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A TEST FOR AUTOCORRELATION IN DYNAMIC PANEL DATA MODELS

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ABSTRACT

This paper presents an autocorrelation test that is applicable to dynamic panel data models with serially correlated errors. The residual-based GMM t-test is a significance test that is applied after estimating a dynamic model by using the instrumental variable (IV) method and is directly applicable to any other consistently estimated residuals. Monte Carlo simulations show that the t-test has considerably more power than the m_2 test or the Sargan test under both forms of serial correlation (*i.e.*, AR(1) and MA(1)).

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1. INTRODUCTION

The main purpose of this paper is to propose a test of serial correlation for dynamic panel models and to compare this test with the m_2 and Sargan tests proposed by Arellano and Bond (1991) (hereafter AB). If the disturbance has an AR(1) structure, the usual approach of using lagged values of the dependent variables as instruments in the differenced equations, applied by, for example, Anderson and Hsiao (1981, 1982) and Arellano and Bover (1995), is no longer valid. Furthermore, an estimator that uses lags as instruments under the assumption of white noise errors is inconsistent if the disturbances are autocorrelated. Thus, the m_2 and Sargan tests are inapplicable because they use inconsistently estimated residuals based on standard first-difference GMM estimation (hereafter GMM), which also uses invalid instruments. To solve this problem, the t-test utilizes consistently estimated residuals based on IV estimation that uses lags of exogenous variables as instruments for the lagged dependent variables.

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The remainder of this paper is organized as follows. In the next section, we present the model and describe the performance of the m_2 and Sargan tests when the disturbances follow an AR(1) process. In Section 3, we propose a t-test for first-order serial correlation and show that the t-test is applicable to both forms of serial correlation (*i.e.*, AR(1) or MA(1)). In Section 4, we present the simulation results.

2. MODELS OF THE TWO AUTOCORRELATION TESTS PROPOSED BY AB (1991)

We consider a simple autoregressive panel data model with $(k-1)$ strictly exogenous variables x_{it}. (*e.g.*, Nerlove, 1971a; Baltagi and Li, 1995).

$$\begin{aligned} y_{it} &= \delta y_{i,t-1} + \beta' x_{it} + u_{it} = \alpha' X_{it} + u_{it} \\ u_{it} &= \mu_i + v_{it} \qquad \text{for} \quad (i = 1, \cdots, N; \quad t = 2, \cdots, T) \end{aligned} \tag{2.1}$$

where α is the $(k \times 1)$ coefficient vector given $|\delta| < 1$. We assume that the unobserved individual-specific effects μ_i and the classical error term v_{it} have a one-way error component structure in which

$$E(\mu_i) = E(v_{it}) = E(\mu_i v_{it}) = 0 \qquad \forall \quad i, t \tag{2.2}$$

where $\mu_i \sim \text{NID}(0, \sigma_\mu^2)$. Adopting the standard assumption that the classical error term, v_{it}, is a white noise error process, AB (1991) noted the validity of the following $p = (T-1)(T-2)/2$ linear moment restrictions for the dynamic model (2.1) given by

$$E\left[(\Delta y_{it} - \delta \Delta y_{i,t-1})\, y_{i,t-j}\right] = 0 \quad \text{for} \quad (j = 2, \cdots, t-1; \quad t = 3, \cdots, T) \tag{2.3}$$

where $\Delta y_{it} = y_{it} - y_{i,t-1}$. However, if the standard assumption of a white noise error for v_{it} is violated, these orthogonality conditions no longer hold. Hence, values of y lagged two periods or more cannot be used as instruments for $\Delta y_{i,t-1}$. Consider two alternative cases of serially correlated disturbances. First, consider the case of AR(1) stationary disturbances in the classical error term, v_{it}:

$$v_{it} = \rho v_{i,t-1} + \epsilon_{it} \qquad 0 < \rho < 1 \tag{2.4}$$

Second, consider an invertible MA(1) disturbance:

$$v_{it} = \epsilon_{it} + \theta \epsilon_{i,t-1} \qquad 0 < \theta < 1 \tag{2.5}$$

where the innovations ϵ_{it} are independent over time and individuals. Since violation of the orthogonality conditions is expected to affect the m_2 and Sargan tests, it is worth considering how these statistics behave when the error follows an AR(1) process.

