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Parameter Selecting in Artificial Potential Functions for Local Path Planning

Dong Hun Kim

Division of Electronic and Electrical Engineering, Kyungnam University, Masan, Kyungnam 631-701, Korea

Abstract

Artificial potential field (APF) is a widely used method for local path planning of autonomous mobile robot. So far, many different types of APF have been implemented. Once the artificial potential functions are selected, how to choose appropriate parameters of the functions is also an important work. In this paper, a detailed analysis is given on how to choose proper parameters of artificial functions to eliminate free path local minima and avoid collision between robots and obstacles. Two kinds of potential functions: Gaussian type and Quadratic type of potential functions are used to solve the above local minima problem respectively. To avoid local minima occurred in realistic situations such as 1) a case that the potential of the goal is affected excessively by potential of the obstacle, 2) a case that the potential of the obstacle is affected excessively by potential of the goal, the design guidelines for selecting appropriate parameters of potential functions are proposed.

Key words : scaling parameters, artificial potential function, path planning, local minimum

1. Introduction

Potential field methods have been studied extensively for path planning of autonomous mobile robot in the past decades [1]-[5] where robots are modeled as moving particles inside an artificial potential field. Negative gradient of the potential field is interpreted as the force acting on the robots which causes variations on their movement. Over the past few years, studies of the potential field method in path planning have been extended to the swarm system for maneuvering group behaviors such as formation, migration and obstacle avoidances [6]. On the other hand, in recent years much attention has also been attracted on the stability analysis of swarms [2]and [3] where potential field methods are implemented. Their focus is on collective convergence and its bound. Yet another challenge in local path planning for mobile robots lies in designing optimal potential functions (PFs). A qualitative approach based on a potential field method has been developed where its potential fields are obtained from relative positions and velocities [8]. In [9], a new method named evolutionary artificial potential field was proposed for real-time robot path planning where potential field method is combined with genetic algorithms [10], to derive optimal potential field functions. [7] presented a systematic criticism of the inherent limitations of potential field methods for the path planning of mobile robot. A novel velocity potential has been proposed for repulsive functions based on fluid dynamics [11]. Nevertheless, as a main presented drawback, these methods may cause a robot to be trapped in local minima generated by the same potential functions. Compared with extensive studies focused on the derivation of optimal potential field functions and their applications, very few of attempts have been made at the proposition of analytical design for RPFs to evade possible local minima. In spite of the challenges to overcome possible local minima problem that may occur in potential fields environment, scaling parameters representing the sizes of attractive and repulsive force have not still been defined well. There are also some unrevealing assumptions, for example, the goal position is usually set relatively far away from obstacles.

In [7], method of using potential functions were systematically analyzed, the inherent problems of artificial potential functions were summarized as: 1) trap situation due to local minima; 2) no passage between closely spaced obstacles; 3) oscillations in the presence of obstacles; and 4) oscillations in narrow passages. In [1], another problem with using artificial functions named GNRON (goal non-reachable problem with obstacle nearby) was described: When the goal and obstacles are located very close due to real implement requirement, the value of total potential at the location of the goal won't be the global minimum and the robots will be trapped in some local minima and can not reach the goal. In the same paper, an algorithm was advanced, which, by adjusting the repulsive potential function, ensures the total potential at the goal position is global minimum. The relationship between the scaling parameters was also analyzed, when this relationship w! as satisfied, the free path local minimum can be eliminated.

Following this idea, it's natural to think following questions: 1)is this algorithm same effective when applied to other types of potential functions; 2) will similar relationship between the

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scaling parameters exist when using different types of potential functions; and one step further 3) does there exist new relationships between scaling parameters to ensure safely implementation of the potential functions adjusted by using the algorithm in [1]. This paper will answers these questions by systematically analyze the performance of the algorithm when it's applied to two pairs of potential functions which are different from previous studies on local path planning based on artificial potential functions [1]-[5]. When a goal is near an obstacle, a robot can not reach the goal due to the larger repulsive force coming from the obstacle, i.e. the goal position is not the global minimum of the total potential. One approach to provide a solution to this problem is to increase the ratio of scaling parameters (attractive force versus repulsive one) beyond a certain threshold. However, in doing so, it will cause problem in the situation that the obstacle lies between the robot and the goal, i.e. the robot may collide with the obstacle due to the large attractive force of the nearby goal. In [1], the problem of a non-reachable goal with obstacles nearby has been presented for the path planning of mobile robot for the first time. Design guidelines for scaling parameters of the attractive and repulsive potential functions are proposed. However, since only the lower boundary of the scaling parameters are considered, the robot may collide with its obstacle nearby due to large attractive force coming from the goal when the obstacle lies between the robot and its own goal. Therefore, the ratio of scaling parameters should be defined for a lower bound and an upper bound.

