Optimal Generation Asset Arbitrage In Electricity Markets

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Abstract - A competitive generating company (GENCO) could maximize its payoff by optimizing its generation assets. This paper considers the GENCO's arbitrage problem using price-based unit commitment (PBUC). The GENCO could consider arbitrage opportunities in purchases from qualifying facilities (QFs) as well as simultaneous trades with spots markets for energy, ancillary services, emission, and fuel. Given forecasted hourly market prices for each market, the GENCO's generating asset arbitrage problem is formulated as a mixed integer program (MIP) and solved by a branch-and-cut algorithm. A GENCO with 54 thermal and 12 combined-cycle units is considered for analyzing the proposed formulation. The proposed case studies illustrate the significance of simultaneous arbitrage by applying PBUC to multi-commodity markets.

Keywords: Arbitrage, price-based unit commitment, mixed integer programming, energy, ancillary services, fuel, emission allowance, qualifying facilities, bilateral contracts

1. Nomenclature

1.1 Variables

$BE(), SE()$	Purchase cost and sales payoff for a type of emission respectively
$BF(), SF()$	Purchase cost and sales payoff for a type of fuel respectively
ET	Emission type
$FP(,)$	Fuel price of a generating unit or a pseudo unit
FT	Fuel type
i	Denote a thermal unit
$I(,)$	Unit status; 1 means on and 0 means off
$I_d(,)$	Indicate whether or not to provide non-spinning reserve when generating unit is off
j	Denote a configuration of combined-cycle units
k	Denote a QF contract
l	Denote a load contract
$N_d(,)$	Non-spinning reserve of a unit when off
$N_u(,)$	Non-spinning reserve of a unit when on
$p_m(,)$	Generation for segment m of the heat curve
$P(,)$	Generation of a unit
$R(,)$	Spinning reserve of a unit
t	Hour index
$TF(,)$	Transition fuel for a configuration
$TP(), TR(), TN()$	Total generation, spinning, and non-spinning reserve sold to the market
$u_m(,)$	Indicate whether segment m in fuel/emission allowance price curve is fully purchased or sold
$v_m(,)$	Startup time indicator at segment m
$x_m(,)$	Traded fuel/emission for segment m in the price curve

2.2 Constants

$b_m()$	Slope for segment m in heat curve
$CP(,)$	Contracted price with QF/load
$\underline{f}()$	Heat rate at the minimum generating capacity
$F(),\ E()$	Consumption of existent fuel and emission by all the units
$\overline{F}()$	Upper limit on the fuel consumption of a unit
$FP_0(),\ EP_0()$	Purchase price for existent fuel and emission
I	Set of thermal units
J	Set of combined-cycle configurations
K	Set of contracted QFs
L	Set of contracted loads
$MSR()$	Maximum sustained ramp rate (MW/min) for a unit
$NSB()$	Number of segments in the purchase price curve for fuel/emission
$NSF()$	Number of segments for the piece-wise linearized heat rate curve
$NSS()$	Number of segments in the selling price curve for fuel/emission

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$\underline{P}_g(), \overline{P}_g()$	Minimum/maximum generating capacity
$QF_0(,), P_0(,)$	Contracted generation with QF and load, respectively
$QSC()$	Quick start capacity
$SG_0()$	Income for bilateral contracts
$BG_0()$	Payment for bilateral contracts
T	Number of hours for the scheduling period
$\rho_g(,)$	Market price for energy
$\rho_{sr}(,)$	Market price for spinning reserve
$\rho_{nr}(,)$	Market price for non-spinning reserve

2. Introduction

In energy markets, arbitrage refers to optimizing payoff by simultaneous trading the same or an equivalent commodity with net zero investment and without undertaking any risks. Arbitrage also refers to any activity that attempts to buy a relatively under-priced commodity and to sell a similar and relatively over-priced commodity for profit[1]. Two types of arbitrage opportunities are exploited in the energy industry which include same-commodity arbitrage and cross-commodity arbitrage. The cross-commodity arbitrage that is aimed at multiple products within a market or multiple markets is also referred to as spark spread. This paper discusses arbitrage opportunities for a GENCO with thermal and combined-cycle units in energy, ancillary service, fuel, and emission markets.

Unit commitment (UC) in vertically integrated electric power systems is to optimize generating resources for supplying the system load while satisfying prevailing constraints, such as minimum on/off time, ramping up/down, minimum/maximum generating capacity, fuel and emission limit [1]-[2]. The UC problem was solved successfully using Lagrangian relaxation method [1]-[4]. In restructured power systems, however, the UC used by individual generating companies (GENCOs) refers to optimizing generation resources that could maximize GENCOs' payoffs. This UC approach, which has a different objective than that of the vertically integrated utilities, is referred to as price-based unit commitment (PBUC). PBUC emphasizes the importance of price signals in a completive environment. In PBUC, satisfying load is no longer an obligation for a GENCO and the signal that would enforce the hourly on/off status of generation assets would be market prices for trading energy, fuel, ancillary service, emission allowance, and other energy related commodities.

