A New Complementary Quadriphase Jacket Sequence with Good Cross Correlation

Jia Hou[1] · Kwang-Jae Lee[2] · Moon-Ho Lee[1]

Abstract

In this paper, a set of new complementary quadriphase sequences based on Jacket matrix is proposed. It is with a good zero cross correlation zone and efficiently eliminates the inter-user interferences for CDMA systems. Unlike the conventional complementary sequence designs, the proposed sequences can be easily extended to large odd and even sizes by using a fast linear transform for multi-user communication systems. The computer simulation shows that the proposed sequences have better performance than conventional multi-user spreading CDMA systems using ZCZ sequence.

Key words : CDMA, Complementary Quadriphase Jacket Sequence, Hadamard.

Ⅰ. Introduction

In the quadrature phase shift keying(QPSK) digital modulation systems, quadriphase sequences with good or perfect periodic cross correlation function are useful to making fast start-up equalization, channel estimation, or synchronization. Therefore, zero cross correlation zone(ZCZ) sequence is proposed by Suehiro and Fan[1] for uplink and downlink signaling design to approach the potential capacity.

Otherwise, a good orthogonal quadriphase transform named Jacket was proposed in [2], [3]. It is extended from Walsh Hadamard transform and center weighted Hadamard. The main idea of Jacket is that the inverse transform could be simply obtained from the element-inverse of the entries.

In this paper, we propose a method to construct new complementary quadriphase sequences for good cross correlation based on Jacket matrices. Promptly, this scheme may be easily applied for other poly-phase sequences by using the generalized Jacket transform[3].

To present our proposed construction, in this section we will first present the basic concept of Jacket matrices and their useful properties. Some results are not included in previous literatures.

A typical $2^n \times 2^n$ symmetric Jacket matrix $[J]_{2^n}$ is defined from a center weighted form as[2]

$$[J]_{2^n} = [J]_{2^{n-1}} \otimes [H]_2,\ n \geq 3, \tag{1}$$

where $[J]_{2^2} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & j & -1 \\ 1 & j & -j & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$, $j=\sqrt{-1}$ and $[H]_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ is Hadamard.

In general, these Jacket matrices have the following characters.

1) The entries of Jacket matrices are complex numbers on the unit circle such as $\{\pm 1,\ \pm j\}$.

2) Jacket matrix has

$$[J]_{2^n}[J]_{2^n}* = 2^n[I]_{2^n},\ n \geq 2, \tag{2}$$

thus the elements $[J_i(m)]_{2^n}$ in $[J]_{2^n}$ has $J_i(m)* = (J_i(m))^{-1}$, $1 \leq i, m \leq 2^n$, * denotes the complex conjugate. Assuming $[J]_{2^n}[J]_{2^n}^{-1} = [I]_{2^n}$, $n \geq 2$ and $[I]_{2^n}$ is a identity matrix, we obtain

$$[J]_{2^n}[J]_{2^n}^{-1} = \frac{1}{2^n}[J]_{2^n}[J]_{2^n}*,$$

and the inverse of the Jacket matrix is

$$[J]_{2^n}^{-1} = \frac{1}{2^n}[J]_{2^n}*,\ n \geq 2. \tag{3}$$

Moreover, the equation (2) implies that

$$\sum_{m=1}^{2^n} J_i(m) J_j*(m) = \begin{cases} N(=2^n), & when\ i=j \\ 0, & when\ i \neq j,\ 1 \leq i, j \leq 2^n \end{cases} \tag{4}$$

Manuscript received July 25, 2005 ; revised October 24, 2005. (ID No. 20050725-026J)

[1]Dept. of Information and Communication, Chonbuk National University, Chonbuk, Korea.

