

# Modeling of an Electricity Market Including Operating Reserve and Analysis of Supplier's Bidding Strategies

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**Abstract** - In an electricity market with imperfect competition, participants devise bidding plans and transaction strategies to maximize their own profits. The market price and the quantity are concerned with the operation reserve as well as the bidding system and demand curves in an electricity market. This paper presents a market model combined by an energy market and an operating reserve market. The competition of the generation producers in the combined market is formulated as a gaming of selecting bid parameters such as intersections and slopes in bid functions. The Nash Equilibrium (NE) is analyzed by using bi-level optimization; maximization of Social Welfare (SW) and maximization of the producers' profits.

**Keywords:** Bidding Strategy, Electricity Market, Game Theory, Market Operator, Nash Equilibrium, Operating Reserve, Social Welfare, Supply Function Model

## 1. Introduction

The electricity industry undergoes a process of fundamental restructuring from the vertically integrated mechanisms to the competitive market with an emphasis on supplying stable electricity. Transactions in a competitive market are accomplished among generation companies (Gencos), consumers, and the market operator (MO) within the market principle. Through the transaction of electric power, Gencos try to maximize profit and consumers try to maximize surplus [1]. On the other hand, the MO determines the quantity and the price that maximizes the market value (social welfare; SW) by utilizing the supply/demand curve [2-6].

Inability to store electricity makes it impossible to cope with irregular changes of electricity demand in real time. So in order to compensate for temporary differences between supply and demand, the backup reserve becomes an essential element [7, 8].

In a market structure wherein the independent systems operator (ISO) and the MO function independently, it is commonly known that the reserve is obtained through a separate market from the energy market [7]. In this divided market structure, Gencos must carry out their reserve transactions in the reserve market through participation in a

bidding process. Based on this process, the MO determines quantities and prices of energy and reserve with simultaneous consideration to the energy and the reserve market (hereinafter referred to as the combined market).

There are some analytical models that explain the strategic bidding behaviors of Gencos in a competitive market. The key parameters of the Cournot, Bertrand, and the Supply Function models respectively are quantity, price and cost function [3]. Among these three models, we will use the supply function model to analyze the reserve market.

The transaction profit functions for Gencos and the SW for the MO, in a combined market, are proposed within this paper. In the profit function for Gencos, the bidding parameters on the energy and the reserve supplies are defined as variables. In regards to the SW for the MO, the price and the quantity are formulated with consideration to the reserve on the basis of constant rate. The bi-level optimization [9] is utilized to calculate the Nash equilibrium in the combined market [10]. We compare the strategic bidding of an energy market with the strategic bidding of a reserve market and analyze the price at the Nash equilibrium.

## 2. Uncombined Market (Energy Market without Reserve)

A reserve market is not considered in conventional studies that analyze the participants' strategy in an electricity market [3]. An energy market without reserve will be referred to as an uncombined market in this

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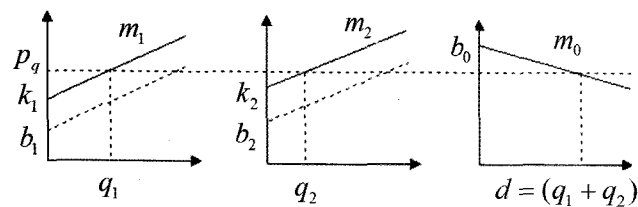
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paper. An uncombined market compared to a combined market is quite structurally simple.

In the uncombined market, Gencos determine the strategic energy supply function for maximizing profit on the demand function. The MO considers the strategic energy supply function that is presented by Gencos as the marginal cost (hereinafter referred to as the pseudo generation cost) and determines the energy quantity and the market price that maximizes SW.

Fig. 1 shows the equilibrium price in the uncombined market. There are no constraints such as generation capacity, etc. in the model. The marginal cost and demand curve are considered as linear functions. The strategy parameter of Gencos is assumed with fixed slope and the variable intercept in the marginal cost function.



