has also minimum distance.

Proof : Let $\boldsymbol{H}_R$ and $\boldsymbol{H}_I$ are real and imaginary part of channel matrix $\boldsymbol{H}$, respectively. Assuming $\boldsymbol{e} = [\ \boldsymbol{e}_1{}^T\ \boldsymbol{e}_2{}^T]$ is error vector, the squared minimum distance is given by

$$\begin{aligned} \| \boldsymbol{Me} \|^2 &= \| \ \boldsymbol{H}_R \boldsymbol{e}_1 - \boldsymbol{H}_I \boldsymbol{e}_2 \|^2 + \| \ \boldsymbol{H}_I \boldsymbol{e}_1 + \boldsymbol{H}_R \boldsymbol{e}_2 \|^2 \\ &= \| - \boldsymbol{H}_R \boldsymbol{e}_2 - \boldsymbol{H}_I \boldsymbol{e}_1 \|^2 + \| - \boldsymbol{H}_I \boldsymbol{e}_2 + \boldsymbol{H}_R \boldsymbol{e}_1 \|^2 \\ &= \| \ \boldsymbol{Me}_p \|^2 \end{aligned} \tag{6}$$

From (6), we can see that $\boldsymbol{e}_p$ is also error vector.

Lemma 3 : There are almost surely four error vectors $\boldsymbol{e}$, $\boldsymbol{e}_p$, $-\boldsymbol{e}$, and $-\boldsymbol{e}_p$ in χ after transformed by random channel $\boldsymbol{M}$.

Proof : Let $\boldsymbol{e} = [\ \boldsymbol{e}_1{}^T\ \boldsymbol{e}_2{}^T]$ and $\boldsymbol{e}' = [\ \boldsymbol{e}_3{}^T\ \boldsymbol{e}_4{}^T]$ are error vectors. Their squared norm after channel transform should be identical, that is,

$$\begin{aligned} \| \boldsymbol{Me} \|^2 &= \| \ \boldsymbol{H}_R \boldsymbol{e}_1 - \boldsymbol{H}_I \boldsymbol{e}_2 \|^2 + \| \ \boldsymbol{H}_I \boldsymbol{e}_1 + \boldsymbol{H}_R \boldsymbol{e}_2 \|^2 \\ &= \| \ \boldsymbol{H}_R \boldsymbol{e}_3 - \boldsymbol{H}_I \boldsymbol{e}_4 \|^2 + \| \ \boldsymbol{H}_I \boldsymbol{e}_3 + \boldsymbol{H}_R \boldsymbol{e}_4 \|^2 \\ &= \| \ \boldsymbol{Me}' \|^2 \end{aligned} \tag{7}$$

Since $\boldsymbol{H}_R$ and $\boldsymbol{H}_I$ are randomly generated, $\boldsymbol{H}_R \neq \boldsymbol{H}_I$ with probability one. In order to satisfy equality, there are only four possible choices for $\boldsymbol{e}'$: $\boldsymbol{e}$, $\boldsymbol{e}_p$, $-\boldsymbol{e}$, and $-\boldsymbol{e}_p$ almost surely. Although the channel condition to induce more error vector than four of them may happen(e.g., unitary channel), the probability of this event is almost zero due to the randomness of channel.

According to ***Lemma 3***, we can certainly say that there exist four error vectors which we will call *joint error vectors* in the sequel. Since each error vector represents the direction of the nearest neighbor, we can infer that each lattice point has four nearest neighbors, and figure out where the nearest one exists from these information. Consider a symbol vector $\boldsymbol{x}_i$ and error vector $\boldsymbol{e}$ induced by random channel. $\boldsymbol{x}_i$ has four neighbors; $\boldsymbol{x}_i + \boldsymbol{e}$, $\boldsymbol{x}_i + \boldsymbol{e}_p$, $\boldsymbol{x}_i - \boldsymbol{e}$, and $\boldsymbol{x}_i - \boldsymbol{e}_p$.

