Closed-form for Bit Error Rate of MSK and OQPSK Systems with a Smart Antenna

Minh-Tuan Le, Van-Su Pham, and Giwan Yoon, *Member, KIMICS*

Abstract—This paper presents closed-form expressions for exact bit error rate of MSK and OQPSK systems employing an adaptive antenna array at base station to eliminate co-channel interference. The channels under consideration are AWGN and one-path flat Rayleigh fading with AWGN. Computer simulation is carried out to confirm the theoretical results.

Index Terms—OQPSK, MSK, LMS, MMSE, Smart antenna

I. INTRODUCTION

A strong point, among many others, of a smart antenna is its capability of eliminating co-channel interference (CCI) based on the deference among the directions of arrival (DOAs) of users. As a consequence, there is an increase in the signal-to-interference-plus-noise ratio (SINR) or a reduction in BER for a given signal-to-noise ratio (SNR) [1]-[2]. This letter provides the readers with close-form equations for computing the BER of the desired user in coherent MSK and OQPSK systems employing an MMSE adaptive beamformer at base station. AWGN channel and one-path slow flat Rayleigh fading channel with AWGN are assumed. The BER expressions are given as functions of the desired SNR per bit, *i.e.*, the transmitted SNR per bit of the desired user, the SNRs per bit of the co-channel interferers, and the DOAs of all users under the assumption that the time delays of all users are approximately equal. It is worth noting that an MSK signal can be thought of as a cosine-weighted OQPSK or can be seen as a special type of a CPFSK [3]-[4]. This letter deals with the former one. The theoretical results are verified by computer simulation.

II. CLOSED-FORM EXPRESSIONS FOR BIT EFFOR RATE OF COHERENT MSK AND OQPSK SYSTEMS WITH A SMART ANTENNA

A. AWGN channel

Let us consider both coherent MSK system, which is viewed as the coherent cosine-weighted OQPSK system, and coherent OQPSK system with an array of M elements at base station to eliminate CCI. If the all users are distributed in such a way that the time delays associated with each signal path are all approximately equal, then the bit error rate of the desired user affected by K co-channel interferers for both systems when the MMSE beamformer is used is given by:

$$P_{e,AWGN}(\gamma_{b0}) = \frac{1}{2} erfc\left(\sqrt{\frac{\left|\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}^{-1}\underline{\mathbf{a}}(\theta_0)\right|^2 \gamma_{b0}}{\left[\sum_{k=1}^{K}\left|\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}^{-1}\underline{\mathbf{a}}(\theta_k)\right|^2 \gamma_{bk} + \left|\underline{\mathbf{a}}^H(\theta_0)\left[\mathbf{R}^{-1}\right]^2\underline{\mathbf{a}}(\theta_0)\right|\right]}} \right) \tag{1}$$

where $|a|$ denotes the module of a complex number a; $\mathbf{R} = \sum_{k=1}^{K}\underline{\mathbf{a}}(\theta_k)\underline{\mathbf{a}}^H(\theta_k)\gamma_{bk} + \mathbf{I}$, $\gamma_{bk} = E_{bk}/N_0$, $k = 0,1,\ldots,K$, is the signal-to-noise ratio per bit for the k^{th} user; N_0 is the one-sided noise power density; $\mathbf{I}$ is the $M \times M$ identity matrix; $\underline{\mathbf{a}}(\theta_k)$, $k = 0,1,\ldots,K$, is the array response or the steering vector corresponding to the k^{th} user or the k^{th} signal path; and θ_k is the DOA of the signal path k. Here we suppose that user $k = 0$ is the desired user.

