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OFDM 및 SC-CP 시스템에 대한 결정지향 방식의 평균위상에러 정정

(Correction of Mean Phase Error for OFDM and SC-CP Systems
using Decision-Directed Method)

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요 약

OFDM(Orthogonal Frequency Division Multiplexing) 및 SC-CP(Single Carrier with Cyclic Prefix) 방식은 무선 분야에서 각기 각광받고 있으며, 두 방식 모두 주파수 영역에서 효율적인 등화를 수행할 수 있는 장점을 지니고 있다. 무선 채널의 도플러 쉬프트 현상이 수반되는 경우 등화기만으로 온전히 왜곡 보상이 여의치 않을 수 있으며, 이에 따라 위상에러 추적회로와 등화기를 연동시켜 성능 향상을 도모할 수 있다. 본 논문에서는 평균위상에러의 효과를 기술하였고, 결정지향 방식의 평균위상에러 추적회로와 proportional 등화기를 연동한 성능을 설계하였다. 아울러 시뮬레이션 결과를 통해 시스템의 성능 저하를 최소화하면서 추적회로의 연산 부담을 줄일 수 있음을 제시하였다.

Abstract

The orthogonal frequency division multiplexing (OFDM) technique and the single carrier with cyclic prefix (SC-CP) scheme are very attractive solutions for wireless applications, being computationally efficient since equalization is performed in the frequency domain. The equalizer could not entirely handle significant mean Doppler shift. This motivates the use of a phase error tracking loop that operates jointly with the frequency equalizer. This paper describes the effect of the mean phase error and the performance of the proportional equalizer coupled with a phase error tracking loop based on decision-directed method. Furthermore, simulation results show that we can reduce the computational load of the tracking loop with minimal performance degradation.

Keywords : Doppler Shift, FDE, Mean Phase Error, OFDM, SC-CP

I. Introduction

The orthogonal frequency division multiplexing (OFDM) technique has been recognized to be one of

the most promising methods for wireless applications. On the other side, the single carrier with cyclic prefix (SC-CP) scheme is also gaining attention as a potential competitor with OFDM. OFDM and SC-CP systems are immune to channel dispersion, and they are computationally efficient since equalization is performed on a block of data at a time in the frequency domain^[1].

In case of a coherent demodulation scheme, the phase error may be fatal for long transmission

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bursts. Thus, a phase error correction scheme must be considered^[2-4]. In this paper, we employ a phase tracking algorithm coupled with the frequency domain equalizer (FDE) in order to correct both the frequency selectivity and the residual mean phase error. Computer simulations show that the phase error tracking loop provides a synergy effect for an equalizer to correct signal distortion. Furthermore, we can reduce the computational load of the tracking loop with minimal performance degradation.

This paper is organized as follows: Section II describes similarities and differences between OFDM and SC-CP. The mean phase error component is derived in section III. In this section, the proportional equalizer operating in the frequency domain is given, and the structure of FDE combined with the phase error tracking loop is also described. Simulation results and conclusions follow in the next sections.

II. System Configurations

1. OFDM and SC-CP

Two fast Fourier transform (FFT)/inverse FFT-based transmission techniques, OFDM and SC-CP, are very attractive solutions to multipath environments, having reasonable signal processing complexity^[1]. Fig. 1 shows simplified baseband block diagrams for two schemes. Serial-to-parallel and parallel-to-serial converters in both schemes and pulse shaping filters in the SC-CP system are dropped for brevity.

OFDM avoids intersymbol interference (ISI) by adding a guard interval, which is the cyclically extended part of an OFDM symbol to prevent intercarrier interference (ICI). The configuration of SC-CP, shown in the lower part of Fig. 1, has similar features with that of OFDM. Similar to the OFDM system, CP is also inserted in the SC-CP system to prevent ISI due to impulse response of a time variant channel. OFDM and SC-CP without an extra adaptation process exhibit the same complexity.

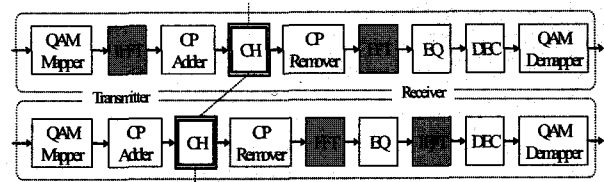


그림 1. OFDM(위)과 SC-CP(아래)

Fig. 1. OFDM(upper part) and SC-CP(lower part).