For simplicity, the first-difference equation for the model (2.1) is now

$$\Delta y = \alpha' \Delta X + \Delta u \tag{2.6}$$

where y is a $(N(T-2)\times 1)$ vector of observations y_{it} and X is a stacked $(N(T-2)\times k)$ matrix of observations on $(y_{i,t-1}, x_{it})$. If we choose Z_i, a $(T-2)\times(p+1)$ block diagonal matrix given by $Z_i = [diag(y_{i1}, \cdots, y_{is}) : (\Delta x_{i3}, \cdots, \Delta x_{iT})']$ where $s = (1, \cdots, T-2)$ and $Z = (Z_1', \cdots, Z_N')$, the AB's first-step GMM estimator of α is

$$\hat{\alpha} = \left[(\Delta X)' Z (Z'(I_N \otimes G)Z)^{-1} Z'(\Delta X)\right]^{-1} \left[(\Delta X)' Z (Z'(I_N \otimes G)Z)^{-1} Z'(\Delta y)\right] \tag{2.7}$$

where G is a $(T-2)$-dimensional square matrix with twos in the main diagonal, minus ones in the first subdiagonals and 0s otherwise. Then, the $N(T-2)\times 1$ vector of residual is given by

$$\Delta\hat{u} = \Delta y - \hat{\alpha}'\Delta X = \Delta u - (\hat{\alpha}-\alpha)'\Delta X = \Delta\hat{v} \tag{2.8}$$

As the consistency of the GMM estimator relies on $E[\Delta u_{it}\Delta u_{i,t-2}] = 0$, a test of the hypothesis that there is no second-order serial correlation in the disturbances of the first-differenced equation is derived by AB(1991).[1]

$$m_2 = \frac{\Delta\hat{u}_{-2}'\Delta\hat{u}_*}{\tilde{u}^{1/2}} \sim \quad \mathrm{N}(0,1) \tag{2.9}$$

To focus only on the effect of the AR(1) serial correlation on the numerator of (2.9) under $H_1 : 0 < \rho < 1$, we obtain

$$\begin{aligned}
E[\Delta u_{it}'\Delta u_{i,t-2}] &= E[(u_{it} - u_{i,t-1})'(u_{i,t-2} - u_{i,t-3})] \\
&= E[(v_{it} - v_{i,t-1})'(v_{i,t-2} - v_{i,t-3})] \\
&= 2\gamma_2 - \gamma_1 - \gamma_3 = \frac{\sigma_\epsilon^2}{\rho^2 - 1}\left[\rho^2 - 2\rho + 1\right]\rho \\
&= \frac{\rho(\rho-1)}{\rho+1}\sigma_\epsilon^2 \neq 0
\end{aligned} \tag{2.10}$$

[1] $\Delta\hat{u}_{-2}$ is the vector of lagged $\Delta\hat{u}$ twice and $\Delta\hat{u}_*$ is a vector of trimmed $\Delta\hat{u}$ to match $\Delta\hat{u}_{-2}$. As $\tilde{u}$ in (2.9) is unrelated to our results, it is not defined explicitly here. See Appendix A in AB(1991) for the complete definition.

where $\gamma_h = E[v_{it}v_{i,t-h}]$, which is an autocovariance function of v_{it} for a fixed i.[2]The above equation reveals that the usual standard-normal asymptotic result in (2.9) cannot be useful and that the power of the test depends on ρ if the error follows an AR(1) process. The invalidity of the orthogonality condition also affects the power of the Sargan test. Using $\Delta\hat{u} = \Delta\hat{v}$ in (2.8), the Sargan test follows:

$$S = \Delta\hat{v}'Z \left(\sum_{i-1}^{N} Z_i'\Delta\hat{v}_i\Delta\hat{v}_i'Z_i \right)^{-1} Z'\Delta\hat{v} \sim \chi^2_{p-k} \tag{2.11}$$

The effect of the autocorrelated errors on $\Delta\hat{v}'Z$ in S can be succinctly expressed as

$$E\left[\Delta\hat{v}_{it}'y_{i,t-s}\right]^2 \cong [\rho(\rho-1)]^2, \cdots, [\rho(\rho-1)]^{2(t-1)} \quad \text{for} \quad s = 2, \cdots, t-1 \tag{2.12}$$

Since the m_2 and Sargan test statistics contain the functions of $[\rho(\rho-1)]$ and $[\rho(\rho-1)]^2$ respectively, the power of these tests is expected to decrease as ρ approaches unity under the AR(1) alternative.

