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ON FUZZY FUNCTIONS

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ABSTRACT. In 1998, Thakur and Singh introduce the concept of fuzzy β-continuity (Fuzzy Sets and Systems, 98(1998), 383–391). In this paper we introduce and study the notion of fuzzy slightly β-continuity. Fuzzy slightly β-continuity generalize fuzzy β-continuity. Moreover, basic properties and preservation therems of fuzzy slightly β-continuous functions are obtained.

1. Introduction and preliminaries

Fuzzy continuity is one of the main topics in fuzzy topology. Various authors introduce various types of fuzzy continuity. One of them is fuzzy β-continuity. In 1998, Thakur and Singh introduce the concept of fuzzy β-continuity.

In this paper, we introduce the notion of fuzzy slightly β-continuity generalizing fuzzy β-continuity. Basic properties and preservation theorems of fuzzy slightly β-continuous are obtained and we study and investigate relationships between fuzzy slightly β-continuity and separation axioms. Moreover, we investigate the relationships between fuzzy slightly β-continuity and fuzzy graphs and the relationships among fuzzy slightly β-continuity and compactness and connectedness.

In the present paper, X and Y are always fuzzy topological spaces. The class of fuzzy sets on a universe X will be denoted by I^X and fuzzy sets on X will be denoted by Greek letters as μ, ρ, η, etc. A family τ of fuzzy sets in X is called a fuzzy topology for X iff

(1) $\emptyset, X \in \tau$,
(2) $\mu \wedge \rho \in \tau$, whenever $\mu, \rho \in \tau$ and
(3) $\bigcup\{\mu_\alpha : \alpha \in I\} \in \tau$, whenever each $\mu_\alpha \in \tau$ $(\alpha \in I)$.

Moreover, the pair (X, τ) is called a fuzzy topological space. Every member of τ is called a open fuzzy set [6].

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Let μ be a fuzzy set in X. We denote the interior and the closure of a fuzzy set μ by $\text{int}(\mu)$ and $\text{cl}(\mu)$, respectively. A fuzzy set in X is called a fuzzy point iff it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at x is α $(0 < \alpha \leq 1)$ we denote this fuzzy point by x_α, where the point x is called its support [6]. For any fuzzy point x_ε and any fuzzy set μ, we write $x_\varepsilon \in \mu$ iff $\varepsilon \leq \mu(x)$.

A fuzzy set μ in a space X is called fuzzy β-open if $\mu \leq \text{cl}(\text{int}(\text{cl}(\mu)))$ [8, 5]. The complement of a fuzzy β-open set is said to be β-closed. Let $f : X \to Y$ a fuzzy function from a fuzzy topological space X to a fuzzy topological space Y. Then the function $g : X \to X \times Y$ defined by $g(x_\varepsilon) = (x_\varepsilon, f(x_\varepsilon))$ is called the fuzzy graph function of f [1].

2. Fuzzy slightly β-continuous functions

In this section, basic properties of slightly β-continuous functions and connectedness and covering properties of slightly β-continuous functions are investigated.

DEFINITION 1. A fuzzy function $f : X \to Y$ is said to be fuzzy β-continuous if $f^{-1}(\rho)$ is fuzzy β-open set in X for every fuzzy open set ρ in Y [8].

DEFINITION 2. A fuzzy function $f : X \to Y$ is said to be fuzzy open if $f(\rho)$ is a fuzzy open set in Y for every fuzzy open set ρ in X [3].

DEFINITION 3. Let (X, τ) and (Y, υ) be fuzzy topological spaces. A fuzzy function $f : X \to Y$ is said to be fuzzy slightly β-continuous if for each fuzzy point $x_\varepsilon \in X$ and each fuzzy clopen set ρ in Y containing $f(x_\varepsilon)$, there exists a fuzzy β-open set μ in X containing x_ε such that $f(\mu) \leq \rho$.