[2]Dept. of Multimedia, Information and Telecommunication, Hanlyo University, Chonnam, Korea.

where the Jacket matrix is denoted by using a sequence form as

$$[J]_{2^n}=\begin{bmatrix} J_1(1) & J_1(2) & \cdots & J_1(m) \\ J_2(1) & J_2(2) & \cdots & J_2(m) \\ \vdots & \vdots & \vdots & \vdots \\ J_{2^n}(1) & J_{2^n}(2) & \cdots & J_{2^n}(m) \end{bmatrix}.$$

3) Hybrid relation of square values: For a $2^n\times 2^n$ Jacket matrix, we have

$$(J_i(m))^2=\begin{cases} 1, & other \\ -1, & 2^{n-2}+1\le m\le \frac{3}{4}\times 2^n, \end{cases}$$

$$2^{n-2}+1\le i\le \frac{3}{4}\times 2^n$$

$$(J_i(m))^2=1,\quad 1\le m\le 2^n,\quad i=others.$$

Ⅱ. Complementary Jacket Sequence

By generalizing the crosscorrelation and autocorrelation values from these Jacket sequences except all +1s rows we can write the complementary correlations of the quadriphase sequences from the central part of Jacket matrices as

$$\sum_{\tau=0}^{2^{n-1}}\sum_{m=1}^{2^n} J_i(m)J_j*(m\oplus\tau_{\bmod 2^n})+J_j(m)J_i*(m\oplus\tau_{\bmod 2^n})=0,$$

when $i\neq j$,

$$2^{n-2}+1\le i\le 2^{n-1},\ 2^{n-1}+1\le j\le \frac{3}{4}\times 2^n, \tag{5}$$

where a condition should be satisfied as

$$j=i+2^{n-2}. \tag{6}$$

These polyphase sequences exist in the central part of the Jacket matrix, and they can be extended similarly as Jacket matrix by using the Kronecker product as

$$center([J]_{2^n})=center([J]_{2^{n-1}})\otimes[H]_2,\ n\ge 3 \tag{7}$$

where the base matrix of the recursive function is $center[J]_4=\begin{bmatrix} J_2(m) \\ J_3(m)\end{bmatrix}=\begin{bmatrix} 1 & -j & j & -1 \\ 1 & j & -j & -1\end{bmatrix}$. Obviously, we would find that

$$J_2(m)=J_3*(m)=[J_3(m)]^{-1}. \tag{8}$$

where these two sequences, which satisfy the equation (5) and (6), are complementary.

Theorem 1: In the case of $[J]_{2^n}$ matrix, there are at least 2^{n-2} complementary sequences(pairs) in the central of $[J]_{2^k}$, and they are satisfied equations (5) and (6).

Proof: As mentioned in (1) and (7), Kronecker product is used to extend the Jacket matrix. Based on the base matrix $[J]_4$, where one complementary sequence $\{J_2(m), J_3(m)\}$ is shown in (8). It implies

$$\sum_{\tau=0}^{3}\sum_{m=1}^{4} J_2(m)J_3*(m\oplus\tau_{\bmod 4})+J_3(m)J_2*(m\oplus\tau_{\bmod 4})$$
$$=\{0,\ -2j,\ 4j,\ -2j\}+(0,\ 2j,\ -4j,\ 2j)=0. \tag{9}$$

Furthermore, we can write the general form for $[J]_{2^n}$ as $\{J_2(m), J_3(m)\}\otimes[H]_2\otimes\cdots\otimes[H]_2$,

$$\text{where}\ [H]_2\otimes[H]_2\otimes\cdots\otimes[H]_2=[H]_{2^{n-2}}. \tag{10}$$

The Kronecker product can linearly extend the complementary sequence $\{J_2(m), J_3(m)\}$, therefore, in the case of $[J]_{2^n}$ matrix, at least we can obtain 2^{n-2} complementary sequences based on $\{J_2(m), J_3(m)\}$. Thus the Theorem 1 is proved.

The correlations values of these complementary sequences as shown in Theorem 1 can be easily calculated by using a simple shift generations. For example, in the case of $[J]_4$, we have

$$J_{23}(\tau=0)=\sum_{m=1}^{4} J_2(m)J_3*(m)=0;$$
$$J_{23}(\tau=1)=\sum_{m=1}^{4} J_2(m)J_3*(m\oplus\tau_{\bmod 4})=-2j;$$
$$J_{23}(\tau=2)=\sum_{m=1}^{4} J_2(m)J_3*(m\oplus\tau_{\bmod 4})=4j;$$
$$J_{23}(\tau=3)=\sum_{m=1}^{4} J_2(m)J_3*(m\oplus\tau_{\bmod 4})=-2j.$$

and

$$J_{32}(\tau=0)=\sum_{m=1}^{4} J_3(m)J_2*(m)=0;$$
$$J_{32}(\tau=1)=\sum_{m=1}^{4} J_3(m)J_2*(m\oplus\tau_{\bmod 4})=2j;$$
$$J_{32}(\tau=2)=\sum_{m=1}^{4} J_3(m)J_2*(m\oplus\tau_{\bmod 4})=-4j;$$
$$J_{32}(\tau=3)=\sum_{m=1}^{4} J_3(m)J_2*(m\oplus\tau_{\bmod 4})=2j.$$