Due to the property of error vectors, all lattice points transformed by the random channel have almost surely four neighbors in infinite lattice. However, we consider the lattice points only in χ, the finite lattice. In finite lattice structure, the number of neighbor is not constant, but variable with respect to the position of lattice point. For extreme example, let $\boldsymbol{x}_i$ be lattice point located on the surface of χ Then, the number of the nearest neighbors for this point may be less than four. This is because some of the nearest neighbors might be out of χ, they are not the nearest neighbors any more.

2. The sum of nearest neighbor numbers

As we know from (5), the sum of nearest neighbor numbers should be calculated to obtain vector symbol error rate. However, since Ne_i is variant with respect to the position of lattice point, enumerating it for all i is extremely difficult. As a matter of fact, we need the sum of nearest neighbor numbers instead of individual Ne_i. Therefore, we propose a systematic approach to obtain the sum of nearest neighbor numbers.

(1) One dimensional case

Let the error vector be $\boldsymbol{e} = 2d$ where d is integer such that $-K+1 \le d \le K-1$. The error vector transfers the lattice points in $\chi(=U)$ to error region U_d, where $U_d = \{u_d | u_d + 2d,\ u \in U\}$. Let us define set of the nearest neighbors as $U_n = U \cap U_d$.

Lemma 4 : The nearest neighbor number introduced by the error vector is equivalent to the number of elements in the set of the nearest neighbors, that is, $Ne(\boldsymbol{e}) = n(U_n)$, where $n(\cdot)$ is the number of the elements in the given set.

Proof : If the lattice points U is transferred by $2d$, they become the elements in U_d. However, since the lattice points of consideration are restricted in U, the lattice points in U_n can be nearest neighbors. Thus, the number of elements in U_n is equivalent to nearest neighbor number.

(2) Multi-dimensional case

Let the lth layer lattice be U_l and the lth layer error vector be $\mathbf{e}_l$. The set of the nearest neighbors for lth layer can be obtained like one dimensional case as $U_{n,l} = U \cap U_{d,l}$, where $U_{d,l}$ is the error region for the lth layer. From ***Lemma 4***, the nearest neighbor number for the lth layer can be given by $Ne(\mathbf{e}_l) = n(U_{n,l})$.

Lemma 5 : The nearest neighbor number introduced by $\mathbf{e}$ is given by

$$Ne(\mathbf{e}) = \prod_{l=1}^{L} Ne(\mathbf{e}_l) = \prod_{l=1}^{L} n(U_{n,l}) \tag{8}$$

where $L = 2M_T$ is the length of lattice vector.

Proof : Since the multi-dimensional lattice is formed by Cartesian product of U_l, the error event at each layer is independent to each other. Therefore, the nearest neighbor number can be obtained by product of each layer's neighbor number.

Lemma 6 : Joint error vectors have identical nearest neighbor number:

$$Ne(\mathbf{e}) = Ne(-\mathbf{e}) = Ne(\mathbf{e}_p) = Ne(-\mathbf{e}_p)$$

Proof : 1) From the symmetric structure of U_l, the polarity of $\mathbf{e}_l$ does not affect on the nearest neighbor numbers $Ne(\mathbf{e}_l)$. 2) The permutation of error vector does not affect on the nearest neighbor numbers. This is because the permutation changes the order of product and the product operation is commutative.

From 1), $Ne(\mathbf{e}) = Ne(-\mathbf{e})$, $Ne(\mathbf{e}_p) = Ne(-\mathbf{e}_p)$ are verified. From 1) and 2), $Ne(\mathbf{e}) = Ne(\mathbf{e}_p)$ is also verified. □

Lemma 5 and ***Lemma 6*** provide insight into the sum of the nearest neighbor numbers. In lattice transformed by random channel, there are four error vectors(joint error vector) almost surely. The nearest neighbor number introduced by each error vector can be obtained by ***Lemma 5***. Since the error to the direction of each error vector is mutually exclusive, from ***Lemma 6***, the sum of the nearest neighbor numbers can be obtained by $\sum_{i=1}^{N} Ne_i = 4Ne(\mathbf{e})$.