The main difference between OFDM and SC-CP is the placement of an IFFT operation. As a result, decisions are made in the frequency domain in OFDM and in the time domain in SC-CP, respectively.

2. Time-Variant Channel

Wireless transmission environments are generally represented by frequency selective fading channels with time variant characteristics. The characteristics of wireless channels are quite variable and depend on a variety of factors. Fig. 2 shows a time variant channel, which is modeled as a Rayleigh fading channel with Doppler shift. In this environment, a system may require some channel equalizations to compensate signal distortion.

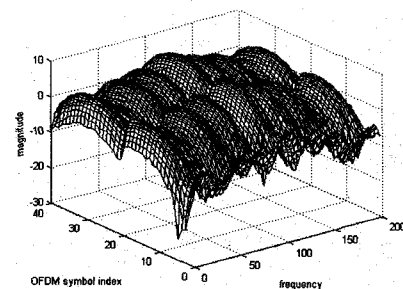


그림 2. 시변 채널

Fig. 2. Time-variant channel.

III. Mean Phase Error

1. Extraction of Mean Phase Error

The OFDM method effectively converts a wide band frequency selective fading channel into a series of narrow band frequency non-selective subchannels by using the parallel transmission scheme. The frequency selective fading channel thus can be

approximated by the sum of flat fading subchannels. As a matter of convenience, considering frequency offsets separately under a channel in OFDM^{[3][4]}, we can express received signal as

$$r(n) = \sum_{k=0}^{N-1} X(k)H(k)e^{j\frac{2\pi}{N}(k+\varepsilon)n}, \quad (1)$$

where $X(k)$ denotes a QAM encoded complex signal which is defined in the frequency domain. N is the number of subcarriers and k is the subcarrier index. $H(k)$ is the channel transfer function and ε is the normalized frequency offset component. CP and additive Gaussian noise are not considered in (1) for simplification. Provided that the correct timing synchronization, after demodulation, received signal becomes as follows:

$$\begin{aligned} R(k) &= \frac{1}{N} \sum_{n=0}^{N-1} r(n) e^{-j\frac{2\pi}{N}kn} \\ &= \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} X(l)H(l) e^{j\frac{2\pi}{N}(l+\varepsilon)n} e^{-j\frac{2\pi}{N}kn} \\ &= \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} X(l)H(l) e^{j\frac{2\pi}{N}(l-k)n} e^{j\frac{2\pi}{N}\varepsilon n}. \end{aligned} \quad (2)$$

In (2), All terms except $l=k$ are ICI components due to the loss of orthogonality and exhibit the appearance of white noise. If N is large and ε is small, we can approximate $e^{j(2\pi/N)\varepsilon n}$ as $1 + j(2\pi/N)\varepsilon n$. So, in case of $l=k$, (2) can be expressed as

$$\begin{aligned} R(k) &= \frac{1}{N} \sum_{n=0}^{N-1} X(k)H(k)e^{j\frac{2\pi}{N}\varepsilon n} \\ &= X(k)H(k) \frac{1}{N} \sum_{n=0}^{N-1} e^{j\theta(n)} \\ &\approx X(k)H(k) \frac{1}{N} \sum_{n=0}^{N-1} \{1 + j\theta(n)\} \\ &= X(k)H(k) + jX(k)H(k) \frac{1}{N} \sum_{n=0}^{N-1} \theta(n) \\ &= X(k)H(k)(1 + j\theta_{avg}) \\ &\approx X(k)H(k)e^{j\theta_{avg}}. \end{aligned} \quad (3)$$

This expression shows that data symbols are distorted by not only the channel transfer function but also the mean phase error. The mean phase deviation causes the rotation of constellation points,

which is multiplied by $j\theta_{avg}$ as shown in (3).

In SC-CP, received signal can be expressed as the circular convolution of the transmission signal and the channel impulse response by virtue of CP. Similar to OFDM, a wide band channel in SC-CP can be treated as the sum of narrow band subchannels. Thus, received signal in the frequency domain can be also expressed in similar manner as (3).

Now, the effect of a channel transfer function $H(k)$ can be compensated by FDE, and additionally the rotation of a constellation due to the residual mean phase error can be detected and corrected by an extra tracking algorithm.