According to (7), the corresponding sequences $center[J]_8=center[J]_4\otimes[H]_2$, thus we write the complementary sequences $\{J_3(m), J_5(m)\}$, and $\{J_4(m), J_6(m)\}$ in $[J]_8$, where the sequences in $[J]_8$ have

$$\begin{bmatrix}[J_3(m)]_8 \\ [J_4(m)]_8 \\ [J_5(m)]_8 \\ [J_6(m)]_8\end{bmatrix}=\begin{bmatrix}[J_2(m)]_4 \\ [J_3(m)]_4\end{bmatrix}\otimes[H]_2 \tag{11}$$

where $[J_i(m)]_{2^n}$ denotes the ith sequence in a $[J]_{2^n}$ matrix. And the correlations of $\{J_3(m), J_5(m)\}$ in $[J]_8$ can be estimated by using the correlations in $[J]_4$ as

$$[J_{35}(\tau=0)]_8 = \sum_{m=1}^{8}[J_3(m)J_5{}^*(m)]_8 = [J_{23}(\tau=0)]_4 + [J_{23}(\tau=0)]_4 = 0;$$

$$[J_{35}(\tau=1)]_8 = \sum_{m=1}^{8}[J_3(m)J_5{}^*(m\oplus\tau_{\mathrm{mod8}})]_8 = [J_{23}(\tau=0)]_4 + [J_{23}(\tau=1)]_4 = 0-2j = -2j;$$

$$[J_{35}(\tau=2)]_8 = \sum_{m=1}^{8}[J_3(m)J_5{}^*(m\oplus\tau_{\mathrm{mod8}})]_8 = [J_{23}(\tau=1)]_4 + [J_{23}(\tau=1)]_4 = -4j;$$

$$[J_{35}(\tau=3)]_8 = \sum_{m=1}^{8}[J_3(m)J_5{}^*(m\oplus\tau_{\mathrm{mod8}})]_8 = [J_{23}(\tau=1)]_4 + [J_{23}(\tau=2)]_4 = -2j+4j = 2j.$$

$$[J_{35}(\tau=4)]_8 = \sum_{m=1}^{8}[J_3(m)J_5{}^*(m)]_8 = [J_{23}(\tau=2)]_4 + [J_{23}(\tau=2)]_4 = 4j+4j = 8j;$$

$$[J_{35}(\tau=5)]_8 = \sum_{m=1}^{8}[J_3(m)J_5{}^*(m\oplus\tau_{\mathrm{mod8}})]_8 = [J_{23}(\tau=2)]_4 + [J_{23}(\tau=3)]_4 = 4j-2j = 2j;$$

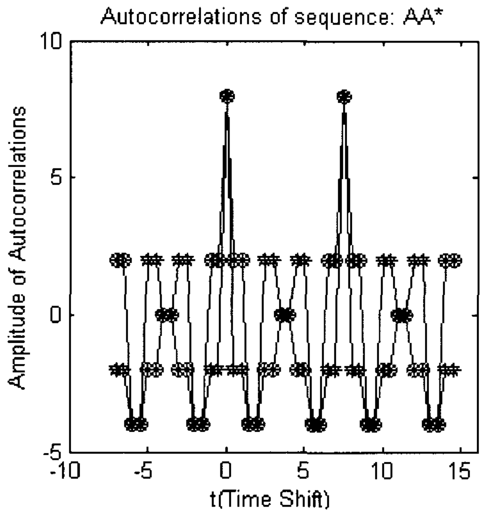
$$[J_{35}(\tau=6)]_8 = \sum_{m=1}^{8}[J_3(m)J_5{}^*(m\oplus\tau_{\mathrm{mod8}})]_8 = [J_{23}(\tau=3)]_4 + [J_{23}(\tau=3)]_4 = -4j;$$

$$[J_{35}(\tau=7)]_8 = \sum_{m=1}^{8}[J_3(m)J_5{}^*(m\oplus\tau_{\mathrm{mod8}})]_8 = [J_{23}(\tau=3]_4 + [J_{23}(\tau=0)]_4 = -2j.$$