This paper is organized as follows: Section 2 will introduce the potential functions which are used for analysis and a brief introduction of the goal unreachable problem. The algorithm for solving this problem is also presented; Section 3 focuses on the analysis of the relationships between scaling parameters when using different types of potential functions. A new relationship is brought forward which ensure no collision between robots and obstacles. Some simulation results were shown and analyzed in section 4 as the support for the argument in section 3, and conclusion is drawn in section 5.

2. Potential Functions and Goal Unreachable Problem

To simplify the problem, all the robots are supposed to have point mass and move on a two-dimensional plane. The motion of robots satisfies Newton's second law and the equations are given as:

$$\dot{\mathbf{P}}_i = \mathbf{v}_i$$
$$m_i \dot{\mathbf{v}}_i = u_i \tag{1}$$

where $\mathbf{P}_i$ and $\mathbf{v}_i$ are the position and velocity of the ith robot, respectively. $u_i = F_i$ is total force acting on individual robot.

Now, suppose there is a velocity damping term of the form $-k_v \mathbf{v}_i$ in u_i, where $k_v > 0$. In other words, assume that we have

$$u_i = -k_v \mathbf{v}_i + \bar{u}_i. \tag{2}$$

$\bar{u}_i$ is the output of controller and described by

$$\bar{u}_i = -\nabla U_i \tag{3}$$

where U_i is the artificial potential energy in the system and is given by Section 2.1.

Now, note that for organism such as bacteria we have m_i very small (i.e. we have $m_i \approx 0$) and the viscosity of the environment for them is high. Therefore, we can take $m_i = 0$. Substituting this in the above system of equations we obtain

$$\dot{\mathbf{P}}_i = -\frac{1}{k_v} \nabla U_i. \tag{4}$$

If we consider in the article with $k_v = 1$, we have the equation of motion of each individual i described by

$$\dot{\mathbf{P}}_i = -\nabla U_i. \tag{5}$$

Each of the individuals in the swarm moves so as to minimize the total artificial potential energy in the system. For a planar formation on multiple vehicles, similar dynamics are used in [12].

In the next subsection, two kinds of APF for solving both goal destination and obstacle avoidance problem are introduced: Gaussian type and quadratic type, which are different from previous studies done by the author [13]-[15].

2.1. Trial Potential Functions 1

In trial 1, we use Gaussian type of functions for molding both the attractive and repulsive potentials. The attractive potential function is described as:

$$U_{i,G}^t = -c_t e^{-\frac{\|\psi_i^t\|^2}{l_t^2}} + c_t \tag{6}$$

where $\psi_i^t = \mathbf{P}_i - \mathbf{P}_{goal}$ represents the relative position vector between each robot and the goal. Subscript G means a Gaussian type of potential function. c_t, l_t are two positive constants.

Its corresponding Force is given by the negative gradient of (6)

$$F_{i,G}^t = -\nabla U_{i,G}^t = -\frac{2c_t \psi_i^t}{l_t^2} e^{-\frac{\|\psi_i^t\|^2}{l_t^2}}. \tag{7}$$

The artificial potential function for obstacle avoidance is

$$U_{i,G}^o = \sum_{j \in N_{oi}} \{c_o e^{-\frac{\|\psi_{ij}^o\|^2}{l_o^2}}\} \tag{8}$$

where $\psi^o_j = \mathbf{P}_i - \mathbf{O}_j$ represent the relative position vectors between the robots and the obstacles. $\mathbf{O}_j$ is the position of an obstacle which is a neighbor of robot i, and N_{oi} denotes the set of labels of those obstacles which are neighbors of robot i at time t.

Its corresponding force is then given by the negative gradient of (8).

$$F^o_{i,G} = -\nabla U^o_{i,G} = \sum_{j\in N_{oi}} \{\frac{2c_o\psi^o_j}{l_o^2} e^{-\frac{\|\psi^o_j\|^2}{l_o^2}}\}. \quad (9)$$

The total potential for both group migration and obstacle avoidance is combined together as the total potential

$$U^{ot}_{i,G} = U^o_{i,G} + U^t_{i,G} = \sum_{j\in N_{oi}} \{c_o e^{-\frac{\|\psi^o_j\|^2}{l_o^2}}\} - c_t e^{-\frac{\|\psi^t_i\|^2}{l_t^2}} + c_t. \quad (10)$$

Its corresponding force is

$$F^{ot}_{i,G} = -\nabla U^o_{i,G} - \nabla U^t_{i,G} = \sum_{j\in N_{oi}} \{\frac{2c_o\psi^o_j}{l_o^2} e^{-\frac{\|\psi^o_j\|^2}{l_o^2}}\} - \frac{2c_t\psi^t_i}{l_t^2} e^{-\frac{\|\psi^t_i\|^2}{l_t^2}}. \quad (11)$$

Fig. 1 illustrates the profile of the potential in 3D coordinate, where two obstacles are positioned at (-0.3,1) and (-0.3,-1) when a goal is placed at (0,0). Potential parameters are used such as $c_t = 1$, $c_o = 3$, $l_t = 2$ and $l_o = 0.5$

2.2. Trial Potential Functions 2

In trial 2, the attractive potential function was molded using a quadratic type function:

$$U^t_{i,Q} = \frac{1}{2} c_t \|\psi^t_i\|^2 \quad (12)$$

where ψ^t_i is same defined as in (6) and c_t is a positive constant. Subscript Q means a quadratic type of potential function.