2. FDE using Proportional Algorithm

FDE has the property of relatively low complexity compared to time domain equalizer (TDE). Moreover, adaptive algorithms in the frequency domain usually converge faster than those in the time domain^[5]. Considering additive noise $w(n)$, we can briefly rewrite received signal as

$$R(k) = X(k)H(k) + W(k), \quad (4)$$

where $W(k)$ is the Fourier transform of $w(n)$. The equalized signal $Y(k)$ in the frequency domain is then given by

$$Y(k) = C(k)R(k) = C(k)X(k)H(k) + C(k)W(k). \quad (5)$$

In the training mode, the equalizer coefficient $C(k)$ may be calculated by the least squares (LS) estimator instead of the minimum mean squares error (MMSE) method^[6]. Thus, we can simply get the equalizer coefficient at a k -th frequency as

$$C(k) = 1/H(k) = X_T(k)/R_T(k), \quad (6)$$

where $X_T(k)$ is a FFT value of the known training signal and $R_T(k)$ is that of the received training signal. After a training block, based on an adaptive algorithm, the receiver can adjust coefficients to compensate for a channel frequency response and apply them in the following blocks. In the information

mode, it is possible to use a conventional least mean squares (LMS) adaptive algorithm operating in the frequency domain^[5].

As shown in (5), equalization affects noise. In the presence of noise, the proportional equalizer in the frequency domain can be considered^[7]. The proportional equalizer has robustness to time variance of channels and its coefficients are adjusted according to

$$\begin{aligned} C(j, k) &= C(j-1, k) + \mu \left[\frac{A(j-1, k)}{R(j-1, k)} - C(j-1, k) \right] \\ &= (1-\mu)C(j-1, k) + \mu \frac{A(j-1, k)}{R(j-1, k)}, \end{aligned} \quad (7)$$

where $C(j, k)$ is a k -th coefficient at a j -th block. $A(j-1, k)$ and $R(j-1, k)$ are the decided symbol and the received signal, respectively. The equalizer constant μ determines the influence of the former coefficient. If μ is zero, the previous coefficients are used for all the following information symbols without adjustment of coefficients. In the initialization mode, μ is usually unity, and in the decision mode, this value can be set between zero and unity. Prior to information blocks, initial coefficients can be obtained by LS, which is the same case that μ is fixed to unity. In the information mode, the equalizer coefficients are derived from the previous coefficients, decision-feedback symbols and received symbols.

As a result, the adaptive FDE could converge as quickly as one block by using the proportional scheme as a kind of decision-feedback equalizer operating in the frequency domain.

3. FDE with Phase Error Tracking Loop

Phase coherent systems are adversely affected by Doppler shift, which is one of the important parameters in tracking channel variations. A simple FDE is sensitive to Doppler shifts and not very capable of handling them.

In OFDM, phase errors cause ICI and common phase error (CPE)^{[2-4][8]}. ICI can be modeled as a Gaussian-like noise, while CPE gives rise to the

rotation of all constellation points. In contrast to the spread of constellation points caused by ICI, the rotation of a constellation can be easily detected and compensated because the rotation error due to CPE occurs in the common direction to all subcarriers.

We can track mean phase errors of OFDM symbols by using the decision-directed method. Using the gradient method, for decided symbol $A(j, k)$ and equalized symbol $Y(j, k)$ in the frequency domain, the update algorithm of a mean phase error at a j -th OFDM block can be induced as

$$\begin{aligned} \Phi(j) &= \Phi(j-1) - \frac{\alpha}{2} \nabla_{\Phi} [MSE] \\ &= \Phi(j-1) - \alpha E \{ \text{Im}(A(j, k) Y^*(j, k)) \} \\ &= \Phi(j-1) - \alpha \frac{1}{N} \sum_{k=0}^{N-1} \{ \text{Im}(A(j, k) Y^*(j, k)) \}, \end{aligned} \quad (8)$$

where $E\{\cdot\}$ is the ensemble averaging operator, $\text{Im}(\cdot)$ is the imaginary part and the asterisk denotes the complex conjugate. The gradient of the mean square error (MSE) is given by averaging all of phase differences. Although the computational load of calculating mean phase errors is proportional to the FFT size, we could reduce the load by dealing with some fraction of all phase errors in a FFT period.

Similarly, the update equation is rewritten as (9) in SC-CP.

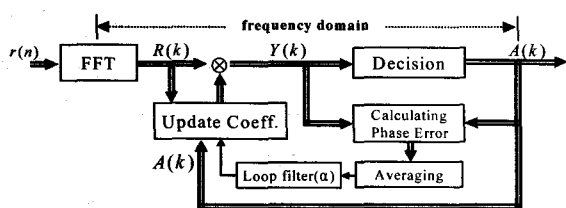
$$\theta(j) = \theta(j-1) - \alpha \frac{1}{N} \sum_{n=0}^{N-1} \{ \text{Im}(a(j, n) y^*(j, n)) \}, \quad (9)$$

where $a(j, n)$ and $y(j, n)$ are respectively the decided symbol and the equalized symbol in the time domain.