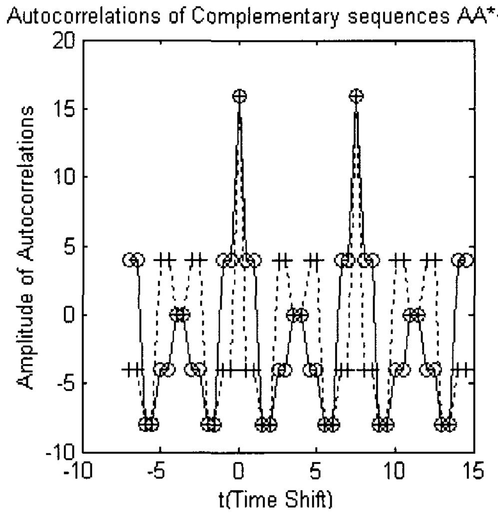
This scheme can be applied to all 2^n size sequences generated from 2^{n-1} size sequences. The simulations of autocorrelations and crosscorrelations of the proposed complementary Jacket sequences demonstrate that they have good autocorrelations and perfect zero crosscorrelations, as shown in Fig. 1.

III. Complementary Jacket Sequence for QPSK CDMA

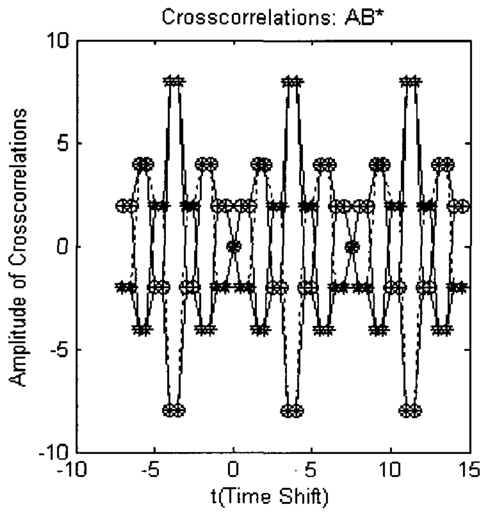
By using the proposed complementary Jacket sequences, we design a QPSK CDMA system. For a transmitted signal of k-th user, the mathematical model of the system is written by



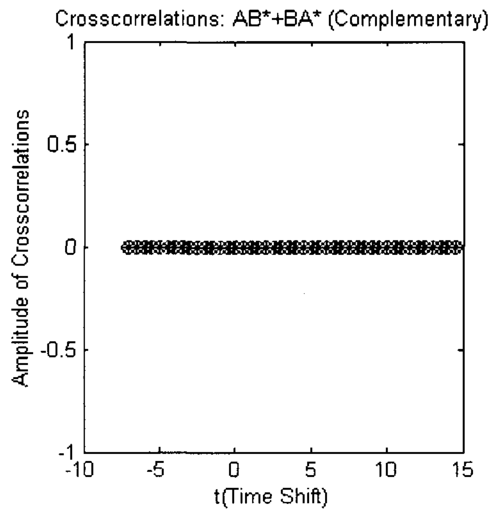
(a) Autocorrelations of the Jacket sequences



(b) Autocorrelations of the complementary Jacket sequences



(c) Crosscorrelations of the Jacket sequences



(d) Crosscorrelations of the complementary Jacket sequences

Fig. 1. (a) and (c) use the Jacket sequences $[J_3(m)]_8$, $[J_4(m)]_8$, $[J_5(m)]_8$, $[J_6(m)]_8$, in $[J]_8$ (b) and (d) use the complementary Jacket sequences $\{[J_3(m)]_8, [J_5(m)]_8\}$ and $\{[J_4(m)]_8, [J_6(m)]_8$ in $[J]_8$.