Equilibrium models such as Supply Function Equilibrium and Cournot Equilibrium were widely applied to develop GENCOs' bidding strategies and analyze market power in energy markets. In [5], supply function equilibrium model was used for developing GENCOs' bidding strategies based on sensitivity functions in energy markets. The competition problem in electricity markets was modeled by the Cournot equilibrium model in [6-8]. However, unit constraints such as minimum on/off time, ramping limits, and startup cost were not considered in most of the equilibrium models because the existence of equilibria could not be proven when integer variables were used in those models. Given forecasted market prices, prevailing unit constraints were considered in PBUC [1]. The PBUC problem was solved using Lagrangian relaxation (LR) and mixed integer programming (MIP) methods in [1] and [9], respectively. The uncertainty in forecasted energy price for PBUC was considered using LR, stochastic dynamic programming, and Benders decomposition in [10]. With uncertain prices of electricity and fuel, PBUC was modeled as a multistage stochastic problem and a solution based on the Monte Carlo simulation and dynamic programming was proposed in [11]. The arbitrage opportunities between energy and ancillary services, fuel, and emission allowance were explored individually using LR in [1]. However, arbitrage opportunities among these products were not considered simultaneously in these markets.

This paper considers arbitrage opportunities in purchases from qualifying facilities (QFs) as well as simultaneous trades with spots markets for energy, ancillary services, emission, and fuel. Based on forecasted market prices, the arbitrage problem of a GENCO with thermal and combined-cycle units is formulated as an MIP problem and solved by a branch-and-cut algorithm. Energy and ancillary service markets are cleared hourly within a day while fuel and emission allowance markets are cleared daily in this paper. It is presumed that forecasting techniques such as time-series and artificial neural network [1] are applied to forecast market prices for various commodities which is outside the scope of this paper.

The advantages of applying a MIP method include global optimality and more flexible and accurate modeling capabilities. The MIP solution could be globally optimal and the MIP approach could model constraints and objective function accurately and efficiently. The disadvantages of MIP approach are mainly represented by its computational complexity. The most successful methods for solving a generalized MIP problem include branch-and-bound and cutting plane algorithms. The branch-and-cut algorithm which is based on conjunction of cutting plane and branch-and-bound methods is much more efficient than using one of the two methods. The current leading MIP solution which utilizes branch-and-cut algorithm is still NP-hard. As shown in [12], however, the computation time of UC by MIP has been improved because of advances in linear programming (LP) and the incorporation of cutting plane techniques to the branch-and-bound algorithm.

This paper is organized as follows: the formulation of arbitrage problem is given in section 3. Section IV gives illustrative examples on a GENCO with 54 thermal and 12 combined-cycle units. The conclusions are provided in section 5.

3. Problem Formulation

The calculation of a GENCO's bidding strategy is discussed in [5] and [13]. Among ancillary services, spinning and non-spinning reserves are primarily considered in this paper for engaging in arbitrage opportunities. However, other types of ancillary services such as regulation up/down and operating reserve could also be modeled here [14]. A good MIP formulation is essential for reducing the computation cost of PBUC in practical cases. We utilize the following schemes for improving the MIP solution [15]-[17]:

(1) Make the corresponding LP relaxation of the MIP problem as tight as possible.
(2) Use integer variables to facilitate the branching process of the branch and bound method. This includes creating a cutting plane by a constraint. Extra binary variables could also be introduced to create meaningful dichotomies.

3.1 Objective function for a GENCO

In restructured power systems, a GENCO intends to maximize its payoff which is given in (1). Although we have considered GENCOs with thermal and combined-cycle units [18]-[20] in this paper, other types of generating units such as cascaded-hydro, and pumped-storage units could easily be modeled for arbitrage [18]. The detailed MIP formulation of unit constraints, such as minimum on/off time, ramping up/ down limits, time-varying startup costs, fuel and emission allowance limits, is given in [18].

$$\begin{aligned}\max : &\sum_t \left\{\rho_g(t)TP(t) + \rho_{sr}(t)TR(t) + \rho_{nr}(t)TN(t)\right\} \\ &-\sum_{FT} F(FT)FP_0(FT) + \sum_t [SG_0(t) - BG_0(t)] \\ &+\sum_{FT}[-BF(FT) + SF(FT)] + \sum_{ET}[-BE(ET) + SE(ET)]\end{aligned} \quad (1)$$

We discuss a GENCO's payoff for arbitraging among multiple markets as follows.