Its corresponding Force F_i is given by the negative gradient of (12)

$$F^t_{i,Q} = -\nabla U^t_{i,Q} = -c_t\psi^t_i. \quad (13)$$

The artificial potential function for obstacle avoidance is

$$U^o_{i,Q} = \sum_{j\in N_{oi}} \{\frac{c_o}{\rho + \|\psi^o_j\|^2}\} \quad (14)$$

where ψ^o_j and N_{oi} are same defined as in (8). ρ is a positive constant.

Its corresponding force is then given by the negative gradient of (14).

$$F^o_{i,Q} = -\nabla U^o_{i,Q} = \sum_{j\in N_{oi}} \{\frac{2c_o\psi^o_j}{(\rho + \|\psi^o_i\|^2)^2}\}. \quad (15)$$

The total potential for both group migration and obstacle avoidance is combined together as the total potential

$$U^{ot}_{i,Q} = U^o_{i,Q} + U^t_{i,Q} = \sum_{j\in N_{oi}} \{\frac{c_o\psi^o_j}{(\rho + \|\psi^o_i\|^2)}\} + \frac{1}{2} c_t \|\psi^t_i\|^2. \quad (16)$$

Its corresponding force is

$$F^{ot}_{i,Q} = \sum_{j\in N_{oi}} \{\frac{2c_o\psi^o_j}{(\rho + \|\psi^o_i\|^2)^2}\} - c_t\psi^t_i. \quad (17)$$

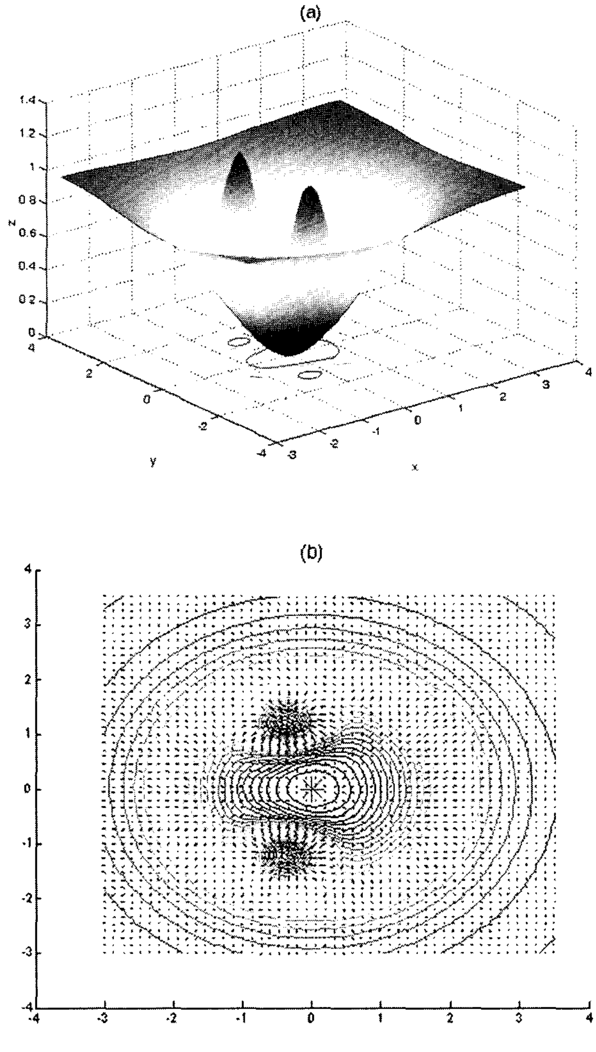

Fig. 1. Trial 1 (a) total potential, (b) contour

2.3. Goal unreachable problem

This problem was first described in [1] as GNRON (goal non-reachable problem with obstacle nearby). The essential cause of this problem is because the total potential is not global minimum at the position of the goal. As can be seen from both eq.(10) and eq.(16), when $\psi^t_i = 0$, the second terms in both equations would disappear and the total potential equals $\sum_{j\in N_{oi}} \{c_o e^{-\frac{\|\psi^o_j\|^2}{l_o^2}}\}$ and $\sum_{j\in N_{oi}} \{\frac{2c_o\psi^o_j}{(\rho+\|\psi^o_i\|^2)^2}\}$ respectively, both values are bigger than zero. This means the global minimum is not located in the position of the goal and the robot will never reach the goal.