THEOREM 1. *For a function $f : X \to Y$, the following statements are equivalent:*

(1) *f is fuzzy slightly β-continuous;*
(2) *for every fuzzy clopen set ρ in Y, $f^{-1}(\rho)$ is fuzzy β-open;*
(3) *for every fuzzy clopen set ρ in Y, $f^{-1}(\rho)$ is fuzzy β-closed;*
(4) *for every fuzzy clopen set ρ in Y, $f^{-1}(\rho)$ is fuzzy β-clopen.*

PROOF. (1) $\Rightarrow$ (2) : Let ρ be a fuzzy clopen set in Y and let $x_\varepsilon \in f^{-1}(\rho)$. Since $f(x_\varepsilon) \in \rho$, by (1), there exists a fuzzy β-open set μ_{x_ε} in X containing x_ε such that $\mu_{x_\varepsilon} \leq f^{-1}(\rho)$. We obtain that $f^{-1}(\rho) = \bigvee_{x_\varepsilon \in f^{-1}(\beta)} \mu_{x_\varepsilon}$. Thus, $f^{-1}(\rho)$ is fuzzy β-open.

(2) $\Rightarrow$ (3) : Let ρ be a fuzzy clopen set in Y. Then, $Y\backslash\rho$ is fuzzy clopen. By (2), $f^{-1}(Y\backslash\rho) = X\backslash f^{-1}(\rho)$ is fuzzy β-open. Thus, $f^{-1}(\rho)$ is fuzzy β-closed.

(3) $\Rightarrow$ (4) : It can be shown easily.

(4) $\Rightarrow$ (1) : Let ρ be a fuzzy clopen set in Y containing $f(x_\varepsilon)$. By (4), $f^{-1}(\rho)$ is β-clopen. Take $\mu = f^{-1}(\rho)$. Then, $f(\mu) \leq \rho$. Hence, f is fuzzy slightly β-continuous. □

REMARK 1. Obviously fuzzy β-continuity implies fuzzy slightly β-continuity. The following example show that this implication is not reversible.

EXAMPLE 1. Let $X = \{a, b\}$, $Y = \{x, y\}$ and λ, μ are fuzzy sets defined as follows:

$$\lambda(a) = 0,3 \quad \lambda(b) = 0,6$$
$$\mu(x) = 0,7 \quad \mu(y) = 0,4$$

Let $\tau_1 = \{X, \emptyset, \lambda\}$, $\tau_2 = \{Y, \emptyset, \mu\}$. Then the fuzzy function $f : (X, \tau_1) \to (Y, \tau_2)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy slightly β-continuous but not fuzzy β-continuous.

THEOREM 2. *Suppose that Y has a base consisting of fuzzy clopen sets. If $f : X \to Y$ is fuzzy slightly β-continuous, then f is fuzzy β-continuous.*

PROOF. Let $x_\varepsilon \in X$ and let ρ be a fuzzy open set in Y containing $f(x_\varepsilon)$. Since Y has a base consisting of fuzzy clopen sets, there exists a fuzzy clopen set β containing $f(x_\varepsilon)$ such that $\beta \leq \rho$. Since f is fuzzy slightly β-continuous, then there exists a fuzzy β-open set μ in X containing x_ε such that $f(\mu) \leq \beta \leq \rho$. Thus, f is fuzzy β-continuous. □

THEOREM 3. *Let $f : X \to Y$ be a fuzzy function and let $g : X \to X \times Y$ be the fuzzy graph function of f, defined by $g(x_\varepsilon) = (x_\varepsilon, f(x_\varepsilon))$ for every $x_\varepsilon \in X$. If g is fuzzy slightly β-continuous, then f is fuzzy slightly β-continuous.*

PROOF. Let β be fuzzy clopen set in Y, then $X \times \beta$ is fuzzy clopen set in $X \times Y$. Since g is fuzzy slightly β-continuous, then $f^{-1}(\beta) = g^{-1}(X \times \beta)$ is fuzzy β-open in X. Thus, f is fuzzy slightly β-continuous. □

DEFINITION 4. A fuzzy filter base Λ is said to be fuzzy β-convergent to a fuzzy point x_ε in X if for any fuzzy β-open set ρ in X containing x_ε, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \leq \rho$.