3.2 Arbitrage between spot market and bilateral contracts of energy

The GENCO could purchase energy from the market to supply its bilateral contracts, which is also viewed as an arbitrage opportunity. A GENCO could sign purchase contracts at its avoided cost with QFs which are small production facilities with capacity not exceeding 80MW. In addition, a GENCO could sign bilateral contracts with loads as given below:

$$\sum_{i\in I} P(i,t) + \sum_{j\in J} P(j,t) + \sum_{k\in K} QF_0(k,t) - TP(t) = \sum_{l\in L} P_0(l,t) \quad \forall t \quad (2)$$

where positive $TP(t)$ is the generation offered to the market and negative $TP(t)$ is the purchased generation from the market. The payment and income for signing contracts with QF contracts and loads are given as:

$$\begin{aligned} SG_0(t) &= \sum_{l\in L} CP(l,t)P_0(l,t) && \forall t \\ BG_0(t) &= \sum_{k\in K} CP(k,t)QF_0(k,t) && \forall t \end{aligned} \quad (3)$$

3.3 Arbitrage between energy and ancillary services markets

Spinning reserve is the unloaded synchronized generation that can ramp up in 10 minutes. Non-spinning reserve is the unsynchronized generating capacity that can ramp up in 10 minutes. Reserves are compensated according to capacity and real-time energy prices. A unit is paid the capacity price for providing reserve capacity and the spot market energy price when called to generate energy in real-time. A probabilistic model of ancillary services was proposed in [21]. For simplicity, we assume spinning and non-spinning reserves are called upon in real time by the ISO. Accordingly, the fuel cost for supplying reserves is included in the objective function.

The market price for a higher quality reserve (spinning reserve) could be higher than that of a lower quality reserve (non-spinning reserve). However, a general case of generating unit constraints for supplying energy and ancillary services in spot market is modeled in (4):

$$\begin{aligned} &P(i,t) + R(i,t) + N_u(i,t) \le \overline{P}_g(i)I(i,t) && \forall i, \forall t \\ &\underline{P}_g(i)I(i,t) \le P(i,t) && \forall i, \forall t \\ &R(i,t) + N_u(i,t) \le 10 * MSR(i)I(i,t) && \forall i, \forall t \\ &N_d(i,t) \le QSC(i)I_d(i,t) && \forall i, \forall t \\ &I_d(i,t) + I(i,t) \le 1 && \forall i, \forall t \end{aligned} \quad (4)$$

We could model the bilateral contracts for supplyingancillary services similarly. However, they are disregarded in this presentation for simplicity. So, the total ancillary services offered to the market are given as:

$$TR(t) = \sum_{i\in I} R(i,t) + \sum_{j\in J} R(j,t) \quad \forall t$$
$$TN(t) = \sum_{i\in I}[N_u(i,t)+N_d(i,t)] + \sum_{j\in J}[N_u(j,t)+N_d(j,t)] \quad \forall t \tag{5}$$

An example of a thermal unit arbitrage is provided here. The market prices and unit data are given in Tables 1 and 2, respectively. The compensation for spinning and non-spinning reserves is based on the energy price and the capacity price. We assume the purchase price of fuel for generating electricity is the same as the spot market price of fuel. The other constraints are relaxed.

Table 1 Market Prices

Energy ($/MWh)	Capacity price for spinning reserve ($/MW)	Capacity price for non-spinning reserve ($/MW)	Fuel ($/MBtu)
22	3	2	1

Table 2 Unit Data

Pmin (MW)	Pmax (MW)	MSR (MW/min)	QSC (MW)	Heat curve (MBtu/h)
5	30	1	30	20*P

The generating unit could choose to sell 30MW of energy to the market with a payoff of $60 (i.e., 30*22 - 30*20*1). The optimal strategy, however, is to shut down the generating unit and provide 30MW of non-spinning reserve with a payoff of $120 (i.e., 30*(22+2) - 30*20*1) if the unit opts to participate in energy and ancillary services market simultaneously. The assumption here is that the entire 30MW will be called upon to provide the non-spinning reserve. Otherwise, the payoff will be lower. The example illustrates that a unit could obtain higher payoff by arbitraging energy and ancillary services markets.

3.4 Arbitrage between electricity and fuel

GENCO could consider an arbitrage between generating electricity and trading fuel for raising its payoffs. The alternatives include purchasing fuel from the market to generate more electricity or selling fuel to the market by turning generating units off. For example, if the thermal unit in section 3.3 has an upper limit of500MBtu on its available fuel, it could sell 25MW of energy to the market with a payoff of $50 (i.e., 25*22 - 25*20*1). Alternatively, it could purchase an additional 100MBtu of fuel to generate additional electricity and offer 30MW of energy to the market with a payoff of $60 (i.e., 30*22 - 500*1 - 100*1).