To solve this problem, an algorithm was brought forward in [1], this algorithm adjusts the form of the repulsive potential function and takes into consideration of the relative distance between robots and the goal. Using similar algorithm, the total potential function in eq.(10) and eq.(16) be adjusted as:

$$U_{i,G}^{ott} = \frac{1}{c_t} U_{i,G}^{o} \cdot U_{i,G}^{t} + U_{i,G}^{t}$$

$$= \sum_{j \in N_{oi}} \{c_o e^{-\frac{\|\psi_j^o\|^2}{l_o^2}}\} \cdot \{-e^{-\frac{\|\psi_i^t\|^2}{l_t^2}} + 1\} - c_t e^{-\frac{\|\psi_i^t\|^2}{l_t^2}} + c_t. \quad (18)$$

$$U_{i,Q}^{ott} = \frac{2}{c_t} U_{i,Q}^{o} \cdot U_{i,Q}^{t} + U_{i,Q}^{t} = \sum_{j \in N_{oi}} \{\frac{c_o \psi_j^o \|\psi_i^t\|^2}{(\rho + \|\psi_i^o\|^2)}\} + \frac{1}{2} c_t \|\psi_i^t\|^2 . \quad (19)$$

Forces corresponding to the adjusted potential functions are

$$F_{i,G}^{ott} = -\nabla U_{i,G}^{ott} = \sum_{j \in N_{oi}} \{\frac{2 c_o \psi_j^o}{l_o^2} e^{-\frac{\|\psi_j^o\|^2}{l_o^2}}\}$$

$$\cdot(-e^{-\frac{\|\psi_i^t\|^2}{l_t^2}} + 1) + \sum_{j \in N_{oi}} \{c_o e^{-\frac{\|\psi_j^o\|^2}{l_o^2}}\}$$

$$\cdot(-\frac{2\psi_i^t}{l_t^2} e^{-\frac{\|\psi_i^t\|^2}{l_t^2}}) - \frac{2 c_t \psi_i^t}{l_t^2} e^{-\frac{\|\psi_i^t\|^2}{l_t^2}}. \quad (20)$$

$$F_{i,Q}^{ott} = -\nabla U_{i,Q}^{ott} = \sum_{j \in N_{oi}} \{\frac{-2 c_o}{(\rho + \|\psi_i^o\|^2)^2}\} [(\rho + \|\psi_i^o\|^2) \psi_i^t$$

$$- \|\psi_i^t\|^2 \psi_o^t] - c_t \psi_i^t. \quad (21)$$

Now, as can be seen in (18)-(21), both the total potential and its force correspondingly equal to 0 when $\psi_i^t = 0$.

3. Relationships Between Scaling Parameters

The target of analyzing the relationships between scaling parameters is to ensure when applying the new potential functions, robots can reach the goal without being trapped at free path local minima, and at the same time, avoid collision with obstacles. In [1], the relationship between two parameters was set up by finding a lower boundary for the ratio of scaling parameters, when the ratio is bigger than this lower bound, there exists no free path local minima. In fact, this lower boundary ensures that when robots and the goal are in the same side, the attractive force coming from the goal will always dominate the repulsive force from the obstacle. But it's also natural to think whether there exits an upper bound for the ratio of the scaling parameters. Since the scaling represents the ratio of attractive and repulsive force, if an arbitrarily big value is chosen, we might wonder if it is possible that the robot will collide with an obstacle due to large attractive force when the obstacle lies between the robot and the goal. In this section the analysis will be focused on these questions.

3.1. Relationship between scaling parameters for potential functions triall

To simplify the problem, suppose the robot, obstacle and goal are collinear, with the robot lying on a different side of the obstacle and the goal in Fig. 2, where $\psi_{1x}^o = -|\psi_{1x}^o|$ and $\psi_{1x}^t = -|\psi_{1x}^t|$. From (18) and (20) we have

$$F_{1,G}^{ott} = 2c_o (\frac{|\psi_{1x}^o|}{l_o^2} + \frac{|\psi_{1x}^t|}{l_t^2}) e^{-\frac{|\psi_{1x}^o|^2}{l_o^2} - \frac{|\psi_{1x}^t|^2}{l_t^2}}$$

$$- \frac{2c_o |\psi_{1x}^o|}{l_o^2} e^{-\frac{|\psi_{1x}^o|^2}{l_o^2}} + \frac{2c_t |\psi_{1x}^t|}{l_t^2} e^{-\frac{|\psi_{1x}^t|^2}{l_t^2}}. \quad (22)$$

3.1.1. Scaling parameters for eliminating free path local minima

To ensure there be no free path local minima and the robot will keep moving until it reaches the goal, F_1^{ott} should be pointing to the goal, i.e., $F_1^{ott} > 0$ until $|\psi^t| = 0$. So the question is under what condition will be satisfied for this requirement.