Transmitter:

$$S_k = d_k A_k + d_k B_k \,, \tag{12}$$

Receiver:

$$\begin{aligned} & S_k A_k^* + S_k B_k^* \\ & = d_k A_k A_k^* + d_k B_k A_k^* + d_k A_k B_k^* + d_k B_k B_k^* \\ & = d_k(|A_k|^2 + |B_k|^2) + d_k(B_k A_k^* + A_k B_k^*) \\ & = d_k(|A_k|^2 + |B_k|^2) \,. \end{aligned} \tag{13}$$

where d_k is the QPSK modulated signal for kth user, A_k and B_k are selected complementary Jacket sequence for k-th user, which has

$$A_k B_k^* + B_k A_k^* = 0. \tag{14}$$

The number of the users is decided by the size of the complementary Jacket sequences from the design criterions as follows.

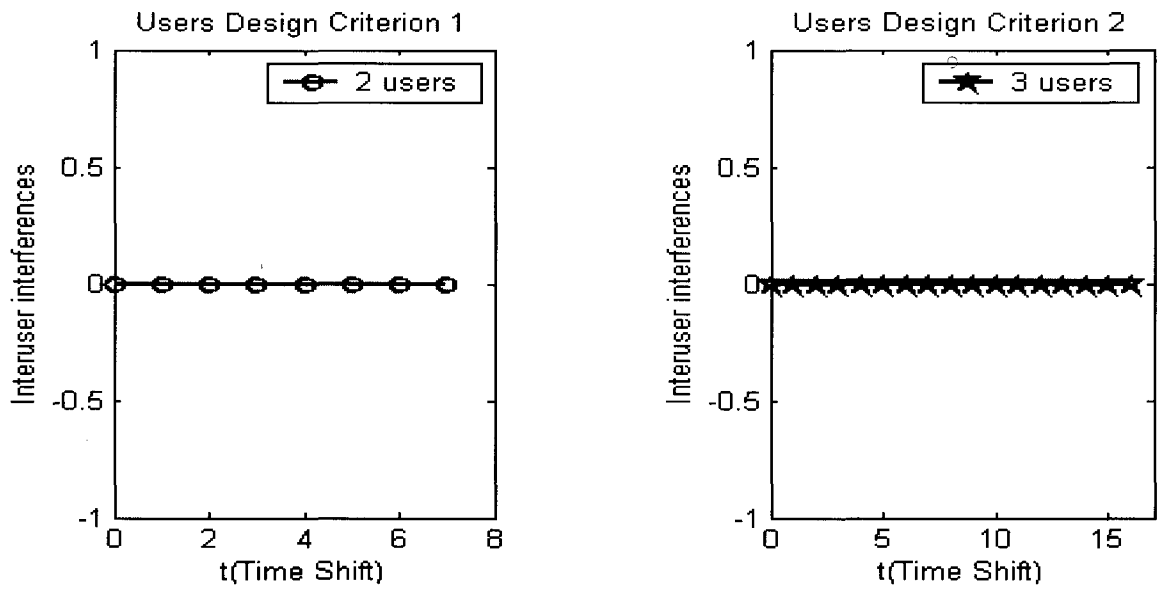
Users Design Criterion 1: Assuming a complementary Jacket sequence $\{J_i(m), J_j(m)\}$ with length 2^n is selected from $[J]_{2^n}$, the transmitted complementary codes for 2^k users with length $2^k \times 2^n$ are given by

$$\begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_{2^k} \end{bmatrix} = [H]_{2^k} \otimes \begin{bmatrix} J_i(m) \\ J_j(m) \end{bmatrix},$$

$$2^{n-2} + 1 \le i \le 2^{n-1}, 2^{n-1} + 1 \le j \le \frac{3}{4} \times 2^n, \tag{15}$$

where the t-th user complementary code $C_t = \{c_{2t-1}, c_{2t}\}$ in (15), c denotes one row in the matrix of (15), and $1 \le t \le 2^k$.