As shown in Fig. 1, the price curve for purchasing the *FT* type of fuel is assumed to be nonconvex and the cost of purchasing the *FT* type fuel is:

$$BF(FT) = \sum_{m=1}^{NSB(FT)} [\rho_m(b,FT) * x_m(b,FT)] \tag{6}$$

Additional integer variables are required to model the nonconvexity of the fuel price curve. So for purchasing fuel:

$$u_m(b,FT)*[NF_m(b,FT)-NF_{m-1}(b,FT)] \le x_m(b,FT) \quad \forall m$$
$$x_m(b,FT) \le u_{m-1}(b,FT)*[NF_m(b,FT)-NF_{m-1}(b,FT)] \quad \forall m \tag{7}$$

It should be noted that $u_0(b,FT)$ is 1 and $NF_0(b,FT)$ is 0 for the first segment of Fig. 1. The selling price curve for the *FT* type of fuel is also assumed to be nonconvex similar to that in Fig. 1. The payoff for selling the *FT* type fuel is:

$$SF(FT) = \sum_{m=1}^{NSS(FT)} [\rho_m(s,FT) * x_m(s,FT)] \tag{8}$$

For selling fuel:

$$u_m(s,FT)*[NF_m(s,FT)-NF_{m-1}(s,FT)] \le x_m(s,FT) \quad \forall m$$
$$x_m(s,FT) \le u_{m-1}(s,FT)*[NF_m(s,FT)-NF_{m-1}(s,FT)] \quad \forall m \tag{9}$$

Again, $u_0(s,FT)$ is 1 and $NF_0(s,FT)$ is 0 for the first segment in the sale price curve for the type *FT* fuel.

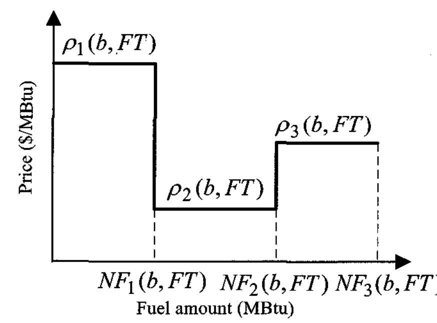

Fig. 1 Nonconvex price curve for purchasing *FT* type fuel

3.5 Arbitrage between electricity and emission allowance

A GENCO could consider the arbitrage between electricity and emission allowance. The price curve for trading emission allowance is also nonconvex and similar to that in Fig. 1. The cost/payoff for trading emission allowance and the corresponding constraints are modeled similar to those in section 3.4.

3.6 Coupling constraints

The upper limit on the *FT* type of fuel consumption is:

$$\sum_{i\in FT}\sum_{t=1}^{T}\left\{\begin{array}{l}[(\underline{f}(i)I(i,t)+\underline{f}(i)I_d(i,t)+\\ \sum\limits_{m=1}^{NSF(i)} p_m(i,t)b_m(i))+\sum\limits_{m=1}^{NS(i)} v_m(i,t)SF_m(i)]\end{array}\right\}+$$
$$\sum_{j\in FT}\sum_{t=1}^{T}\left\{[(\underline{f}(j)I(j,t)+\sum_{m=1}^{NSF(j)} p_m(j,t)b_m(j))+TF(j,t)\right\}$$
$$=F(FT)+\sum_{m}x_m(b,FT) \tag{10}$$
$$F(FT)+\sum_{m}x_m(s,FT)\le \overline{F}(FT)$$

The upper limit on emission allowance is modeled similarly. The upper limits on energy and ancillary services supplied by a GENCO are:

$$\begin{aligned} TP(t) &\le \overline{P}(t)\\ TR(t) &\le \overline{R}(t)\\ TN(t) &\le \overline{N}(t) \end{aligned} \tag{11}$$

3.7 Solution procedure

Based on the formulation presented in sections 3.1~3.6, the MIP-based arbitrage for a GENCO is solved by a commercial branch-and-cut algorithm solver such as CPLEX.

4. Numerical Examples

A GENCO with 54 thermal and 12 combined-cycle units is considered for studying arbitrage opportunities. The detailed data on units, QFs, and bilateral loads are given in http://motor.ece.iit.edu/Data/GenAssetData.pdf. Daily market prices for trading gas and NOx allowance are given in Tables 3 and 4, respectively. We assume the purchase/ sale prices are the same for fuel and for emission allowance. The price for the GENCO's gas in storage is $1/MBtu.The hourly market price for energy and the capacity price for ancillary services are given in Fig. 2. The compensation for spinning and non-spinning reserves is based on the hourly energy price and the capacity price.