By substituting the sum of the nearest neighbor numbers into (5), the vector symbol error rate can be given by

$$P_{e|\mathbf{M}} \approx \frac{4Ne(\mathbf{e})}{N} Q\left(\frac{d_{\min}}{2\sqrt{N_o/2}}\right) \tag{9}$$

III. Performance analysis

1. Channel classification

We classify the MIMO random channel into three categories: unitary channel, dense channel, and sparse channel. In the case of the unitary channel, the channel matrix is unitary: $\mathbf{H}^H\mathbf{H} = \mathbf{I}$. Since the unitary channel just rotates the lattice points in χ, the regular structure of lattice is preserved even after the channel transformation, so that ML performance over this channel does not affected by the channel. However, this favor channel hardly happens in real random channel environments, so we will consider this channel just to verify our works. Actually, the real random channel can be classified to the dense and the sparse channels. Let us call a channel as 'dense', when the next nearest neighbors are close to

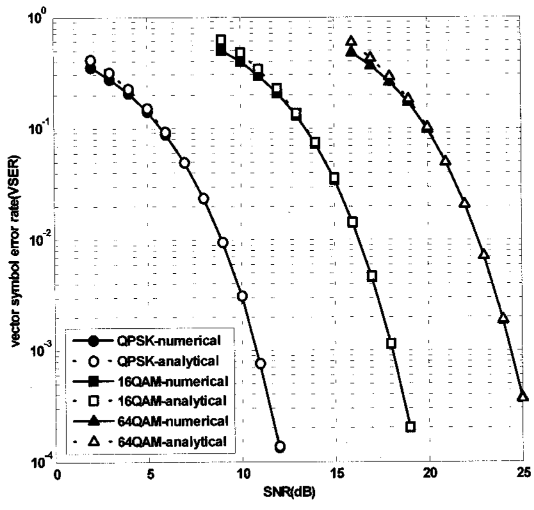

그림 1. 유니터리 채널에서의 성능 비교
Fig. 1. Performance comparisons over the unitary channel.

the nearest neighbors, and call a channel as 'sparse', when the next nearest neighbors are sufficiently far away from the nearest neighbors. Let us define channel density as $D_c = d_n^2 / d_{\min}^2$, where d_n denotes the distance from the given lattice point to the next nearest neighbor. We will define the dense channel as the channel with $1 < D_c < 2$ and the sparse channel as the channel with $D_c \geq 2$. We have found the distribution of the channel density for the random channel by exhaustive search. From this simulation, we can see that the dense channel occurs with probability about 0.6544 and the sparse channel with probability about 0.3456.

2. Simulation results

We assumed a MIMO system in which transmitter and receiver are equipped with 2 antennas and the MIMO channel is spatially uncorrelated. The results of the analytic performance and numerical performance were compared in terms of averaged transmit SNR for the given random channel. We used uncoded QPSK, 16QAM, and 64QAM constellations for the symbol set.

(1) The unitary channel

When the MIMO channel is the unitary matrix, the regular structure of lattice in χ is preserved after the channel transformation, so that the error vectors are not changed by the channel transformation. Assuming 2X2 unitary MIMO channel, there are eight joint error vectors: $\pm[2\ 0\ 0\ 0]^T$, $\pm[0\ 2\ 0\ 0]^T$, $\pm[0\ 0\ 2\ 0]^T$, and $\pm[0\ 0\ 0\ 2]^T$. Thus, (9) should be modified by changing the number of the joint error vectors 4 into $8\,(=4M_t)$. The performance of ML detector over the unitary channel is given in Fig. 1, where the required SNR to get 10^{-3} VSER is around 10.7dB for QPSK, 18.1dB for 16QAM, and 24.5dB for 64QAM. The SNR gabs between modulation types come from the difference between $d_{\min}$ of each. From this result, we clearly demonstrate that the (9) is very good analytic approximation of VSER performance for the ML detection over the unitary channels.