- $p_q$  : Market price for energy,  $d$  : Energy demand
- $b_0$  : Intercept of demand curve,  $m_0$  : Slope of demand curve
- $b_i$  : Intercept of marginal cost function for Genco  $i$
- $m_i$  : Slope of marginal cost function for Genco  $i$
- $k_i$  : Strategic bidding of Genco  $i$  in an energy market
- $q_i$  : Energy quantity of Genco  $i$

Fig. 1 Equilibrium in an Uncombined Market

Gencos try to raise the  $k_i$  to increase profit in an uncombined market. In response, the prices increase causing demands to fall in the demand curve. Accordingly, the supply falls and the attempts to increase profit converge into an equilibrium as shown in Fig. 1. The energy quantities of Gencos are determined dependent upon the supply function and the resulting identical price [1-6]. We can confirm this principle by calculating the optimum point of the maximized SW [3].

### 3. Combined Market

#### 3.1 The Interaction between the MO and the Gencos

An electricity market fundamentally operates with emphasis on a steady supply and high standard of quality electricity. The demand for electricity fluctuates dependent upon various factors from time to time. As electric power cannot be stored, supply of electricity in relation to irregular demand is difficult to adjust in real time. Hence,

securing the reserve is an essential element for successfully operating a stable electricity market [7, 8]. In this study, the reserve rate is set at 10% of the entire energy quantity and modeled as a constraint for the MO to solve the required reserve.

In a competitive structure, it is better for the reserve to be obtained in a market that differs from an energy market [7]. Due to the separation of the energy market and the reserve market, Gencos participate in independent bidding in both markets. Therefore, Gencos strive to choose strategic bidding parameters that maximize total profit resulting from an energy market and a reserve market. On the other hand, the MO determines the quantity and price of energy and reserve that maximize the market value in the combined market. Fig. 2 represents the interaction between the MO and the Gencos in an uncombined market.

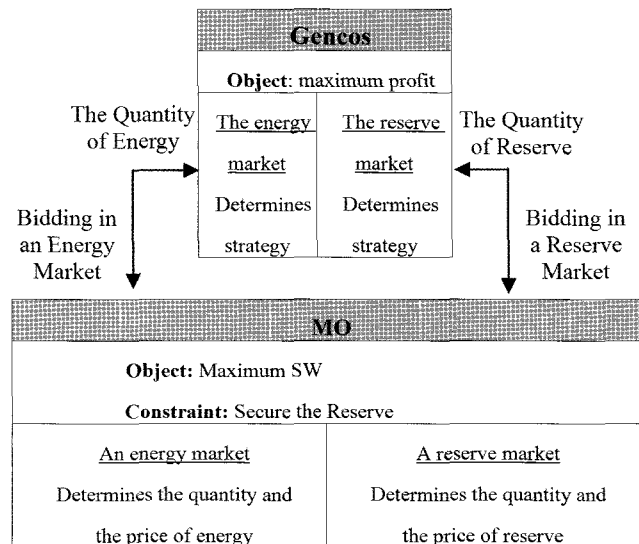


Fig. 2 Participants in a Combined Market Model

The MO determines the quantity and the price in the energy and the reserve markets that will maximize the market value on the basis of the bids by Gencos. The quantities and the market prices, in a combined market, influence the profit of Gencos directly. Therefore, Gencos need to analyze how the strategic bidding affects quantities and prices of the energy and the reserve market. Through these analyses, Gencos choose the bidding parameters that maximize profit. These strategic biddings of the Gencos also affect the MO on the prices and the quantities. This interaction between the MO and the Gencos continues until it reaches a state of equilibrium.

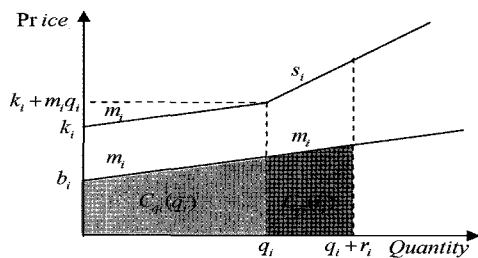
#### 3.2 Modeling of bidding function

The supply function of the energy and reserve markets that Gencos submit and the demand function are all assumed as linear functions. In this study, the strategic