Table 3 Market Price Curve for trading Gas

Segment #	From (MBtu)	To (MBtu)	Price ($/MBtu)
1	0	4000	1.20
2	4000	50000	1.10
3	50000	500000	1.15

Table 4 Market Price Curve for Trading NOx Allowance

Segment #	From (lb)	To (lb)	Price ($/lb)
1	0	30000	0.75
2	30000	70000	0.65
3	70000	500000	0.70

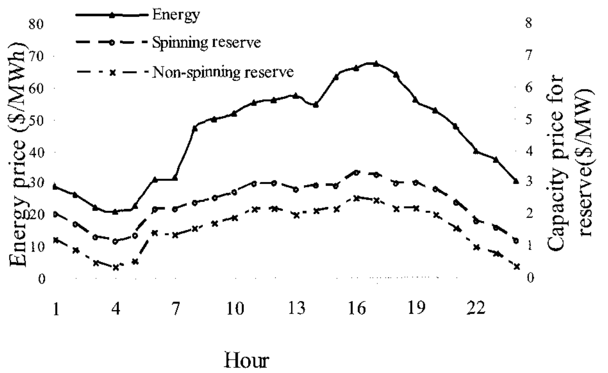

Fig. 2 Market prices for energy and ancillary services for 24 hours

4.1 Arbitrage in multi-commodity markets

The following five cases are analyzed in this study:

- Case 0: GENCO will participate in energy market;
- Case 1: GENCO will consider the arbitrage between energy market and bilateral contracts (QFs);
- Case 2: GENCO will consider the arbitrage among energy, bilateral contract, and ancillary service markets;
- Case 3: GENCO will consider the arbitrage among energy, bilateral contract, ancillary service, and fuel markets;
- Case 4: GENCO will consider the arbitrage among energy, bilateral contract, ancillary service, fuel, and NOx allowance trading markets.

The GENCO assumes energy and ancillary services markets are cleared hourly while fuel and emission allowance markets are cleared daily in this study. The case studies are executed on a Pentium-4 1.8GHz personal computer. We have used the CPLEX 9.0 software with the convergence tolerance of 0.001%. The GENCO's storage of coal, oil, and gas are 1.8e+6MBtu, 5e+5MBtu, 3.3e+

5MBtu, respectively and its NO_x allowance is 1.2e+5lb. The GENCO's upper limits on SO_2 allowance and its limits on generation, spinning reserve, and non-spinning reserve quantities offered to the market are relaxed.

Table 5 illustrates arbitrage payoffs and computation times. Table 6 shows the purchased quantities of gas and NOx allowance for the arbitrage in Cases 3 and 4. Table 7 demonstrates the GENCO's consumption of fuel and NOx allowance. Fig. 3 shows the offered generation to the market in which Cases 0 and 1 follow market prices closely. The reason is that additional units are dispatched when hourly market prices are higher.

Table 5 Arbitrage Payoff and Computation TiME

Case #	Payoff ($)	Computation time (s)
0	4.94770e+6	715
1	4.96178e+6	669
2	5.02206e+6	359
3	5.07265e+6	30
4	5.23957e+6	48

Table 6 Purchases of Market Gas and NOx Allowance

Case #	Gas (MBtu)	NOx (lb)
3	80770.5	-
4	146248	80887.3

The comparison of Cases 0 and 1 in Table 5 shows that a GENCO could increase its payoff by arbitraging between energy market and QF contracts. The arbitrage opportunity in Case 1 is because the hourly market price of energy is higher than contract price of QFs. Additional QF case studies are presented later in this section. The reason for getting the same PBUC schedule in Cases 0 and 1 is that QF contracts would only increase the total generation offered to the market as verified in Fig. 3. In this figure, the difference between the total generation offered in Cases 0 and 1 (i.e.,top of the figure) is QF contracts.

Table 7 Consumption of Fuel and Emission According to Arbitrage

Case#	Coal (MBtu)	Oil (MBtu)	Gas (MBtu)	NOx (lb)
0	1.8e+6	107363	330000	120000
1	1.8e+6	107363	330000	120000
2	1.8e+6	108010	330000	120000
3	1.8e+6	108010	410770.5	120000
4	1.8e+6	136338	476248	200887.3

The comparison of Cases 2 and 3 in Table 5 shows that a GENCO could profit additionally by arbitraging among energy, bilateral contracts, ancillary services, and fuel markets simultaneously. Table 5 demonstrates that a GENCO could increase its payoff by 5.90% (i.e., (5.23957e+6 – 4.94770e+6) / 4.94770e+6) by arbitraging among five markets as compared to participating in the energy market alone. The comparison of Cases 3 and 4 in Table 5 shows that a GENCO could increase its payoff by arbitraging simultaneously among energy, bilateral contract, ancillary services, fuel, and emission allowance markets.

The reasonable computation times in Table 5 verify the efficiency of our arbitrage formulation. Much of the computation time in Cases 0 to 2 of Table 5 is devoted to identifying a better solution after finding a feasible solution with a gap less than 0.02%. The analyses of Cases 0 to 4 illustrate the significance of arbitrage opportunities when considered simultaneously in all markets.