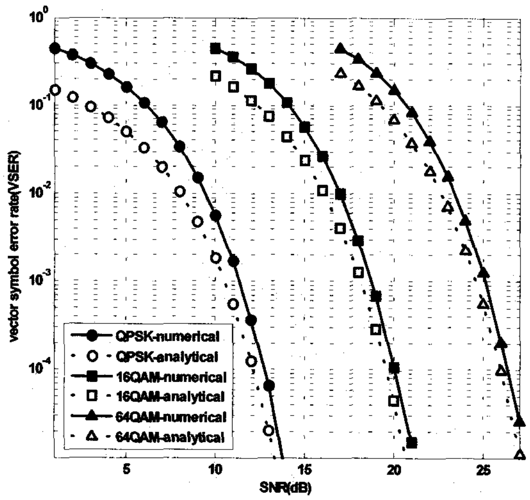

그림 2. Dense 채널에서의 성능비교

Fig. 2. Performance comparisons over the dense channel.

(2) The dense channel and the sparse channel

Though the analytic performance for the unitary channel gives a good approximation for ML detector, this channel hardly happens in real environment. To show the validity of our results in the real random channel, we performed simulations over the dense channel and the sparse channel which contain the case of all random channels. For the dense channel, we generated a random channel whose channel density is 1.002.

Fig. 2 shows the analytical and the numerical performances over the dense channel. The SNR gaps with respect to 10^{-3} VSER between analytical and numerical results are less than 1dB and decrease as SNR increases. In the dense channel, the next nearest neighbors are close to the nearest neighbors, so that they may act as the nearest neighbors. In low SNR region, since the noise variance is larger than the squared minimum distance, the error occurs into the next nearest neighbors as well as the nearest neighbor. However, we did not take into account the errors to the next neighbor in our approximation. This is reason why the analytical result is relatively loose in low SNR region. On the contrary, as SNR increases, the error events happen dominantly into the directions of the nearest neighbors, the analytical result is tightened to numerical results as we expected. Note that since channel considered in this

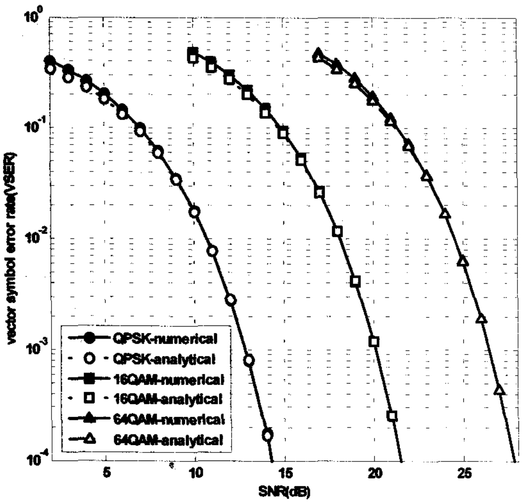

그림 3. Sparse 채널에서의 성능비교
Fig. 3. Performance comparisons over the sparse channel.

figure was severely dense, this analytical results in this channel might be looser than those in any other moderate dense channel.

We also evaluated the numerical performance to compare with analytical performance for the sparse channel. We generated a sparse channel whose density is 2. Fig. 3 gives the analytical and the numerical performance over the sparse channel. With compared to the results in Fig. 2, the analytical performance converges more quickly to the numerical performance. This comes from that since the next nearest neighbors are further away from transmit vector than the nearest ones, the error events tend to happen dominantly into the nearest neighbors even in low SNR region.

From these results, we can declare that our works provide a desirable measure for the performance of ML detection over the various random MIMO channels.

III. Conclusion

In this paper, we have provided the analytical performance of ML detection for the given random MIMO channel. The derived analytical performance has been verified in various random channels: the unitary channel, the dense channel, and the sparse channel, which represent all types of random channels. We have shown by simulations that our analytical result can approximate the numerical performance in the unitary channel and in the sparse channel quite well. In the dense channel, though the analytical performance has the difference of less than 1dB with compared to the numerical result at 10^{-3} VSER, it becomes tight to the numerical results as SNR increases. As a result, we have verified that our results provide a good approximation for the performance of ML detection in random MIMO channels. The analytical performance we have derived can be of use for various applications using ML detector, such as adaptive modulation with ML detection, antenna subset selection and quantized multi-stream beamforming, etc.

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