Proposition 1 For $|\psi_{1x}^t| < \rho_o^t$ in the force (22), there exist positive constants c_t, l_t, c_o and l_o satisfying following inequality.

$$c_o (\frac{\|\psi_j^o\|}{l_o^2} + \frac{\|\psi_i^t\|}{l_t^2}) e^{-\frac{\|\psi_j^o\|^2}{l_o^2} - \frac{\|\psi_i^t\|^2}{l_t^2}} - \frac{c_o \|\psi_j^o\|}{l_o^2} e^{-\frac{\|\psi_j^o\|^2}{l_o^2}}$$

$$+ \frac{c_t \|\psi_i^t\|}{l_t^2} e^{-\frac{\|\psi_i^t\|^2}{l_t^2}} \geq 0. \quad (23)$$

Sketch of proof: For $F_{1x}^{ott} > 0$ and $F_{1y}^{ott} = 0$, the following inequality can be obtained.

$$c_{t/o} \geq [\frac{|\psi_{1x}^o|}{l_o^2} e^{-\frac{|\psi_{1x}^o|^2}{l_o^2}} - (\frac{|\psi_{1x}^o|}{l_o^2} + \frac{|\psi_{1x}^t|}{l_t^2}) \cdot e^{-\frac{|\psi_{1x}^o|^2}{l_o^2} - \frac{|\psi_{1x}^t|^2}{l_t^2}}] / (\frac{|\psi_{1x}^t|}{l_t^2} e^{-\frac{|\psi_{1x}^t|^2}{l_t^2}})$$

$$\geq \frac{l_t^2}{|\psi_{1x}^t|} e^{-\frac{|\psi_{1x}^o|^2}{l_o^2}} \alpha^t(\psi_{1x}^t, \psi_{1x}^o) \quad (24)$$

where $c_{t/o} = c_t / c_o$ and $\alpha(\psi_{1x}^t, \psi_{1x}^o) = [\frac{|\psi_{1x}^o|}{l_o^2} e^{\frac{|\psi_{1x}^t|^2}{l_t^2}} - (\frac{|\psi_{1x}^o|}{l_o^2} + \frac{|\psi_{1x}^t|}{l_t^2})]$. Let d^{to} be a positive constant for the distance between the goal and the obstacle. We have $|\psi_{1x}^o| = |\psi_{1x}^t| + d^{to}$. Let d_m^{to} donated as the minimum distance of d^{to}, i.e., the diameter of permissible goal region. Since (24) demands large $c_{t/o}$ for small d^{to}, we can replace d^{to} into d_m^{to} after substituting $|\psi_{1x}^t|$ for $|\psi_{1x}^o|$ in (24). Through some simple algebraic manipulations, we have

$$\alpha^t(|\psi_{1x}^t|) = \frac{|\psi_{1x}^t| + d_m^{to}}{l_o^2} e^{\frac{|\psi_{1x}^t|^2}{l_t^2}} - (\frac{|\psi_{1x}^t| + d_m^{to}}{l_o^2} + \frac{|\psi_{1x}^t|}{l_t^2}) . \begin{cases} \leq 0, if\ |\psi_{1x}^t| \leq \rho_m^t \\ > 0, if\ \rho_m^t < |\psi_{1x}^t| < \rho_o^t \end{cases} \quad (25)$$

where ρ_o^t is the distance of influence of the goal and ρ_m^t is the distance when the total of repulsive force from the obstacle and attractive force from the goal is zero. The lower bound of $c_{t/o}$ is chosen when $\rho_m^t < |\psi_{1x}^t| < \rho_o^t$. **Q.E.D**

Now, the lower bound of the ratio for the scaling parameters of the potential functions can be found by analyzing process described above. We will try to find if there also exists an upper bound.

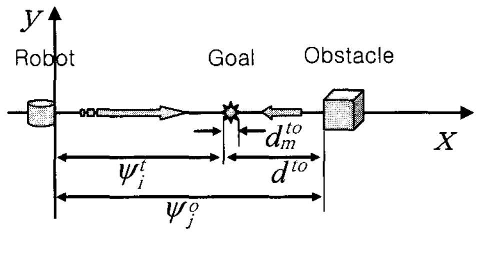

Fig. 2. When the robot, goal and obstacle are collinear

3.1.2. scaling parameters for avoiding collision between a robot and obstacles

Suppose that the robot, obstacle and goal are collinear, with the obstacle lying between the robot and the goal in Fig. 3.