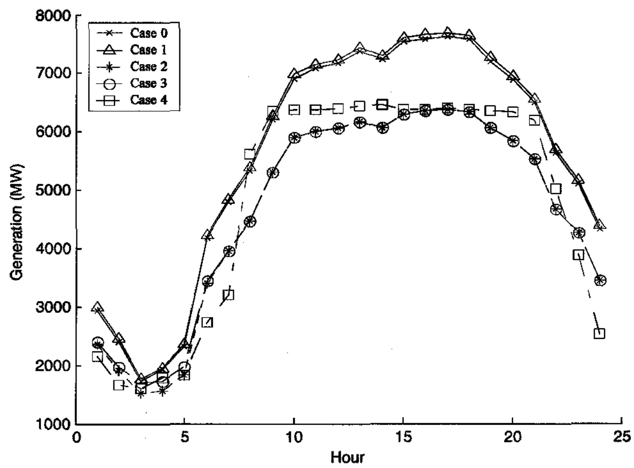

Fig. 3 Generation offered to market

Table 6 shows the arbitrage opportunity in Case 3 for purchasing additional gas which enables the GENCO to generate more and consider further arbitrage between energy and ancillary services markets. Accordingly, more ancillary services are provided in comparison to Case 2 as shown in Figs. 4 and 5. The emission arbitrage in Case 4 of Table 6 would enhance the arbitrage among gas, energy, and ancillary services markets. Accordingly, more ancillary services are provided in comparison to Case 3 as shown in Figs. 4 and 5.

The arbitrage opportunity in Case 2 stems from that fact that the price of ancillary services is higher than that of energy when ancillary services are compensated at both energy and capacity prices. Accordingly, ancillary services in Case 2 are shown in Figs. 4 and 5. The Case 2's curve in Fig. 5 shows that additional non-spinning reserve is offered when market price is high. The non-spinning reserve is not offered at hour 14 because the energy price at that hour has dropped noticeably as shown in Fig. 2. In addition, gas-fired units 1001-1003, 1006, 1012- 1013 have small capacity and fast-response capability so they are dispatched to provide non-spinning reserve when market price is high and shut down quickly when market price is low.

The Case 2's spinning reserve in Fig. 4 demonstrates that the GENCO follows fluctuations in the hourly market price of energy and spinning reserve closely. Compared to Case 1, GENCO has reduced its generation offered in Fig. 3 in order to satisfy its limited fuel (coal and gas) and NOx allowance constraint listed in Table 7. Coal- and gas- fired units are dispatched to provide ancillary services in arbitrage.

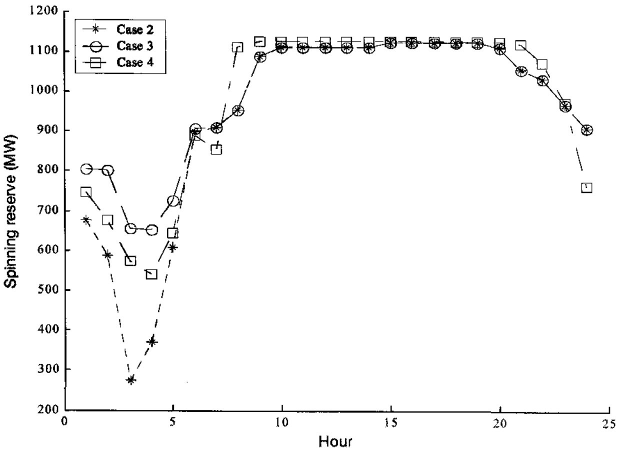

Fig. 4 Spinning reserve offered to market

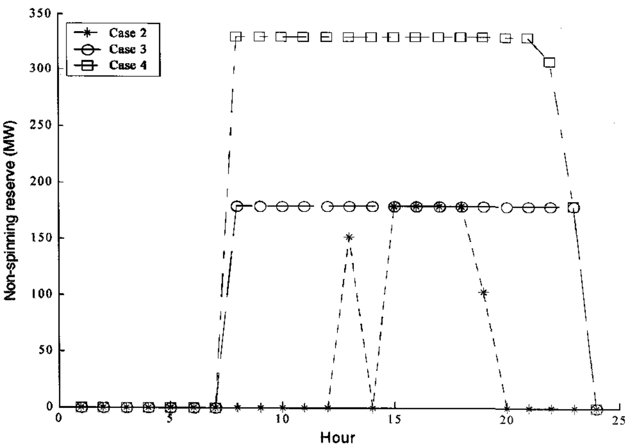

Fig. 5 Non-spinning reserve offered to market

Table 8 shows the PBUC for Cases 0 and 1. In this table, 0 means off and 1 means on. For combined-cycle units, the values in Table 8 denote the corresponding operating modes of the generating unit. Units that are not shown in Table 8 are off during the scheduling period. Table 8 shows that cheaper thermal units are committed during the entire scheduling period while more expensive thermal units are committed merely when market prices are high.