DEFINITION 5. A fuzzy filter base Λ is said to be fuzzy co-convergent to a fuzzy point x_ε in X if for any fuzzy clopen set β in X containing x_ε, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \leq \beta$.

THEOREM 4. *If a fuzzy function $f : X \to Y$ is fuzzy slightly β-continuous, then for each fuzzy point $x_\varepsilon \in X$ and each fuzzy filter base Λ in X β-converging to x_ε, the fuzzy filter base $f(\Lambda)$ is fuzzy co-convergent to $f(x_\varepsilon)$.*

PROOF. Let $x_\varepsilon \in X$ and Λ be any fuzzy filter base in X β-converging to x_ε. Since f is fuzzy slightly β-continuous, then for any fuzzy clopen set λ in Y containing $f(x_\varepsilon)$, there exists a fuzzy β-open set μ in X containing x_ε such that $f(\mu) \leq \lambda$. Since Λ is fuzzy β-converging to x_ε, there exists a $\rho \in \Lambda$ such that $\rho \leq \mu$. This means that $f(\rho) \leq \lambda$ and therefore the fuzzy filter base $f(\Lambda)$ is fuzzy co-convergent to $f(x_\varepsilon)$. □

DEFINITION 6. A fuzzy space X is said to be fuzzy β-connected if it cannot be expressed as the union of two nonempty, disjoint fuzzy β-open sets.

DEFINITION 7. A fuzzy space X is said to be fuzzy connected if it cannot be expressed as the union of two nonempty, disjoint fuzzy open sets [7].

THEOREM 5. *If $f : X \to Y$ is fuzzy slightly β-continuous surjective function and X is fuzzy β-connected space, then Y is fuzzy connected space.*

PROOF. Suppose that Y is not fuzzy connected space. Then there exists nonempty disjoint fuzzy open sets β and μ such that $Y = \beta \vee \mu$. Therefore, β and μ are fuzzy clopen sets in Y. Since f is fuzzy slightly β-continuous, then $f^{-1}(\beta)$ and $f^{-1}(\mu)$ are fuzzy β-closed and β-open in X. Moreover, $f^{-1}(\beta)$ and $f^{-1}(\mu)$ are nonempty disjoint and $X = f^{-1}(\beta) \vee f^{-1}(\mu)$. This shows that X is not fuzzy β-connected. This is a contradiction. By contradiction, Y is fuzzy connected. □

DEFINITION 8. A fuzzy space X is called hyperconnected if every fuzzy open set is dense [2].

REMARK 2. The following example shows that fuzzy slightly β-continuous surjection do not necessarily preserve fuzzy hyperconnectedness.

EXAMPLE 2. Let $X = \{x, y, z\}$ and μ, β, ρ be fuzzy sets of X defined as follows:

$$\mu(x) = 0,2 \quad \mu(y) = 0,2 \quad \mu(z) = 0,5$$
$$\beta(x) = 0,8 \quad \beta(y) = 0,8 \quad \beta(z) = 0,4$$
$$\rho(x) = 0,8 \quad \rho(y) = 0,7 \quad \rho(z) = 0,6$$

We put $\tau_1 = \{X, \emptyset, \rho\}$, $\tau_2 = \{X, \emptyset, \mu, \beta, \mu \wedge \beta, \mu \vee \beta\}$ and let $f : (X, \tau_1) \to (X, \tau_2)$ be a fuzzy identity function. Then f is fuzzy slightly β-continuous surjective. (X, τ_1) is hyperconnected. But (X, τ_2) is not hyperconnected.

Definition 9. A fuzzy space X said to be

(1) fuzzy β-compact if every fuzzy β-open cover of X has a finite subcover [4].

(2) fuzzy countably β-compact if every fuzzy β-open countably cover of X has a finite subcover.

(3) fuzzy β-Lindelof if every cover of X by fuzzy β-open sets has a countable subcover.

(4) fuzzy mildly compact if every fuzzy clopen cover of X has a finite subcover.

(5) fuzzy mildly countably compact if every fuzzy clopen countably cover of X has a finite subcover.

(6) fuzzy mildly Lindelof if every cover of X by fuzzy clopen sets has a countable subcover.