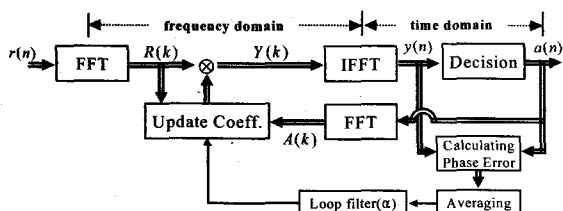
Finally, combining FDE with the phase error tracking loop, we can describe the equalizer coefficient as

$$C(j, k) = \left[(1-\mu)C(j-1, k) + \mu \frac{A(j-1, k)}{R(j-1, k)} \right] e^{-j\theta_{\text{avg}}(j)}, \quad (10)$$

where $\theta_{\text{avg}}(j)$ is updated by (8) in OFDM and by (9) in SC-CP for a FFT period. Although decisions are made in the frequency domain in OFDM and in the time domain in SC-CP, we could expect the improvement of the performance by calculating and



(a) FDE for OFDM



(b) FDE for SC-CP

그림 3. 위상에러 추적회로와 주파수 등화기의 구조

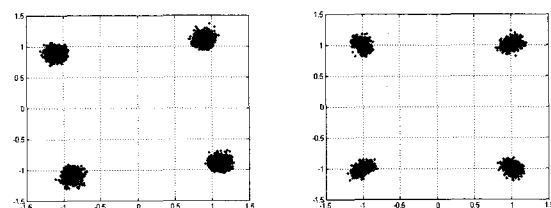
Fig. 3. Structure of FDE with phase error tracking loop.

correcting the mean phase error in each domain. Additionally, we may reduce the number of phase errors for calculation of the mean rotation error in case of channels in which each phase does not change rapidly.

Fig. 3 shows the structure of the proportional equalizer with phase error tracking loop, where α is a small positive constant called the loop gain. The optimum loop gain is subject to factors of system environments. In case of an adaptive FDE for SC-CP of Fig. 3(b), we require an additional FFT, which may not significantly add complexity to the receiver. The savings by the use of FDE are substantial, even if one takes into account the load of an additional FFT processing. If we used special pilots per information block, an additional FFT could be eliminated. Instead, this scheme requires interpolation to estimate a channel transfer function. We do not deal with those problems in this paper.

IV. Simulation Results

In the simulations, we have used the Rayleigh fading channel with 6 taps and chosen the guard interval to be greater than the maximum delay



(a) Without correction of mean phase errors

(b) With correction of mean phase errors

그림 4. 4QAM-OFDM의 신호 성상도

Fig. 4. Signal constellations of 4QAM-OFDM.

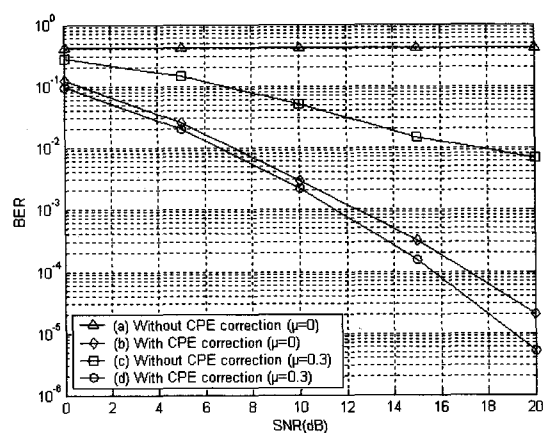


그림 5. 4QAM-OFDM의 BER 성능

Fig. 5. BER performances for 4QAM-OFDM.

spread to combat ISI. We did not consider channel coding and interleaving to improve the performance, since the aim was to observe mainly the performance of FDE combined with mean phase error tracking loop.

The signal constellations and BER results for 4QAM-OFDM schemes are shown in Figs. 4 and 5, respectively. The number of subcarriers was 256, and the filter gain of the phase error tracking loop was fixed to 0.7. We have set $f_d T_s = 6 \times 10^{-5}$, where f_d is the Doppler frequency and T_s is the sampling time. As shown in these results, in case of applying only FDE without correcting mean phase errors, the rotation of constellation points appears since residual phase errors remain. The simulation results show that the gap of the BER performance increases as SNR increases.