B. One-path flat Rayleigh fading channel

In such kind of channel, the BER of the desired user for the MSK as well as the OQPSK system under consideration is given by:

$$P_{e,FADING}(\bar{\gamma}_{b0}) = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + 1/\eta}}\right) \tag{2}$$

where,

$$\eta = \frac{\left|\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}^{-1}\underline{\mathbf{a}}(\theta_0)\right|^2 \bar{\gamma}_{b0}}{\left[\sum_{k=1}^{K}\left|\underline{\mathbf{a}}^H(\theta_0)\mathbf{R}^{-1}\underline{\mathbf{a}}(\theta_k)\right|^2 \bar{\gamma}_{bk} + \left|\underline{\mathbf{a}}^H(\theta_0)\left[\mathbf{R}^{-1}\right]^2\underline{\mathbf{a}}(\theta_0)\right|\right]}, \tag{3}$$

$\mathbf{R} = \sum_{i=1}^{K}\underline{\mathbf{a}}(\theta_i)\underline{\mathbf{a}}^H(\theta_i)\bar{\gamma}_{bi} + \mathbf{I}$; $\bar{\gamma}_{bk} = E\left[\alpha_k^2\right]E_{bk}/N_0$, $k = 0,1,\ldots,K$, is the average signal-to-noise ratio per bit

---

Manuscript received November 25, 2005.

Minh-Tuan Le is Ph.D course student in School of Engineering, Information and Communications University (ICU), 119, Munjiro, Yuseong-gu, Daejeon, 305-732, Republic of Korea, (e-mail : letuan@icu.ac.kr)

Van-Su Pham is Ph.D course student in School of Engineering, Information and Communications University (ICU), 119, Munjiro, Yuseong-gu, Daejeon, 305-732, Republic of Korea, (e-mail : vansu_pham@icu.ac.kr)

Giwan Yoon is associate professor in Information and Communications University (ICU), 119, Munjiro, Yuseong-gu, Daejeon, 305-732, Republic of Korea, (e-mail : gwyoon@icu.ac.kr)

for the k^{th} user; $E[.]$ denotes the ensemble average; and α_k is the fading coefficient, which is Rayleigh distributed, associated with k^{th} signal path.

III. SIMULATION RESULTS AND COMPARISON

Some main simulation parameters are as follows. A uniform linear half-wavelength array is used. The symbol rate is $R_s = 256 \times 10^3 bauds/s$. The maximum Doppler shift is $f_D = 160Hz$. The co-channel interference powers all are equal and are 10dB less than that of the desired signal. The DOAs of all the signal paths are chosen from the set (5°, 20°, -20°, 30°, -25°, 25°, 50°, 29°, 18°, 23°, 36°, 70°, -40°, 35)

We assume that DOA of the desired signal path is 5°. The well-known LMS beamforming algorithm [5] is used. For the OQPSK system, a pulse-shaping root raised-cosine filter with roll-off factor of 0.5 is used to eliminate ISI. Fig.1 and Fig. 2 illustrate the theoretical results and the simulation results for both the MSK and OQPSK systems in AWGN channel and in one-path flat Rayleigh fading channel with AWGN, respectively, for different number of elements and users. As can be seen from these figures, the simulation results show a very good agreement with the theoretical results.

IV. CONCLUSION

This letter demonstrates the theoretical bit error rates of coherent MSK and OQPSK systems employing an adaptive array to eliminate co-channel interference and verifies the analytical results by computer simulation. Results show that whether or not there exists co-channel interference the bit error rates of the MSK and OQPSK systems are identical for the same operating environment. However, the results are shown only for the case in which the time delays associated with each user are approximately equal. We will extend and report the results for the case with different time delays in our future work.

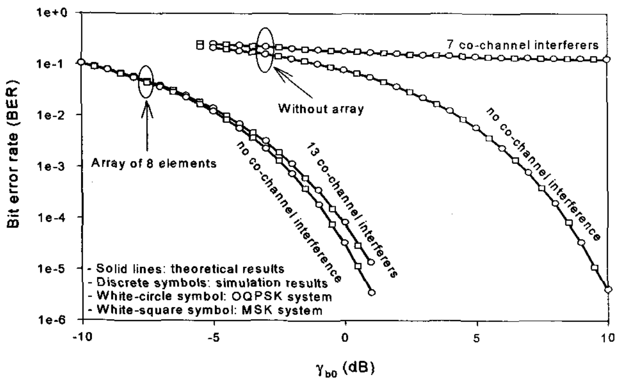


Fig. 1 Theoretical and simulation bit error rates versus SNR per bit of the desired user for different number of array elements and number of co-channel interferers in AWGN channel.