Theorem 6. *Let $f : X \to Y$ be a fuzzy slightly β-continuous surjection. Then the following statements hold:*

(1) *if X is fuzzy β-compact, then Y is fuzzy mildly compact.*

(2) *if X is fuzzy β-Lindelof, then Y is fuzzy mildly Lindelof.*

(3) *if X is fuzzy countably β-compact, then Y is fuzzy mildly countably compact.*

Proof. (1) Let $\{\mu_\alpha : \alpha \in I\}$ be any fuzzy clopen cover of Y. Since f is fuzzy slightly β-continuous, then $\{f^{-1}(\mu_\alpha) : \alpha \in I\}$ is a fuzzy β-open cover of X. Since X is fuzzy β-compact, there exists a finite subset I_0 of I such that $X = \vee\{f^{-1}(\mu_\alpha) : \alpha \in I_0\}$. Thus, we have $Y = \vee\{\mu_\alpha : \alpha \in I_0\}$ and Y is fuzzy mildly compact.

The other proofs are similarly. □

Definition 10. A fuzzy space X said to be

(1) fuzzy β-closed-compact if every β-closed cover of X has a finite subcover.

(2) fuzzy countably β-closed-compact if every countable cover of X by β-closed sets has a finite subcover.

(3) fuzzy β-closed-Lindelof if every cover of X by β-closed sets has a countable subcover.

Theorem 7. *Let $f : X \to Y$ be a fuzzy slightly β-continuous surjection. Then the following statements hold:*

(1) *if X is fuzzy β-closed-compact, then Y is mildly compact.*
(2) *if X is fuzzy β-closed-Lindelof, then Y is mildly Lindelof.*
(3) *if X is fuzzy countably β-closed-compact, then Y is mildly countably compact.*

PROOF. It can be obtained similarly as the previous theorem. □

3. Fuzzy properties

In this section, we investigate the relationships between fuzzy slightly β-continuous functions and separation axioms and the relationships between fuzzy slightly β-continuity and fuzzy graphs.

DEFINITION 11. A fuzzy space X is said to be fuzzy β-T_1 if for each pair of distinct fuzzy points x_ε and y_ν of X, there exist fuzzy β-open sets β and μ containing x_ε and y_ν respectively such that $y_\nu \notin \beta$ and $x \notin \mu$.

DEFINITION 12. A fuzzy space X is said to be fuzzy co-T_1 if for each pair of distinct fuzzy points x_ε and y_ν of X, there exist fuzzy clopen sets β and μ containing x_ε and y_ν respectively such that $y_\nu \notin \beta$ and $x_\varepsilon \notin \mu$.

THEOREM 8. *If $f : X \to Y$ is a fuzzy slightly β-continuous injection and Y is fuzzy co-T_1, then X is fuzzy β-T_1.*

PROOF. Suppose that Y is fuzzy co-T_1. For any distict fuzzy points x_ε and y_ν in X, there exist fuzzy clopen sets μ, ρ in Y such that $f(x_\varepsilon) \in \mu$, $f(y_\nu) \notin \mu$, $f(x_\varepsilon) \notin \rho$ and $f(y_\nu) \in \rho$. Since f is fuzzy slightly β-continuous, $f^{-1}(\mu)$ and $f^{-1}(\rho)$ are β-open sets in X such that $x_\varepsilon \in f^{-1}(\mu)$, $y_\nu \notin f^{-1}(\mu)$, $x_\varepsilon \notin f^{-1}(\rho)$ and $y_\nu \in f^{-1}(\rho)$. This shows that X is fuzzy β-T_1. □

DEFINITION 13. A fuzzy space X is said to be fuzzy β-T_2(β-Hausdorff) if for each pair of distinct fuzzy points x_ε and y_ν in X, there exist disjoint fuzzy β-open sets β and μ in X such that $x_\varepsilon \in \beta$ and $y_\nu \in \mu$.

DEFINITION 14. A fuzzy space X is said to be fuzzy co-T_2 (co-Hausdorff) if for each pair of distinct fuzzy points x_ε and y_ν in X, there exist disjoint fuzzy clopen sets β and μ in X such that $x_\varepsilon \in \beta$ and $y_\nu \in \mu$.