Users Design Criterion 2: Assuming the complementary sequences which is extended from (10) has the



(a) Interuser interferences from design criterion 1 (b) Interuser interferences from design criterion 2

Fig. 2. (a) is from $[H]_2 \otimes \begin{bmatrix} 1 & -j & j & -1 \\ 1 & j & -j & -1 \end{bmatrix}$, where the complementary codes are defined as $\begin{cases} A_1 = \{1 \ -j \ j \ -1 \ 1 \ -j \ j \ -1\} \\ B_1 = \{1 \ j \ -j \ -1 \ 1 \ j \ -j \ -1\} \end{cases}$; $\begin{cases} A_2 = \{1 \ -j \ j \ -1 \ -1 \ j \ -j \ 1\} \\ B_2 = \{1 \ j \ -j \ -1 \ -1 \ -j \ j \ 1\} \end{cases}$.

(b) is from

$$[H]_2 \otimes \begin{bmatrix} 1 & -j & j & -1 \\ 1 & j & -j & -1 \end{bmatrix} \otimes [H]_2 = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \\ S_7 \\ S_8 \end{bmatrix},$$ and $C_l = \{S_1, S_3\}$, $C_3 = \{S_5, S_7\}$, $C_4 = \{S_6, S_8\}$, $\begin{bmatrix} user1 \\ user2 \\ user3 \end{bmatrix} = \begin{bmatrix} C_l \\ C_3 \\ C_4 \end{bmatrix}$.

form as

$$\begin{bmatrix} CJ_1 \\ CJ_2 \\ \vdots \\ CJ_{2^{n+1}} \end{bmatrix} = \begin{bmatrix} 1 & -j & j & -1 \\ 1 & j & -j & -1 \end{bmatrix} \otimes [H]_{2^n},$$

where the complementary Jacket sequence with length 2^{n+2} is given by

$$\{CJ_i, CJ_j\},\ 1 \le i \le 2^n, 2^n+1 \le j \le 2^{n+1} \text{ and } j = i + 2^n. \tag{16}$$

thus we write the complementary codes as

$$\begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_{2^k \times 2^{n+1}} \end{bmatrix} = [H]_{2^k} \otimes \begin{bmatrix} CJ_1 \\ CJ_2 \\ \vdots \\ CJ_{2^{n+1}} \end{bmatrix}, \tag{17}$$

where one complementary sequence $C_1=\{CJ_i, CJ_j\}$, $1 \le i$, $j \le 2^{n+1}$ should be selected to construct the complementary codes for $(2^k-1)\times 2^n+1$ users with length $2^k \times 2^{n+2}$ as the form as

$$[C_1 \quad C_2 \quad \cdots \quad C_{(2^k-1)\times 2^n} \quad C_{(2^k-1)\times 2^n+1}] \tag{18}$$

where the other $(l\times t+1)$th user $C_{l\times t}=\{S_{2^{n+1}+l\times i}, S_{2^{n+1}+l\times j}\}$ $1 \le i \le 2^n$, $2^n+1 \le j \le 2^{n+1}$, and $j=i+2^n$, $1 \le l \le 2^k-1$, $1 \le t \le 2^n$. The simulations results demonstrate that the interuser interferences can be perfectly rejected by using these two proposed users design criterions as shown in Fig. 2. Unlike the traditional complementary codes[4], the proposed complementary quadriphase Jacket sequences can be easily generated by using the fast Jacket transform[2],[3] and simple index mapping for 2^k and $(2^k-1)\times 2^n+1$ users, as shown in Table 1. The complementary Jacket sequences have perfect zero crosscorrelations and perfect interuser correlations to combat interferences from CDMA systems as the same as the conventional complementary codes. As shown in Fig. 3, the numerical results demonstrate that the proposed CDMA system has about 0.5 dB improved from ZCZ codes for multiuser CDMA system[1], especially for low SNR.

Table 1. Compaing of the Jacket complementary codes and conventional designs.