Proposition 2 For $d^{ro}_m < |\psi_{1x}^o| \le \rho_m^o$ in the force (22), there exist positive constants c_t, l_t, c_o and l_o satisfying following inequality.

$$c_o\left(\frac{\|\psi_j^o\|}{l_o^2}+\frac{\|\psi_i^t\|}{l_t^2}\right)e^{-\frac{\|\psi_j^o\|^2}{l_o^2}-\frac{\|\psi_i^t\|^2}{l_t^2}}-\frac{c_o\|\psi_j^o\|}{l_o^2}e^{-\frac{\|\psi_j^o\|^2}{l_o^2}}+\frac{c_t\|\psi_i^t\|}{l_t^2}e^{-\frac{\|\psi_i^t\|^2}{l_t^2}}\le 0 \tag{26}$$

where d^{ro} is a positive constant for the distance between the robot and obstacle, and d_m^{ro} is the minimum distance of d^{ro}, i.e., the minimum distance avoiding collision between the robot and the obstacle.

Sketch of proof: Let us denote d^{ot} as a positive constant for the distance between the obstacle and the goal nearby, and d_m^{ot} as the minimum distance of d^{ot}, i.e., the diameter of the obstacle. We have $|\psi_{1x}^t| = |\psi_{1x}^o| + d^{ot}$.

After the robot reaches the point where the total of repulsive force from the obstacle and attractive force from the goal is zero, the robot does not approach the obstacle any more. The closer the goal is to the obstacle, the higher the probability that the robot collides with the obstacle is. Thus we can replace d^{ot}_m into d^{ot} and substitute $|\psi_{1x}^o| + d^{ot}_m$ for $|\psi_{1x}^t|$ as $|\psi_{1x}^t| = |\psi_{1x}^o| + d^{ot}$. Using the same procedure as Proposition 3.1.1, similarly we have

$$c_{t/o} \le \frac{l_t^2}{|\psi_{1x}^o| + d^{ot}_m} e^{-\frac{|\psi_{1x}^o|^2}{l_o^2}}\left[\frac{|\psi_{1x}^o|}{l_o^2} e^{\frac{(|\psi_{1x}^o|+d^{ot}_m)^2}{l_t^2}} -\left(\frac{|\psi_{1x}^o|}{l_o^2}+\frac{|\psi_{1x}^o|+d^{ot}_m}{l_t^2}\right)\right]. \tag{27}$$

The smaller $|\psi_{1x}^o|$ is, (27) demands smaller $c_{t/o}$. For this reason, we can replace d_m^{ro} into $|\psi_{1x}^o|$ in (27).

$$c_{t/o} \le \frac{l_t^2}{d^{ro}_m + d^{ot}_m} e^{-\frac{{d^{ro}_m}^2}{l_o^2}}\left[\frac{d^{ro}_m}{l_o^2} e^{\frac{(d^{ro}_m+d^{ot}_m)^2}{l_t^2}} -\left(\frac{d^{ro}_m}{l_o^2}+\frac{d^{ro}_m+d^{ot}_m}{l_t^2}\right)\right]. \tag{28}$$

Q.E.D

The purpose of proposition 2 is to ensure that each robot does not collide with the obstacle by the effect of excessive goal's potential fields. In fact, in some implementation environment such as in a swarm system, robots which fall into local may escape under the influence of potential fields by the other agents. Hence, it is more important for us to choose appropriate values for the parameters in the potential functions for avoiding collision with obstacles.

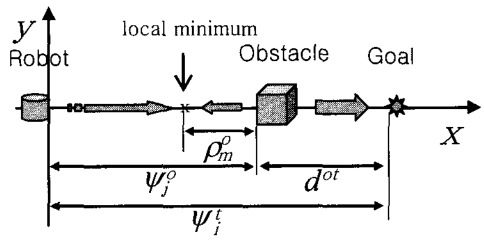

Fig. 3. When the robot, obstacle and goal are collinear

3.1.3. example of finding both the lower and upper bound

Suppose some constants are given: $d_m^{to} = 0.2$, $l_t = 2$ and $l_o = 0.2$. By using (24), we can get $\rho_m^t = 0.122$ and $c_{t/o} > 0.022$. If there is no permissible region of the goal. i.e. $d_m^{to} = 0$, we can get $c_{t/o} > 0.151$. If the robot is far from the obstacle, i.e. in the case of $|\psi_{1x}^o| > \rho_o^t$, $c_{t/o}$ becomes to zero due to $e^{-\frac{|\psi_{1x}^o|^2}{l_o^2}}$ in (24). In (28), we can get $\rho_m^o = 0.281$ and $c_{t/o} < 0.383$ for $d_m^{ro} = 0.2$ and $d_m^{ot} = 0.2$. Thus we can choose c_o and c_t satisfying to $0.022 < c_t/c_o < 0.383$. Finally, we choose $c_t = 1$ and $c_o = 3$. The implementation result can be seen in simulation results in Section 4.

3.2. Relationship between scaling parameters for potential functions trial2

Similarly to the method obtained in Section 3.1, we can find the relationships between the parameters of the potential functions in trial 2. Some of analyzing process are simplified due to the similarity to Section 3.1.