THEOREM 9. *If $f : X \to Y$ is a fuzzy slightly β-continuous injection and Y is fuzzy co-T_2, then X is fuzzy β-T_2.*

PROOF. For any pair of distict fuzzy points x_ε and y_ν in X, there exist disjoint fuzzy clopen sets β and μ in Y such that $f(x_\varepsilon) \in \beta$ and $f(y_\nu) \in \mu$. Since f is fuzzy slightly β-continuous, $f^{-1}(\beta)$ and $f^{-1}(\mu)$ is fuzzy β-open in X containing x_ε and y_ν respectively. We have $f^{-1}(\beta) \wedge f^{-1}(\mu) = \emptyset$. This shows that X is β-T_2. □

DEFINITION 15. A space is called fuzzy co-regular (respectively fuzzy strongly β–regular) if for each fuzzy clopen (respectively fuzzy β-closed) set η and each fuzzy point $x_\varepsilon \notin \eta$, there exist disjoint fuzzy open sets β and μ such that $\eta \leq \beta$ and $x_\varepsilon \in \mu$.

DEFINITION 16. A fuzzy space is said to be fuzzy co-normal (respectively fuzzy strongly β-normal) if for every pair of disjoint fuzzy clopen (respectively fuzzy β-closed) sets η_1 and η_2 in X, there exist disjoint fuzzy open sets β and μ such that $\eta_1 \leq \beta$ and $\eta_2 \leq \mu$.

THEOREM 10. *If f is fuzzy slightly β-continuous injective fuzzy open function from a fuzzy strongly β-regular space X onto a fuzzy space Y, then Y is fuzzy co-regular.*

PROOF. Let η be fuzzy clopen set in Y and be $y_\varepsilon \notin \eta$. Take $y_\varepsilon = f(x_\varepsilon)$. Since f is fuzzy slightly β-continuous, $f^{-1}(\eta)$ is a fuzzy β-closed set. Take $\lambda = f^{-1}(\eta)$. We have $x_\varepsilon \notin \lambda$. Since X is fuzzy strongly β-regular, there exist disjoint fuzzy open sets β and μ such that $\lambda \leq \beta$ and $x_\varepsilon \in \mu$. We obtain that $\eta = f(\lambda) \leq f(\beta)$ and $y_\varepsilon = f(x_\varepsilon) \in f(\mu)$ such that $f(\beta)$ and $f(\mu)$ are disjoint fuzzy open sets. This shows that Y is fuzzy co-regular. □

THEOREM 11. *If f is fuzzy slightly β-continuous injective fuzzy open function from a fuzzy strongly β-normal space X onto a fuzzy space Y, then Y is fuzzy co-normal.*

PROOF. Let η_1 and η_2 be disjoint fuzzy clopen sets in Y. Since f is fuzzy slightly β-continuous, $f^{-1}(\eta_1)$ and $f^{-1}(\eta_2)$ are fuzzy β-closed sets. Take $\beta = f^{-1}(\eta_1)$ and $\mu = f^{-1}(\eta_2)$. We have $\beta \wedge \mu = \emptyset$. Since X is fuzzy strongly β-normal, there exist disjoint fuzzy open sets λ and ρ such that $\beta \leq \lambda$ and $\mu \leq \rho$. We obtain that $\eta_1 = f(\beta) \leq f(\lambda)$ and $\eta_2 = f(\mu) \leq f(\rho)$ such that $f(\lambda)$ and $f(\rho)$ are disjoint fuzzy open sets. Thus, Y is fuzzy co-normal. □

Recall that for a fuzzy function $f : X \to Y$, the subset $\{(x_\varepsilon, f(x_\varepsilon)) : x_\varepsilon \in X\} \leq X \times Y$ is called the graph of f and is denoted by $G(f)$.

DEFINITION 17. A graph $G(f)$ of a fuzzy function $f : X \to Y$ is said to be fuzzy β-co-closed if for each $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy β-open set β in X containing x_ε and a fuzzy clopen set μ in Y containing y_ν such that $(\beta \times \mu) \wedge G(f) = \emptyset$.

