# 얼굴 인식을 위한 2D DLDA 알고리즘

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# 2D Direct LDA Algorithm for Face Recognition

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## 요 약

본 논문에서는 얼굴 인식을 위한 새로운 저차원 특징 표현 기법을 제안하였다. 선형판별기법(LDA)는 인기있는 특징추출 기법이다. 하지만 고차원 데이터의 경우에 계산적인 복잡도가 높고 샘플의 개수가 적은 경우 역행렬을 구할 수 없는 특이행렬문제에 직면한다. 이러한 문제들을 해결하기 위해 일반적인 선형판별기법과 다르게 우리는 이차원 이미지 공분산 행렬을 구한 다음 직접선형판별기법(dirct LDA)을 적용하였으며 이것을 2D-DLDA라고 부른다. ORL 얼굴데이터베이스를 사용하여 실험한 결과 기존의 직접선형판별기법보다 성능이 우수함을 확인하였다.

**Key Words:** PCA, LDA, DLDA, 2D-DLDA, face recognition

## ABSTRACT

A new low dimensional feature representation technique is presented in this paper. Linear discriminant analysis is a popular feature extraction method. However, in the case of high dimensional data, the computational difficulty and the small sample size problem are often encountered. In order to solve these problems, we propose two dimensional direct LDA algorithm, which directly extracts the image scatter matrix from 2D image and uses Direct LDA algorithm for face recognition. The ORL face database is used to evaluate the performance of the proposed method. The experimental results indicate that the performance of the proposed method is superior to DLDA.

## Ⅰ. Introduction

Over the past 20 years, face recognition (FR) has been an active research. Various methods have been proposed for FR[1]. Especially, the appearance-based methods have been successfully employed. Principal Component Analysis (PCA) and Linear Discriminant Analysis (LDA) are well known methods among them. PCA seeks directions that have the largest variance associated with it. On the other hand, LDA seeks directions that are efficient for discrimination between classes. Turk and Pentland presented the Eigenfaces method for face recognition[2]. Since then, PCA based methods have been developed. Recently, Yang used Kernel PCA for FR[3]. Although the Kernel PCA provides better performance, it requries mor computational complexity than PCA's. Yang et al. proposed 2DPCA[4]. While previous methods use 1D image vector, 2DPCA makes directly the scatter matrix from 2D image matrices. 2DPCA deals with the small size scatter matrix

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논문번호 : KICS2005-06-237, 접수일자 : 2005년 6월 8일

than traditional PCA-based methods and evaluates the scatter matrix accurately. For example, an image vector of 112×92 forms 10304 dimensional vector and the size of the scatter matrix is 10304×10304. On the other hand, the covariance of 2DPCA forms only 92×92 matrix. To avoid computational problem to get the eigenvectors, traditional PCA-based methods use SVD techniques. However, the eigenvectors which are acquired in this way may not be evaluated accurately since the eigenvectors are statistically determined by the scatter matrix. Also, 2DPCA is more suitable for small sample size problems because its scatter matrix is small. The demerit of 2DPDA is that it requires more coefficients for image representation than PCA. It needs more storage and more time for recognition.

Belhumeur et al. proposed Fisherfaces method based on LDA[5]. In general, it is believed that LDA-based pattern classification methods outperform PCA-based ones. However, Traditional LDA has small sample size (SSS) problem. Also, it is difficult to directly apply to high dimensional matrix because of computational complexity. To solve the problem, Belhumer et al proposed dimensionality reduction using PCA before LDA. A potential problem is that PCA step may discard dimensions that contain important discriminative information. Chen et al. have proved that the null space of within-class scatter matrix contains the most discriminative information[6]. In reality, PCA discards the null space of the within-class scatter matrix. To prevent the null space from discarding, Yu et al. proposed Direct LDA (DLDA) method[7]. DLDA directly processes data in the original high dimensional vectors. By simultaneous diagonalization, DLDA is able to discard the null space of between-class scatter matrix and to keep the null space of within-class scatter matrix, which contains very important discriminative information. But DLDA still uses SVD technique to obtain eigenvectors of scatter matrix. This does not imply that the eigenvectors can be evaluated accurately.