supply function in an energy market is  $k_i + m_i q_i$  that uses the intercept of the marginal cost as the strategic bidding parameter [9]. In a reserve market, the slope of the supply function is the strategic bidding parameter  $s_i$  in  $k_i + m_i q_i + s_i r_i$  with the energy price fixed at the intercept parameter as  $k_i + m_i q_i$ . Fig. 3 depicts the strategic supply function of Gencos that participate in an uncombined market. The latter function is also represented as the following (1):

$$C_i'(q_i, r_i) = k_i + m_i q_i + s_i r_i \quad (1)$$

Gencos can get two different revenues from an energy market and a reserve in a combined market. As seen in Fig. 3, the energy price is  $k_i + m_i q_i$  and the reserve price is  $k_i + m_i q_i + s_i r_i$ . So the revenue in an energy market is  $q_i \cdot (k_i + m_i q_i)$  and the revenue in a reserve market is calculated as  $r_i \cdot (k_i + m_i q_i + s_i r_i)$ . The generation cost combines the energy cost  $C_{qi}(q_i)$  in the energy market and the reserve cost  $C_{ri}(r_i)$  in the reserve market.



- $s_i$  : The strategic bidding in a reserve market
- $r_i$  : The quantity of reserve
- $C_{qi}(q_i)$  : The generation cost of energy
- $C_{ri}(r_i)$  : The generation cost of reserve
- $C_i'(q_i, r_i)$  : The bidding function

**Fig. 3** Bid Function of Energy and Reserve

Fig. 3 shows that  $k_i$  influences the energy quantity  $q_i$ . It also shows that  $k_i$  influences the demand quantity of the reserve because the energy demand quantity  $\sum q_i$  is directly related to the reserve demand quantity  $\sum r_i$  by 10%. Also  $s_i$  influences not only the energy quantity but the reserve demand quantity as well. Thus, Gencos pursuing the maximum profit strategy must analysis the interactions of  $k_i$  and  $s_i$  and the energy/reserve quantity.

There are cases that consider the reserve cost  $C_{ri}$  [11] and cases that do not [12] consider it in calculating the profit on the reserve transaction. If it doesn't consider  $C_{ri}$  the transaction in the reserve market brings the profit to

Gencos and has nothing to do with the marginal cost. As a result, Gencos prefer the supply in a reserve market rather than in an energy market. We also cannot ignore the cost because the reserve can create a situation that needs the generation of electric power in real time. This is the reason that  $C_{ri}$  is considered in this study.

The Korea Power Exchange (KPX) performs the role of the ISO and the MO. In the Korean electricity power market, the reserve regulation of the electricity market operation rules is not yet perfect [8]. However, the rules are clear as to the necessity of securing a reserve. This demand curve expresses the buyers' willingness to pay for the energy. In this case, the energy price is determined by the demand curve. The energy quantity is calculated as  $b_0 - m_0 \sum q_i$  as represented in Fig. 1. Securing of the reserve that comes under the reserve rate as the constraint of the MO is also considered in this study.

## 4. Formulation and the Nash Equilibrium

### 4.1 Optimization Formulation

The problem of the MO in an uncombined market, attempting to maximize the market value in terms of SW, is represented in the following set of equations (2)~(5):

$$\text{Max } SW = B(d) - \sum C_{qi}'(q_i) - \sum C_{ri}'(r_i) \quad (2)$$

$$\text{s.t. } d = \sum q_i \quad (3)$$

$$\eta \sum q_i = \sum r_i \quad (4)$$

$$q_i + r_i \leq P_i^{\text{max}} \quad (5)$$

$SW$  : Social Welfare,

$B_i(d_i)$  : Consumer Benefit

$\eta$  : Reserve Ratio,

$P_i^{\text{max}}$  : Upper limit of generation,

$C_{qi}'(q_i)$  : Pseudo cost function of energy generation,

$C_{ri}'(r_i)$  : Pseudo cost function of reserve.

The objective function of the MO is calculated as the consumer benefit subtracted by the generation costs of energy  $C_{qi}'$  and of reserve  $C_{ri}'$  as shown in (2). Equation (3) represents the matching of the energy supply with the demand. The reserve ratio is represented in (4) where  $\eta = 0.1$  in this paper. Equation (5) indicates the inequality constraint of the upper generation limit. With the latter equations (3)~(5), the MO determines the  $q_i$  and  $r_i$  to maximize (2).