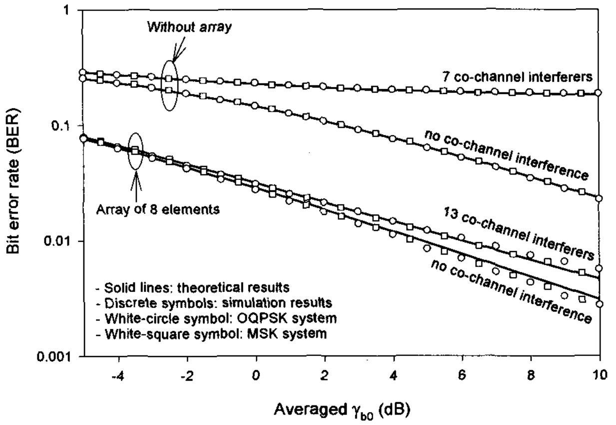


Fig. 2 Theoretical and simulation bit error rates versus averaged SNR per bit of the desired user for different number of array elements and number of co-channel interferers in flat Rayleigh fading channel with AWGN.

ACKNOWLEDGEMENT

This work was supported by the Korea Science and Engineering Foundation (KOSEF) under ERC program through the Intelligent Radio Engineering Center (IREC) at ICU, Republic of Korea.

REFERENCES

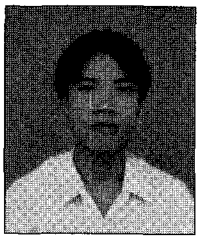
[1] Gupta I. and Ksienski A., "Adaptive antenna arrays for weak interfering signals," IEEE Trans. Antennas Propagat., vol. 34, pp. 420-426, Mar. 1986.

[2] Winters J., "Optimum Combining in Digital Mobile Radio with Cochannel Interference," IEEE J. Select. Areas Commun., vol. 2, pp. 528-539, Jul. 1984.

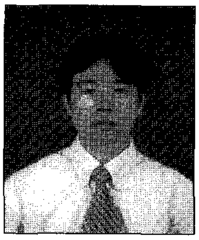
[3] Rappaport T. S., "Wireless Communications- Principle and Practice," Published by Prentice Hall PTR, 1996

[4] Harada H. and Prasad R., "Simulation and Software Radio for Mobile Communications," Artech House Publishers, 2002.

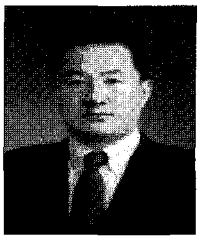
[5] Jr. Joseph C. Liberti and Theodore S. Rappaport, "Smart Antenna for Wireless Communications – IS-95 and 3rd Generation CDMA Applications," Prentice Hall, 1999.



Van-Su Pham,
Member KIMICS Received B. S. degree in Electronic Engineering, Hanoi University, Vietnam, in 1999. M.S. degree in Electrical Engineering from Information and Communications University (ICU), Taejon, Korea in 2003. Since February 2004, he has been Ph.D. student in Communication and Electronics Lab, Information and Communications University (ICU), Taejon, Korea.



Minh-Tuan Le,
Member KIMICS Received his B. S. degree in Electronics and Telecommunication from Hanoi University of Technology, Vietnam in 1999, M.S. degree in Electrical Engineering from Information and Communications University (ICU), Taejon, Korea in 2003. From 1999 to summer 2001, he was lecturer of Posts and Telecommunications Institute of Technology, Vietnam. Currently, he is working toward Ph.D. degree in Communication and Electronics Lab., ICU, Taejon, Korea. His research interest includes smart antenna, space-time coding and MIMO systems.



Giwan Yoon
Received B.S. degree from Seoul National University (SNU) in 1983 and M.S. degree from The Korea Advanced Institute of Science and Technology (KAIST) in 1985, in Korea, and Ph.D. degree from The University of Texas (UT) at Austin, USA in 1994. From 1985 to 1990, he was an associate engineer in Semiconductor Research Center of LG group, Seoul, Korea. From 1994 to 1997, he was a senior engineer of Digital Equipment Corporation (DEC), USA. Since 1997, he has been a professor of Information & Communications University (ICU), Daejeon, Korea. Presently, he is an associate professor and his areas of interest are intelligent communication electronics and RF technology for wireless applications.