In this paper, we introduce a new low dimensional feature representation method, called two dimensional direct linear discriminant analysis (2D-DLDA). The method combines the merits of the image scatter matrix like 2DPCA and DLDA approaches. The image scatter matrix reduces the chance of singularity caused by SSS problem. Furthermore, our method does not use SVD technique to get eigenvectors of the image scatter matrix. It can evaluate the image scatter matrix accurately. And then DLDA method is used for obtaining the feature matrix. It maximizes Fisher's criterion.

The remainder of this paper is organized as follows. In Section 2, the proposed 2D-DLDA algorithm is described. Experimental results and comparisons with DLDA are presented in Section 3. Finally, conclusions are offered in Section.

## Ⅱ. 2D Direct LDA

Let $X$ denotes a m by n image, and $W$ is an n dimensional column vector. $X$ is projected onto $W$ by the following linear transformation

$$Y = XW \tag{1}$$

Thus, we get an m dimensional projected vector $Y$, called the feature vector of the image $X$. Suppose there are $C$ known pattern classes in the training set, and $M$ denotes the size of the training set. The jth training image is denoted by a m by n matrix $X_j$ ($j$=1, 2, ..., $M$), and the mean image of all training sample is denoted by $\overline{X}$ and $\overline{X}_i$($i$=1, 2, ..., $C$) denoted the mean image of class $T_i$ and $N_i$ is the number of samples in class $T_i$, the projected class is $P_i$. After the projection of training image onto $W$, we get the projected feature vector

$$Y_j = X_j W, \quad (j=1,\ 2,\ \ldots,\ M) \tag{2}$$

LDA attempts to seek a set of optimal discrimination vectors to form a transform $W$ by maximizing the Fisher criterion denoted as

$$J(W) = \frac{tr(\widetilde{S}_b)}{tr(\widetilde{S}w)} \quad (3)$$

where tr( ) denotes the trace of matrix, $\widetilde{S}_b$ denotes the between class scatter matrix of projected feature vectors of training images, and $\widetilde{S}_w$ denotes the within class scatter matrix of projected feature vectors of training images. So,

$$\begin{aligned}\widetilde{S}_b &= \sum_{i=1}^{C} N_i(\overline{Y_i} - \overline{Y})(\overline{Y_i} - \overline{Y})^T \\ &= \sum_{i=1}^{C} N_i[(\overline{X_i} - \overline{X})W][(\overline{X_i} - \overline{X})W]^T \quad (4)\end{aligned}$$

$$\begin{aligned}\widetilde{S}_w &= \sum_{i=1}^{C} \sum_{Y_k \in P_i} (Y_k - \overline{Y_i})(Y_k - \overline{Y_i})^T \\ &= \sum_{i=1}^{C} \sum_{X_k \in T_i} [(X_k - \overline{X_i})W][(X_k - \overline{X_i})W]^T (5)\end{aligned}$$

So,

$$tr(\widetilde{S}_b) = W(\sum_{i=1}^{C} N_i(\overline{X_i} - \overline{X})^T(\overline{X_i} - \overline{X}))W \quad (6)$$

$$tr(\widetilde{S}_w) = W(\sum_{i=1}^{C} \sum_{X_k \in T_i} (X_k - \overline{X_i})^T(X_k - \overline{X_i}))W \quad (7)$$

Let us define the following matrix

$$G_b = \sum_{i=1}^{C} N_i(\overline{X_i} - \overline{X})^T(\overline{X_i} - \overline{X}) \quad (8)$$

$$G_w = \sum_{i=1}^{C} \sum_{X_k \in T_i} (X_k - \overline{X_i})^T(X_k - \overline{X_i}) \quad (9)$$

The matrix $G_b$ is called the image between class scatter matrix and $G_w$ is called the image within class scatter matrix.

Alternatively, the criterion can be expressed by

$$J(W) = \frac{W^T G_b W}{W^T G_w W} \quad (10)$$

Now, we try to find a matrix that simul- taneously diagonalizes both $G_b$ and $G_w$.

$$AG_w A^T = I,\ AG_b A^T = \Lambda \quad (11)$$

Where $\Lambda$ is a diagonal matrix with diagonal elements sorted in decreasing order. First, we find eigenvectors $V$ that diagonalizes $G_b$.

$$V^T G_b V = \Lambda \quad (12)$$

Where $V^T V = I$. $\Lambda$ is a diagonal matrix sorted in decreasing order, i.e. each column of $V$ is an eigenvector of $G_b$ and $\Lambda$ contains all the eigenvalues.