Conversely, Gencos determine the strategic bidding  $k_i$  and  $s_i$  that maximizes profit in each market. The profit of Gencos is the sum of the profit ( $\Pi_{q_i}$ ) from the energy market and the profit ( $\Pi_{r_i}$ ) from the reserve market. The total profit of Gencos ( $\Pi_i$ ) is expressed as (6):

$$Max \Pi_i(k_i, s_i, q_i, r_i) = \Pi_{q_i}(k_i, s_i, q_i, r_i) + \Pi_{r_i}(k_i, s_i, q_i, r_i) \quad (6)$$

Where  $k_i, s_i$  influences the quantity of the energy and the reserve. The quantities of the energy and the reserve are also directly related to the profit of Gencos. The Gencos profit is determined by way of  $k_i, s_i, q_i$  and  $r_i$  as represented in (6).

### 4.2 Computation of the Nash equilibrium

The MO determines the quantities  $q_i$  and  $r_i$  that maximize SW with respect to the bidding parameters of the Gencos. Gencos determine the  $k_i, s_i$  that maximize profits on the basis of the quantity that is determined by the MO. So the variables of the MO and of the Gencos to be determined have direct correlation with each other.

In order to solve the problem of different objectives influencing each other, we use a bi-level optimization technique in computing the Nash equilibrium [9]. The MO's objective is maximizing the SW within the reserve rate constraint. Equations (2)~(5) combine to form the Lagrange's function as shown in (7).

In the sample system, two Gencos, G1 and G2, compete in an electricity market with the demand curve as a linear function:

$$L = SW + \lambda \{r_1 + r_2 - \eta(q_1 + q_2)\} \quad (7)$$

Where,

$$SW = b_0(q_1 + q_2) - 0.5m_0(q_1 + q_2)^2 - \{k_1q_1 + 0.5m_1q_1^2 + k_2q_2 + 0.5m_2q_2^2 + (k_1 + m_1q_1)r_1 + 0.5s_1r_1^2 + (k_2 + m_2q_2)r_2 + 0.5s_2r_2^2\}$$

$\lambda$  in (7) is a multiplier for adding the constraint of reserve ratio into the Lagrange's function, and has a relation to the degree of binding of the constraint.

The optimal conditions of MO that maximize the SW within the constraints are as shown in (8). By solving these conditions, we can obtain  $q_1, q_2, r_1, r_2$  and  $\lambda$ .

$$\frac{\partial L}{\partial q_1} = 0, \quad \frac{\partial L}{\partial q_2} = 0, \quad \frac{\partial L}{\partial r_1} = 0, \quad \frac{\partial L}{\partial r_2} = 0, \quad \frac{\partial L}{\partial \lambda} = 0 \quad (8)$$

Since the MO determines the energy quantity and the reserve quantity at the same time, the following conditions are satisfied:  $\partial q_i / \partial r_i = 0, \partial r_i / \partial q_i = 0, \partial q_i / \partial r_j = 0$  and  $\partial r_i / \partial q_j = 0$ . By the way of developing (8), the optimum conditions are arranged as a matrix-vector form as in (9).

$$\begin{bmatrix} m_0 + m_1 & m_0 & m_1 & 0 & \eta \\ m_0 & m_0 + m_2 & 0 & m_2 & \eta \\ m_1 & 0 & s_1 & 0 & -1 \\ 0 & m_2 & 0 & s_2 & -1 \\ \eta & \eta & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ r_1 \\ r_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} b_0 - k_1 \\ b_0 - k_2 \\ -k_1 \\ -k_2 \\ 0 \end{bmatrix} \quad (9)$$

In a reserve market, Gencos submit a slope of the supply function as a bid parameter. The variable  $s_i$  in (9) is the bid parameter of  $G_i$ . The quantities  $q_i$  and  $r_i$  have nonlinear relations with  $s_i$  as shown in the form of (9). On the contrary, the quantities  $q_i$  and  $r_i$  have linear relations with the bid parameter  $k_i$  in an energy market as indicated in (9).