	Conventional complementary code[4]	Jacket complementary code
BER 10^{-4} (Performance)	2.3 dB	2.3 dB
Size of users	Even 2^k	Even and Odd 2^k or $(2^k-1)\times 2^n+1$

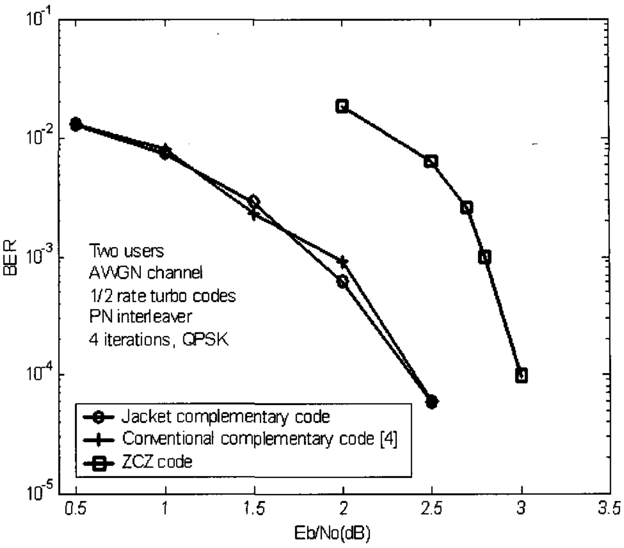


Fig. 3. Performances of the complementary Jacket sequences for QPSK modulations.

Ⅳ. Conclusion

In this paper, a complementary quadriphase sequence based on Jacket matrix was introduced. It can offer a perfect zero crosscorrelations and good auto correlations to combat interferences in multi-user CDMA systems. Also, based on the users design criterions in this paper, the interuser interferences can be perfectly canceled. In the proposed CDMA system, a large size of users can be applied, which includes 2^k and $(2^k-1)\times 2^n+1$. The performances demonstrate that the complementary Jacket sequences have potential capacity to be applied for CDMA mobile communications.

This work was supported by The International Cooperation Research Program of the Ministry of Science & Technology, KOTEF and Chonbuk National University.

References

[1] P. Z. Fan, N. Suehiro, N. Kuroyanagi, and X. M. Deng, "Class of binary sequences with zero correlation zone", *Elect. Letters*, vol. 35, no. 10, pp. 777-779, May 1999.

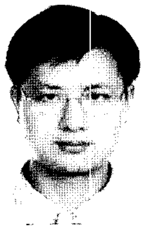
[2] M. H. Lee, "A new reverse jacket transform and its fast algorithm", *IEEE Trans. on Circuit and System*, vol. 47, no. 1, Jan. 2000.

[3] M. H. Lee, B. S. Rajan, and J. Y. Park, "A generalized reverse jacket transform", *IEEE Trans. on*

Circuits System, vol. 48, no. 7, Jul. 2001.

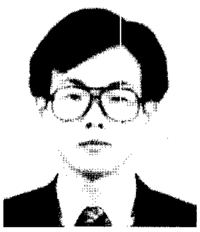
[4] H. H. Chen, J. F. Yeh, and Suehiro, N., "A multicarrier CDMA architecture based on orthogonal complementary codes for new generations of wideband wireless communications", *IEEE Communications Magazine*, vol. 39, no. 10, pp. 126-135, Oct. 2001.

Jia Hou



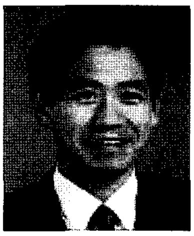
received the M.S. and Ph.D. degrees in electronic & information engineering from Chonbuk National University in 2003, 2005 respectively. He is currently an associate professor in Soochow University. His research interests include the areas of mobile communications, space-time coding and channel coding.

Kwang-Jae Lee



received the B.S. and M.S. degrees in electronic engineering from Chonbuk National University in 1986, 1990 respectively. He is currently a full time lecturer in Hanlyo University. His research interests include the areas of mobile communications, RFID, powerline communications, and channel coding.

Moon-Ho Lee



received the B.S. and M.S. degree both in electrical engineering from the Chonbuk National University, Korea, in 1967 and 1976, respectively, and the Ph.D. degree in electronics engineering from the Chonnam National University in 1984 and the University of Tokyo, Japan, in 1990. From 1970 to 1980, he was a chief engineer with Namyang Moonhwa Broadcasting. Since 1980, he has been a professor with the department of information and communication and a director with the Institute of Information and Communication, both at Chonbuk National University. From 1985 to 1986, he was also with the University of Minnesota, as a Postdoctoral Feller. He has held visiting positions with the University of Hannover, Germany, during 1990, the University of Aachen, Germany, during 1992 and 1996, and the University of Munich, Germany, during 1998. His research interests include the multidimensional source and channel coding, mobile communication, and image processing.