In Table 8, large and cheap coal-fired units 1004-1005, 1020, 1025, 1027-1028, 1039-1040, 1043-1045 are committed for the entire scheduling period while others are committed only at high-price periods. The limits on coal consumption and NOx allowance also contribute to the commitment of certain coal-fired units at high-price periods alone. Gas- and oil-fired thermal units are expensive so that certain units are dispatched only at high-price periods while others are not committed at all.

In Table 8, although gas-fired thermal units have identical heat curves, units 1008-1009, 1015, 1017-1018 are not dispatched even at high-price periods because of limitations on gas consumption and NOx allowance shown in Table 7. Combined-cycle units are operating at mode 4 for most periods. The reason is that market price is higher than the marginal cost of combined-cycle units at most periods and mode 4 is the most efficient configuration for each combined-cycle unit.

Table 8 Generation Units Schedule in Case s 0 and 1

Unit	Hours (0-24)																								
1001-1003	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0
1004-1005	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1006	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0
1007	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1010	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
1011	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
1012-1013	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
1014	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1016	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
1019	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1020	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1021	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
1022-1023	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
1024	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
1025	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1026	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1027-1028	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1029	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
1030	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
1031	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
1032	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
1034-1035	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
1036	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
1037	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1038	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1039-1040	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1041	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
1042	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
1043-1045	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1046	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
1047-1048	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1049	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
1050	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
1051-1053	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1054	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
4001	0	0	0	0	1	1	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4002	0	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4003	0	1	1	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4004-4005	0	1	1	1	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4006	0	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4007-4009	0	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4010-4011	0	1	1	1	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4012	0	2	2	2	2	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

Table 9 shows a portion of schedule in Case 2 that is different from that of Case 1. Table 9 also shows that gas-fired units 1001-1003, 1006, 1012-1013 are shut down starting at hour 12 to either provide non-spinning reserve or stay off-line because of arbitrage between energy and ancillary services markets.

Table 9 shows that the generating unit 1016 is shut down at hours 13 and 14 to save the available coal for supplying coal-fired units at higher price periods since the availability of coal and emission allowance is limited. Compared to Case 1, combined-cycle unit 4001 is started earlier to provide spinning reserve and energy while units 4010-4011 are scheduled to operate at a higher-capacity mode to offer more spinning reserve and energy.

A portion of the schedule in Case 3 that is different from that of Case 2 is shown in Table 10. Table 10 shows that combined-cycle units 4001, 4004-4005, 4010-4011 are committed at higher capacity modes earlier to offer additional spinning reserve and energy as verified in Figs. 3 and 4. Meanwhile, Fig. 5 shows that gas-fired thermal units 1001-1003, 1006, 1012-1013 are scheduled to provide non-spinning reserve starting at an earlier hour 8. Accordingly to the gas arbitrage, gas-fired thermal and combined-cycle units would offer additional energy and ancillary services to the market as shown in Figs. 3~5. The total consumption of gas is shown in Table 7. The schedule of coal- and oil- fired units remains the same as that in Case 2 because of the limited availability of coal and NOx allowance. In addition, combined-cycle units are already operating at their most efficient mode starting at hour 6. Figs. 3 and 4 verify that, beyond hour 6, generation and spinning reserve quantities remain the same as those in Case 2.

Table 9 Generation Units Schedule in Case 2 in Comparison to Case 1

Unit	Hours (0-24)
1001-1003	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1006	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1012-1013	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1016	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0
4001	0 0 0 1 1 3 3 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4010-4011	0 1 1 3 3 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4

Table 10 Generation Units Scheduled in Case 3 in Comparison to Case 2

Unit	Hours (0-24)
4001	0 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4004-4005	0 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
4010-4011	0 2 2 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4

Table 11 Units Schedule in Case 4 That is different from Case 3

Unit	Hours (0-24)
1007	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
1010	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 0
1011	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1014	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
1016	1 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1019	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
1021	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1022-1023	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
1024	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1026	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
1029	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0
1030	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0
1031	1 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1032	1 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1033	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
1034-1035	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1036	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0
1037	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
1047-1048	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
1051-1053	1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0

Table 11 shows a portion of schedule in Case 4 that is different from that of Case 3. The table shows that oil-fired units 1031-1033 are committed for additional hours to supply energy and spinning reserve. Accordingly, Table 6 shows that the purchase of NOx allowance enables generating units to increase their generation which is also verified by the additional oil consumption shown in Table 7.