The profit  $\Pi_i$  of  $G_i$  in the combined market is expressed as (10).

$$\Pi_i = (k_i + m_i q_i) q_i + \{(k_i + m_i q_i) + s_i r_i\} r_i - \{b_i(q_i + r_i) + 0.5m_i(q_i + r_i)^2\} \quad (10)$$

The bid parameters  $k_i$  and  $s_i$  providing maximum profit of Gencos, are obtained by solving  $\partial \Pi_i / \partial k_i = 0$  and  $\partial \Pi_i / \partial s_i = 0$ . The equations are rearranged as a matrix-vector form as in the following (11).

$$\begin{bmatrix} \frac{\partial q_i}{\partial k_i} + \frac{\partial r_i}{\partial k_i} & 2r_i \frac{\partial r_i}{\partial k_i} \\ \frac{\partial q_i}{\partial s_i} + \frac{\partial r_i}{\partial s_i} & 2r_i \frac{\partial r_i}{\partial s_i} \end{bmatrix} \begin{bmatrix} k_i \\ s_i \end{bmatrix} = \begin{bmatrix} \frac{\partial q_i}{\partial k_i} & \frac{\partial r_i}{\partial k_i} \\ \frac{\partial q_i}{\partial s_i} & \frac{\partial r_i}{\partial s_i} \end{bmatrix} \begin{bmatrix} b_i - m_i q_i \\ b_i + m_i r_i \end{bmatrix} - \begin{bmatrix} q_i + r_i \\ r_i^2 \end{bmatrix} \quad (11)$$

In order for the Gencos to maximize profit, the parameters  $k_i$  and  $s_i$  need to be computed by evaluating the following sensitivity terms:  $\partial q_i / \partial k_i, \partial r_i / \partial k_i, \partial q_i / \partial s_i, \partial r_i / \partial s_i$  as shown in the matrix in (11). The terms are derived in the following process. Equation (9) is expressed in (12) as symbols of a matrix and vectors containing the bid parameters.

$$A(s_i) \cdot x(q_i, r_i) = b(k_i) \quad (12)$$

Matrix  $A$  involves  $s_i$ , the vector  $x$  involves  $q_i$  and  $r_i$ , and the vector  $b$  involves  $k_i$ . Therefore,  $\partial q_i / \partial k_i$  and  $\partial r_i / \partial k_i$  can be calculated by  $A^{-1} \cdot b$ .

The other sensitivities  $\partial q_i / \partial s_i$  and  $\partial r_i / \partial s_i$  are calculated in a different way. Since the  $k_i$  in the right hand side is independent of  $s_i$  vector  $b$  has no direct relation with the bid parameter  $s_i$ . Therefore, (12) can be expressed as  $[A + dA] \cdot [x + dx] = b$  by denoting the changes of  $A$  and  $x$  as  $dA$  and  $dx$ . Under the assumption of infinitesimal change of  $s_i$ , the equation is arranged as  $dA \cdot x + A \cdot dx = 0$ , because  $dA \cdot dx$  can be ignored. Written in another way, the equation is expressed as (13).

$$-A^{-1} \cdot dA \cdot x = dx \quad (13)$$

Let the (i,j) element  $A_{ij}$  in the matrix  $A$  change slightly. The latter form (13) can be expressed as the following element-wise form in (14) by taking into consideration the  $dA$ 's structure for a non-zero term.

$$\begin{bmatrix} 0 \cdots 0 & -A_{1j}^{-1} \times dA_{ij} & 0 \cdots 0 \\ 0 \cdots 0 & -A_{2j}^{-1} \times dA_{ij} & 0 \cdots 0 \\ 0 \cdots 0 & -A_{3j}^{-1} \times dA_{ij} & 0 \cdots 0 \\ \vdots & \vdots & \vdots \\ 0 \cdots 0 & -A_{mj}^{-1} \times dA_{ij} & 0 \cdots 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \\ \vdots \\ dx_m \end{bmatrix} \quad (14)$$

In the element-wise equations, the change of  $x_k$  with regard to  $A_{ij}$  is expressed as  $dx_k / dA_{ij} = -A_{ki}^{-1} \cdot x_j$  and this is defined as a partial differential equation as shown in (15).

$$\partial x_k / \partial A_{ij} = -[A^{-1}]_{ki} \times x_j \quad (15)$$

The initial values of the variables in the matrix  $A$  and the vector  $x$  are required to start the computation of (15).