Let $Y$ be the first m columns of $V$ (n by n matrix, n being the column numbers of image). In general, $G_b$ is not singular in contrast with conventional DLDA. So m is equal to n. Now

$$Y^T G_b Y = D_b \quad (13)$$

Where $D_b$ is the m by m principal sub-matrix of $\Lambda$. Further let $Z = YD_b^{-1/2}$ to unitize $G_b$.

$$(YD_b^{-1/2})^T G_b (YD_b^{-1/2}) = I,\ Z^T G_b Z = I \quad (14)$$

Next, we find eigenvectors $U$ to diagonalize $Z^T G_w Z$.

$$U^T Z^T G_w Z U = D_w \quad (15)$$

Where $U^T U = I$. $D_w$ may contain zeros in its diagonal.

To maximize $J(W)$, we can sort the diagonal elements of $D_w$ and discard some high eigenvalues with the corresponding eigenvectors.

Let the optimal projection matrix, $W$

$$W = (D_w^{-1/2} U^T Z^T)^T \quad (16)$$

The low dimensional transformed $X^*$ is $X^* = (X - \overline{X})W$.

The Frobenius norm is used for classification. The distance between $X_1^*$ and $X_2^*$ is defined by

$$D_F(X_1^*, X_2^*) = \| X_1^* - X_2^* \|_F \quad (17)$$

## Ⅲ. Experimental Results & Observation

The proposed method is tested on the ORL face image database. The ORL database consists of 40 distinct persons. There are 10 images per person. The images are taken at different times and contain various facial expressions (open/closed eyes, smiling/not smiling) and facial details (glasses or no glasses). The size of image is 92 by 112 pixels with 256 gray levels. For the FR experiments, first five images are chosen for training from each person and the other five images are used for testing. Thus the total number of training images and testing images are both 200.

In the proposed mehtod, the size of image scatter matrix $G_b$ and $G_w$ are both 92 by 92. In the 112 by 92 image matrix, the best result is 93.5%. when the images is down-sampled to 28 by 23 matrix to reduce the computational complexity, the size of image scatter matrix is 23 by 23 and the best result is 94.5%.

To evaluate performance of 2D_DLDA, it is compared with DLDA. Table 1 presents a comparison of performance of the two algorithms for different image matrix sizes. The experimental results show that 2D-DLDA is more efficient than DLDA in terms of recognition rate.

In 2D-DLDA, Image scatter matrix is smaller than scatter matrix of DLDA so that we can avoid computational complexity of feature extraction. But 2D image based methods have a weak point. The extracted feature matrix of 2D-DLDA is larger than DLDA. For instance, the extracted feature matrix forms 112 by 87 to get the best performance when the size of image matrix is 112 by 92.

Therefore it needs the dimensional reduction of image. Table 1 shows that the dimensional reduction has little influence on the performance.

Table 1. The Comparison of recognition rate

| Image matrix size | DLDA | 2D-DLDA |
|---|---|---|
| 112 x 92 | 87.5% | 93.5% |
| 56 x 46 | 86.0% | 93.0% |
| 39 x 30 | 86.5% | 92.5% |
| 28 x 23 | 85.5% | 94.5% |

## Ⅳ. Conclusion

In this paper, 2D-DLDA algorithm is proposed. The method combines the merits of the image scatter matrix and DLDA approaches. Since the size of the image scatter matrix is smaller than the conventional method, SSS problem can be avoided and eigenvectors can be efficiently computed. Furthermore it achieves the better performance by using DLDA algorithm since DLDA preserves the null space of within class scatter matrix, which contains very important discriminative information and the experimental results show that the dimensional reduction has little influence on the recognition rate. In future, we intend to develop dimensional reduction method.

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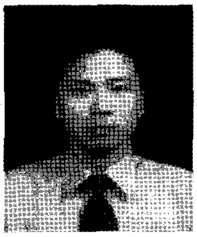

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