Figs. 6 and 7 present the constellations and BER

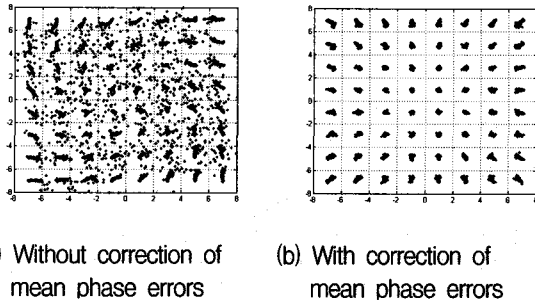


그림 6. 64QAM-SC-CP의 신호 성상도

Fig. 6. Signal constellations of 64QAM-SC-CP.

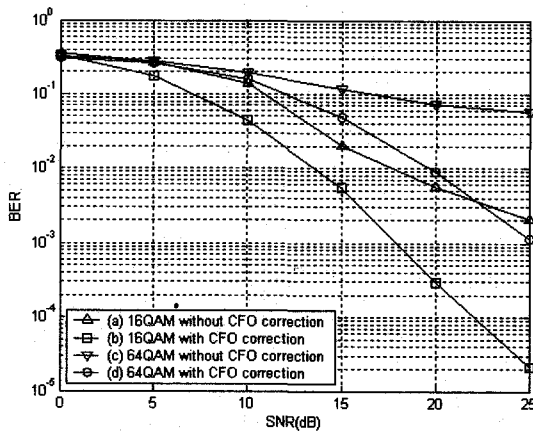


그림 7. 16/64QAM-SC-CP의 BER 성능

Fig. 7. BER performances for 16/64QAM with SC-CP.

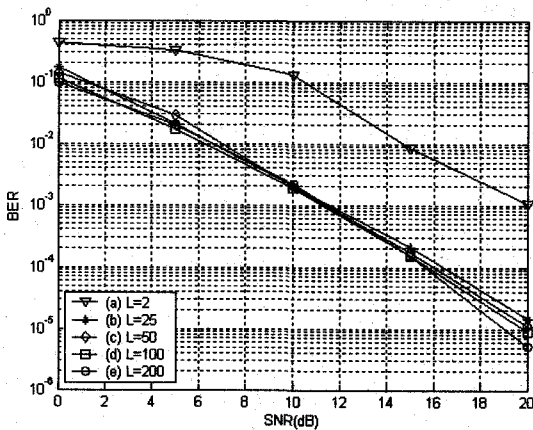


그림 8. 4QAM-OFDM에서 평균 개수에 따른 BER 성능 비교

Fig. 8. Comparison of BER performances for different averaging numbers in 4QAM-OFDM.

curves of SC-CP, respectively. As the order of QAM increases, the tailing of constellation points becomes more serious. In SC-CP, the simulations were carried out using FDE with 128 taps. For pulse shaping

filtering in the transmitter and receiver for the SC-CP system, we used the root-raised cosine filter with the roll-off factor of 0.25, upsampled by four times. The equalizer constant μ in SC-CP was 0.6. The performance was enhanced dramatically by correcting mean phase errors likewise.

Furthermore, we could reduce the number of phase errors for calculation of the mean phase error, since FFT size is usually large and the rotation due to the mean phase error is in the common direction for a FFT period. Fig. 8 shows BER performances for the number of phase errors averaged in the 4QAM-OFDM scheme.

As shown in these results, the degradation of the performance is not serious, even if the number of subchannels for averaging phase errors is considerably reduced. In our results, even if the number of phase differences for calculation of the mean phase error was twenty or more, the improvement on BER performance was not remarkable. This number may depend on channel environments. In conclusion, we could ease the computational burden for correcting mean phase errors and thus implement FDE with phase error tracking loop having low complexity.

V. Conclusions

In this paper, we have investigated the effect of mean phase errors and the performance of the frequency equalizer coupled with the mean phase error tracking loop. We have adopted the proportional equalizer as an adaptive FDE in order to cope with time variant channels and combined it with the phase error tracking loop based on decision-directed method. The equalizer could not solely handle significant mean Doppler shift. This motivated the use of a mean phase error tracking loop that operates jointly with FDE. As a result, the performance improvement could be achieved by using FDE coupled with the phase error tracking loop for OFDM

and SC-CP. Furthermore, we could significantly reduce the computational load of the tracking algorithm with negligible effect on performance. Simulation results showed that the BER performance of the mean phase error tracking loop with low complexity could be allowable.

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