In this study, we set up the initial value  $k_i$  and  $s_i$  of the Gencos, and then determine  $q_i$  and  $r_i$  by solving (9). The values  $q_i$  and  $r_i$  are substituted into (15) in order to calculate  $\partial q_i / \partial s_i$  and  $\partial r_i / \partial s_i$ . These sensitivities are used to solve (11) to obtain the new values  $k_i$  and  $s_i$  instead of the initial values. These new values are utilized again to solve (9). This computation process continues while the bid parameters  $k_i$  and  $s_i$  are converged as constant values called equilibrium strategy.

## 5. Case Studies

In the sample system, two Gencos G1 and G2 compete in a combined market with the demand curve as a linear function. Coefficients of the marginal cost function of G1 and G2 are as follows respectively:  $m_1 = 0.25$ ,  $b_1 = 10$  and  $m_2 = 0.45$ ,  $b_2 = 5$ . In the demand curve, the coefficients of slope and intersection are assumed as follows:  $m_0 = 0.5$  and  $b_0 = 100$ . The reserve rate has the value of  $\eta = 0.1$  as a constraint between the quantities of energy and reserve. No other constraint, such as a transmission flow limit, is included except the reserve rate.

As a reserve market is introduced, Gencos have chances to increase profit in the reserve market. Therefore, gaming in the combined market appears in a more complicated fashion [13]. The Nash equilibrium of the competition in the sample system is obtained using the method proposed in this study. The results, as given in Table 1, differ depending on whether or not a reserve market is included in the simulation.

Table 1 shows that the energy price is higher in the combined market (46.92[\$/MW]) than in the uncombined market (40.29[\$/MW]). However, the total energy quantity is less in the combined market (106.15[MW]) than in the uncombined market (119.4[MW]). These results lead to less consumer benefit (B) in the combined market than in the uncombined market. Table 1 also shows that an inclusion of a constraint of a reserve market results in a decrease of SW from 8377 to 7798.

**Table 1** Comparisons between Combined and Uncombined Markets

| Classification    |    | $k$  | $s$  | $q$ [MW] | $r$ [MW] | $p_q$ [\$/MW] | $p_r$ [\$/MW] | profit | $B$    | $SW$   |
|-------------------|----|------|------|----------|----------|---------------|---------------|--------|--------|--------|
| Uncombined Market | G1 | 24.7 | x    | 62.2     | x        | 40.29         | x             | 1400.3 | x      | 4786.2 |
|                   | G2 | 14.5 | x    | 57.2     | x        | 40.29         | x             | 1282.4 | x      |        |
|                   | D  | x    | x    | 119.4    | x        | 40.29         | x             | x      | 8377.1 |        |
| Combined Market   | G1 | 23.9 | 5.97 | 56.31    | 6.07     | 37.98         | 74.20         | 1479.1 | x      | 7798.0 |
|                   | G2 | 15.0 | 8.08 | 49.84    | 4.55     | 37.46         | 74.20         | 1266.7 | x      |        |
|                   | D  | x    | x    | 106.15   | 10.62    | 46.92         | x             | x      | 8377.1 |        |