LEMMA 12. *A graph $G(f)$ of a fuzzy function $f : X \to Y$ is fuzzy β-co-closed in $X \times Y$ if and only if for each $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy β-open set β in X containing x_ε and a fuzzy clopen set μ in Y containing y_ν such that $f(\beta) \wedge \mu = \emptyset$.*

THEOREM 13. *If $f : X \to Y$ is fuzzy slightly β-continuous and Y is fuzzy co-Hausdorff, then $G(f)$ is fuzzy β-co-closed in $X \times Y$.*

PROOF. Let $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, then $f(x_\varepsilon) \neq y_\nu$. Since Y is fuzzy co-Hausdorff, there exist fuzzy clopen sets β and μ in Y with $f(x_\varepsilon) \in \beta$ and $y_\nu \in \mu$ such that $\beta \wedge \mu = \emptyset$. Since f is fuzzy slightly β-continuous, there exists a β-open set ρ in X containing x_ε such that $f(\rho) \leq \beta$. Therefore, we obtain $y_\nu \in \mu$ and $f(\rho) \wedge \mu = \emptyset$. This shows that $G(f)$ is fuzzy β-co-closed. □

THEOREM 14. *If $f : X \to Y$ is fuzzy β-continuous and Y is fuzzy co-T_1, then $G(f)$ is fuzzy β-co-closed in $X \times Y$.*

PROOF. Let $(x_\varepsilon, y_\nu) \in (X \times Y)\backslash G(f)$, then $f(x_\varepsilon) \neq y_\nu$ and there exists a fuzzy clopen set μ in Y such that $f(x_\varepsilon) \in \mu$ and $y_\nu \notin \mu$. Since f is fuzzy β-continuous, there exists a β-open set β in X containing x_ε such that $f(\beta) \leq \mu$. Therefore, we obtain that $f(\beta) \wedge (Y\backslash\mu) = \emptyset$ and $Y\backslash\mu$ is fuzzy clopen containing y_ν. This shows that $G(f)$ is fuzzy β-co-closed in $X \times Y$. □

THEOREM 15. *Let $f : X \to Y$ has a fuzzy β-co-closed graph $G(f)$. If f is injective, then X is fuzzy β-T_1.*

PROOF. Let x_ε and y_ν be any two distinct fuzzy points of X. Then, we have $(x_\varepsilon, f(y_\nu)) \in (X \times Y)\backslash G(f)$. By definition of fuzzy β-co-closed graph, there exist a fuzzy β-open set β in X and a fuzzy clopen set μ in Y such that $x_\varepsilon \in \beta$, $f(y_\nu) \in \mu$ and $f(\beta) \wedge \mu = \emptyset$; hence $\beta \wedge f^{-1}(\mu) = \emptyset$. Therefore, we have $y_\nu \notin \beta$. This implies that X is fuzzy β-T_1. □

DEFINITION 18. A fuzzy function is called always fuzzy β-open if the image of each fuzzy β-open set in X is fuzzy β-open set in Y.

THEOREM 16. *Let $f : X \to Y$ has a fuzzy β-co-closed graph $G(f)$. If f is surective always fuzzy β-open function, then Y is fuzzy β-T_2.*

PROOF. Let y_ν and y_ξ be any distinct points of Y. Since f is surjective $f(x_\nu) = y_\nu$ for some $x_\nu \in X$ and $(x_\nu, y_\xi) \in (X \times Y)\backslash G(f)$. By fuzzy β-co-closedness of graph $G(f)$, there exists a fuzzy β-open set β

in X and a fuzzy clopen set μ in Y such that $x_\nu \in \beta$, $y_\xi \in \mu$ and $(\beta \times \mu) \wedge G(f) = \emptyset$. Then, we have $f(\beta) \wedge \mu = \emptyset$. Since f is always fuzzy β-open, then $f(\beta)$ is fuzzy β-open such that $f(x_\nu) = y_\nu \in f(\beta)$. This implies that Y is fuzzy β-T_2. □

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