According to Table 11, certain coal-fired units prolong their committed hours while others are shut down to satisfy the limited availability of coal shown in Table 7. Used in this way, cheaper and dirtier coal-fired units would supply the market with additional energy and ancillary services when the market price is high at the expense of purchasing additional emission allowance from the market. This strategy explains the decrease in energy and spinning reserve production at low-price hours and additional production at high-price hours in Figs. 3 and 4, as compared to Case 3. This strategy results from the arbitrage of NOx allowance by purchasing additional NOx allowance from the market. Table 11 also shows that the combined-cycle units schedule remains the same since the schedule in Case 3 is already optimal. The additional non-spinning reserve in Fig. 4 is because of gas arbitrage when additional gas is purchased in comparison to Case 3. The gas-fired thermal units 1008-1009, 1015, 1017-1018 are dispatched additionally for supplying non-spinning reserve as verified by Fig. 5.

4.2 Impact of price and quantity on QF contracts

In this section, we study the impact of price and quantity on QF contracts. The upper limit on total generation offered by the GENCO is enforced and the other conditions are the same as those in Case 1.

When the quantity of QF contract is fixed, the payoff vs. the contract price is shown in Fig. 6 where the price coefficient is relative to the contract price in Case 1. When the price of QF contract is fixed, the payoff vs. the quantity of QF contracts is shown in Fig. 7 where the MW coefficient is relative to QF MW in Case 1. The dotted line in Figs. 6 and 7 depicts the payoff without QF contracts.

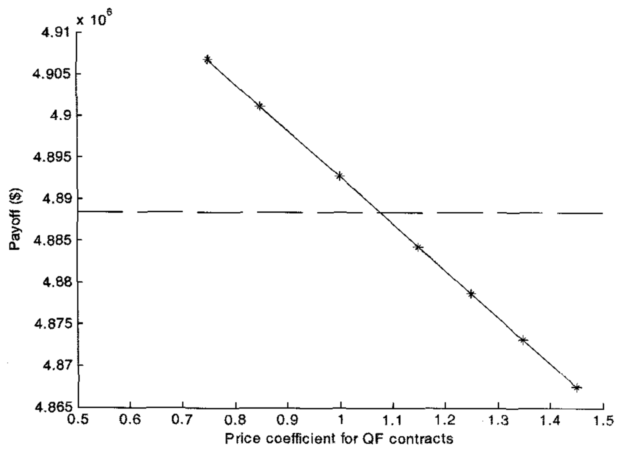

Fig. 6 Payoff vs. price for QF contracts

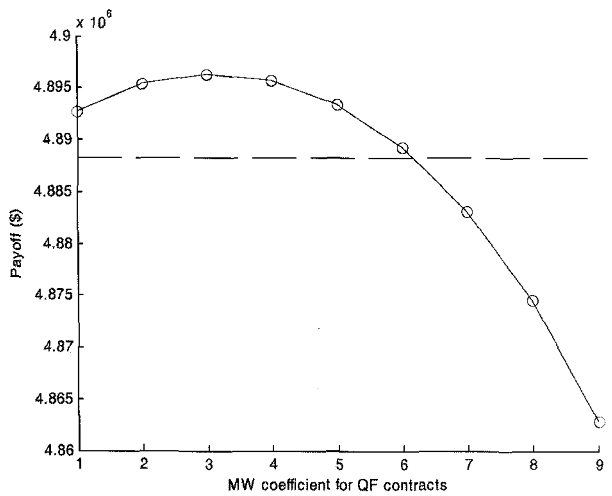

Fig. 7 Payoff vs. MW Quantity for QF contracts

Fig. 6 could be used to figure out the optimal price for QF contracts. The payoff curve is linear because the price of QF contracts would not impact the operation of GENCO's own units and the worst option is to merely sell QF quantities to the market.

Fig. 7 shows the QF quantity which should be contracted by GENCO for a given contract price. The payoff would increase first and then drop by increasing the QF quantity. The reason is that the QF contract would impact the operation of GENCO's own units when the total generation offered to the market is fixed. At first, additional QF quantities would reduce the generation of expensive units in the GENCO. Then, the additional QF quantity would reduce the generation of GENCO's cheap units since QF contracts are utilized first.

5. Conclusions

A competitive GENCO tends to maximize its payoff regardless of other participants' possible payoffs. Given purchases from QFs and market prices for energy, ancillary services, fuel, and emission allowance, the arbitrage opportunities among the five markets are discovered by using PBUC. The arbitrage problem for a GENCO with thermal and combined-cycle units is modeled in this paper as an MIP problem. The MIP method exhibits a globally optimal solution with enhanced modeling capabilities. The GENCO results for 54 thermal and 12 combined-cycle generating units show the efficiency of our proposed formulation and the significance of arbitraging the purchases from QFs as well as simultaneous spot market trades for energy, ancillary services, emission, and fuel.

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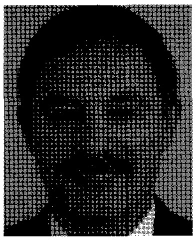

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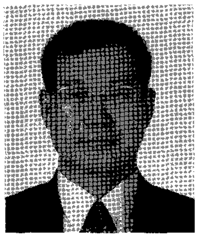

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