3.2.1. Scaling parameters for eliminating free path local minima

Similarly, the force expressed in (21) can be adjusted as:

$$F_{i,Q}^{ott} = \{\frac{-2c_o}{(\rho + |\psi_i^o|^2)^2}\}[(-|\psi_i^t|)(\rho + |\psi_i^o|^2) + |\psi_i^t|^2 |\psi_i^o|] + c_t |\psi_i^t|. \tag{29}$$

To ensure there be no free path local minima, it's required that:

$$c_{t/o} \geq \{\frac{2}{(\rho + |\psi_i^o|^2)^2}\}[(-(\rho + |\psi_i^o|^2) + |\psi_i^t||\psi_i^o|]. \tag{30}$$

Since $|\psi_{1x}^o| = |\psi_{1x}^t| + d^{to}$. Plug into (30) and rearrange, we have:

$$c_{t/o} \geq \{\frac{2}{(\rho + |\psi_i^o|^2)^2}\}[(-(\rho + (|\psi_i^t| + d^{to})^2) + |\psi_i^t|(|\psi_i^t| + d^{to})]$$
$$= \{\frac{2}{(\rho + |\psi_i^o|^2)^2}\}[-\rho - |\psi_i^t| d^{to} - d^{to2}]. \tag{31}$$

As can be seen that the right half of the inequality is negative, so as long as $c_{t/o} > 0$, there would be no free path local minima, and the lower bound is simply 0 for the scaling.

3.2.2. scaling parameters for avoid collision between a robot and obstacles

Similarly, in order to ensure that robot does not collide with the obstacle, $F_{1,Q}^{ott}$ is required to be positive until the robot reaches a point near the obstacle where $F_{1,Q}^{ott} = 0$. Using (30), we have

$$c_{t/o} \leq \{\frac{2}{(\rho + |\psi_i^o|^2)^2}\}[(-(\rho + |\psi_i^o|^2) + |\psi_i^t||\psi_i^o|]. \tag{32}$$

Since now $|\psi_{1x}^t| = |\psi_{1x}^o| + d^{ot}$. Plug into (32) and rearrange, we have:

$$c_{t/o} \leq \{\frac{2}{(\rho + |\psi_i^o|^2)^2}\}[(-(\rho + |\psi_i^o|^2) + |\psi_i^o|(|\psi_i^o| + d^{ot})]$$
$$= \{\frac{2}{(\rho + |\psi_i^o|^2)^2}\}[-\rho + |\psi_i^t| d^{ot}]. \tag{33}$$

As can be seen when $|\psi_i^t| > \rho/d^{ot}$, the right half of the inequality will be a positive value. Suppose we want the robot to stop at the position which is $d_{safe}(d_{safe} \geq \rho/d^{ot})$, we can just plug the value into (33) and find the upper bound of the scaling parameters.

4. Simulation results

The environment of the simulation is a $7m \times 5m$ workspace where a goal is situated at (0,0) and five obstacles are situated at (-0.3,1), (-0.3,-1), (-0.2,0), (-0.3,1.6), (-0.3,-1.6) respectively. And the initial positions of 10 robots are (-3,0), (-2.5 0.5), (-2.5 -0.5), (-2 1), (-2 -1), (-1.5,1.5), (-1.5,-1.5), (-1,-2), (1.5,1.5), (1.5,-1.5). Basic parameters for all the simulation program are the same: $d_m^{to} = 0.2$, $l_t = 2$ and $l_o = 0.2$ as in Section 3.1.3.

Fig. 4 shows the scenario of goal unreachable problem by selecting the parameters that satisfies Proposition 2 but not Proposition 1. From the trajectories of robots, it's clear to see when the goal is placed close to obstacles, all the robots will be trapped at local minima. The local minimum which lies on the right side of the goal is called the free path local minimum because there be no obstacles between the robot and goal. Parameters of the potential functions are: $c_t = 1$, $c_o = 5$,

In Fig. 5, $c_t = 1$ and $c_o = 2$ are used by selecting parameters that satisfies Proposition 1 but not Proposition 2. As can be seen in Fig. 5, the free path local minimum is eliminated that is, Robots on the same side of goal reach the goal. However, one robot coming from the other side of the obstacles bumped into the obstacle which is collinear with the robot and target, due to improperly chosen parameters. This shows when the ratio of the scaling parameters exceeds the upper bound, repulsive force coming from the obstacles can not counteract the attractive force coming from the target, and collision happens.

Fig. 6 shows the result of proper ration of scaling parameters based on Proposition 1 and 2, where $c_t = 1$ and $c_o = 3$ are used. The robot coming from the same side as the target reaches the goal, since all the free path local minima already eliminated. The robot coming from the other side of the goal which may collide with the obstacle stops at some distance near the obstacle.