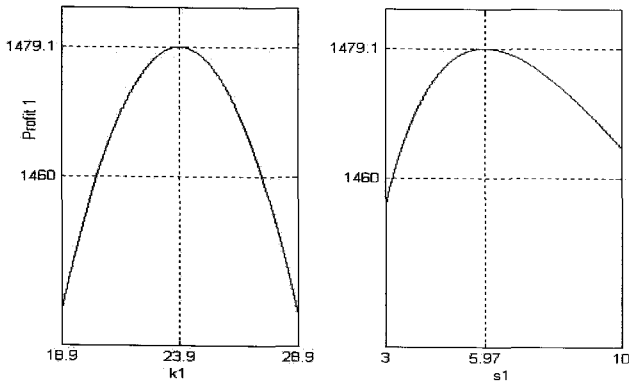


Fig. 4 Marginal Profit of G1 at NE

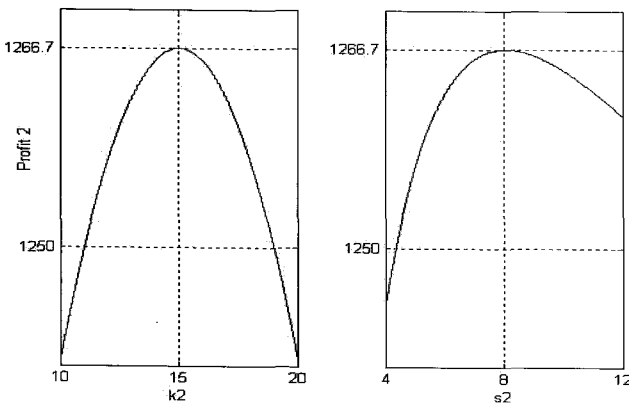


Fig. 5 Marginal Profit of G2 at NE

The curves in Figs 4 and 5 indicate that profits of both Gencos are maximum at  $k_1 = 23.9$ ,  $s_1 = 5.97$ ,  $k_2 = 15.0$  and  $s_2 = 8.08$ . This means there is no incentive to Gencos that increase profit by changing the strategy parameters. It also corresponds to the definition of NE condition.

Gencos determine their strategic bid parameters to maximize profit on the basis of marginal cost. And the bid parameters of Gencos influence the quantities of energy and reserve in the MO's optimization. In order to observe the relation between the marginal costs and the quantities at the NE, some parametric simulations are tested. The parametric combination corresponds to the two values of the two coefficients in the marginal cost of the two Gencos ( $b_i = 5, 10$ ,  $m_i = 0.25, 0.45$ ,  $i = 1, 2$ ).

Table 2 indicates the results of the parametric simulation. The quantity ratio of a reserve quantity over an energy quantity ( $r_1/q_1$ ) of G1 varies around 0.1.

The quantity ratio ( $r_1/q_1$ ) is interpreted as an incentive of G1 to take part in a reserve rather than an energy market in a combined market. In Table 2, the elements in the shaded second column are bigger than those in the first column. The elements in the shaded fourth column are also bigger than those in the third column. This means that if a generator has a bigger intercept  $b_1$  in the marginal cost of

G1, then this company's incentive to transact electricity in a reserve market increases.

Table 2 Quantity Ratio of G1 in Energy and Reserve Markets

| $r_1/q_1$    |            | $m_1 = 0.25$ |            | $m_1 = 0.45$ |            |
|--------------|------------|--------------|------------|--------------|------------|
|              |            | $b_1 = 5$    | $b_1 = 10$ | $b_1 = 5$    | $b_1 = 10$ |
| $m_2 = 0.25$ | $b_2 = 5$  | 0.1          | 0.1128     | 0.0991       | 0.1078     |
|              | $b_2 = 10$ | 0.0896       | 0.1        | 0.0912       | 0.1012     |
| $m_2 = 0.45$ | $b_2 = 5$  | 0.0991       | 0.1078     | 0.1          | 0.1096     |
|              | $b_2 = 10$ | 0.0914       | 0.0991     | 0.0918       | 0.1        |

### 6. Conclusion

Reserve power is an essential element for an electricity system to operate securely. As the electricity industry transforms into a competitive system, the electricity market revolves into a combined market consisting of usable energy and reserved energy. This paper proposes an analysis method for the combined market, and presents the result of the analysis for a sample system.

In a combined market, Gencos have the strategic bidding parameters in an energy market and in a reserve market. The bid parameter in an energy market is modeled as an intercept in a marginal cost function. Alternatively, the parameter in a reserve market is modeled as a slope in a marginal cost function. The mechanism of the combined market is simply formulated as an optimization problem where the MO's objective is the maximization of SW and the Gencos' objectives are profit maximization. The NE of the problem is calculated by the bi-level optimization technique.

In the case study, the combined and the uncombined market are compared at the NE from the viewpoints of market price, transaction quantities, consumer benefits, and SW. Parametric combination of the bidding parameter is simulated to observe an incentive to take part in a reserve market. The incentive studies lead to the result that the incentive to transact electricity in a reserve market increases if an intercept in the marginal cost is bigger.

The model for the electricity market in this paper can be applied to the study for the security of the reserve through the reserve market in our country, which is in need of a study regarding a reserve market.

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