It can be seen that when choosing proper scaling parameters, the problem of goal unreachable and obstacle collision can be solved. But in real implements, when choosing exact values for the parameters, information such as initial positions and maximum acceleration of the robots all need to be carefully considered.

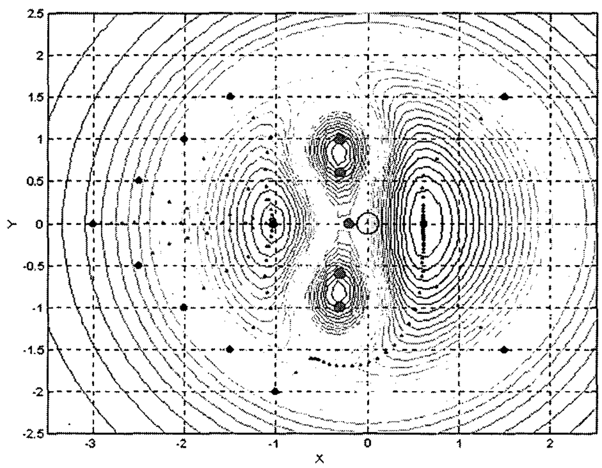

Fig. 4. Goal unreachable problem

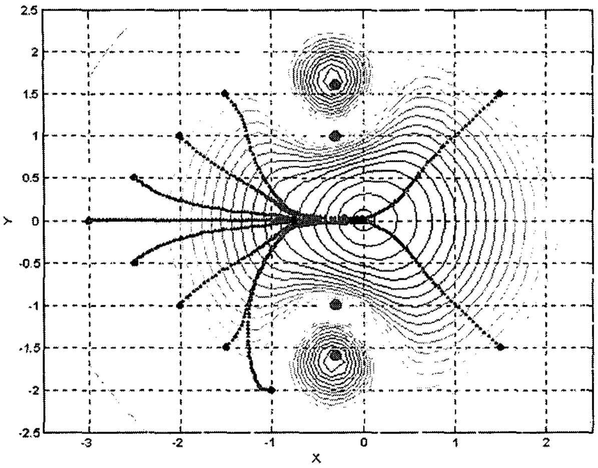

Fig. 5. Obstacle collision problem

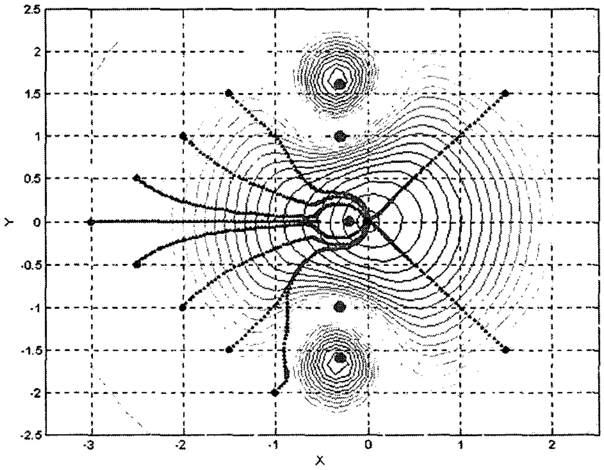

Fig. 6. Path planning by selected APF parameters

5. Conclusions

In the paper, the algorithm of solving goal unreachable problem is tested with two new pairs of potential functions which are different from previous studies. Relationships between the scaling parameters are analyzed with new relationship brought forward to ensure both elimination of free path local minima and no collision with obstacles, differently from [1]. As well, the design guidelines of potential functions for avoiding local minima occurred in more realistic situations are proposed such as 1) a case that the potential of the goal is affected excessively by potential of the obstacle, 2) a case that the potential of the obstacle is affected excessively by potential of the goal. Simulation results show that new relationship is also important for implementation of the algorithm and that the design guideline is effective.

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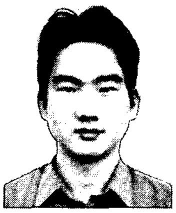

Dong Hun Kim

Dong Hun Kim received his B.S., M.S. and Ph.D. degrees from the Department of Electrical Engineering, Hanyang University, Korea, in 1995, 1997 and 2001, respectively. From 2001 to 2003, he was a Research Associate under several grants in the Department of Electrical and Computer Engineering, Duke University, NC, USA. He was a Visiting Assistant Professor under several grants in the Department of Aerospace and Mechanical Engineering, Boston University, MA, USA in 2003. In 2004, he was engaged in post-doctoral research at School of Information Science and Technology, the University of Tokyo, Japan. Currently, he is an Assistant Professor with the Division of Electronic & Electrical Engineering, Kyungnam University, Korea. His research interests include self-organization of swarm systems, mobile robot path planning, decentralized control of autonomous vehicles, intelligent control, and adaptive nonlinear control.

Phone : 055-249-2629
E-mail : dhkim@kyungnam.ac.kr