PROPOSITION 2.1. *The power of the m_2 and Sargan tests is maximized at $\rho = 0.5$ and approximately $\rho = 0.7$, respectively, under the AR(1) alternative. Consequently, for these tests, the probabilities of Type II errors increase as ρ approaches unity, which suggests misspecification.*

3. A RESIDUAL-BASED GMM t-TEST

3.1. The AR(1) Case

The poor performance of the two standard tests, the m_2 and the Sargan test, in the presence of AR(1) disturbances motivates the discussion in this section. Whether the unobserved disturbances v_{it} follow an AR(1) or an MA(1) process, we are able to obtain the consistent IV estimator $\hat{\alpha}_{IV}$ and the consistently estimated residuals $\hat{u}_{it}$, by using lagged exogeneous variable $x_{i,t-1}$ for the instruments of $y_{i,t-1}$. We consider adopting GMM estimation to the first-differenced residuals, $\Delta\hat{u}_{it}$, to test whether the coefficient ρ is significantly different from 0. On the

[2]This equation equals 0 if the errors in the model in levels are not autocorrelated or if they follow a random walk.

other hand, the simple relationship between $\Delta v_{it} = \Delta u_{it}$ is essential for deriving our t-test. In the case of an AR(1), as in (2.4), the first-differenced equation is

$$\Delta v_{it} = \rho \Delta v_{i,t-1} + \Delta \epsilon_{it} \tag{3.1}$$

or equvalently

$$\Delta u_{it} = \rho \Delta u_{i,t-1} + \Delta \epsilon_{it}. \tag{3.2}$$

Consequently, as the latter equation (3.2) has the same AR(1) dynamic random-effects specification, we are able to test $H_0 : \rho = 0$ after obtaining the GMM estimator, $\hat{\rho}$, and its t-value, $t_{\hat{\rho}}$. Thus, the significance test for ρ in (3.2) can be an autocorrelation test on the classical error term in (2.1). To perform this test, Δu_{it} in (3.2) is replaced with the estimated first-differenced residual, $\Delta \hat{u}_{it}$. Using (2.8) and (3.2), we obtain

$$\begin{aligned} \Delta \hat{u}_{it} &= \rho \Delta \hat{u}_{i,t-1} + \Delta \epsilon_{it} - (\hat{\alpha}_{IV} - \alpha)'(\Delta X_{it} - \rho \Delta X_{i,t-1}) \\ &= \rho \Delta \hat{u}_{i,t-1} + \Delta \eta_{it} \qquad \text{say} \end{aligned} \tag{3.3}$$

For $T \geq 3$, this new derived AR(1) dynamic model implies that the linear moment restrictions in vector form, $E[W'_{ui} \Delta \eta_i] = \mathbf{0}$, are satisfied where $\eta_i = (\eta_{i3} \cdots \eta_{iT})'$ and

$$W_{ui} = \begin{bmatrix} [\hat{u}_{i1}] & & & \mathbf{0} \\ & [\hat{u}_{i1}, \hat{u}_{i2}] & & \\ & & \ddots & \\ \mathbf{0} & & & [\hat{u}_{i1}, \cdots, \hat{u}_{i,T-2}] \end{bmatrix} \tag{3.4}$$

The one-step GMM estimator, based on the sample moments $N^{-1} \sum_{i=1}^{N} W'_{ui} \Delta \eta_i$ is obtained by $\hat{\rho} = \text{argmin}_{\rho} (\Delta \eta' W_u) V_N (W'_u \Delta \eta)$, where $\Delta \eta = (\Delta \eta'_1, \cdots, \Delta \eta'_N)$ and $W_u = (W'_{u1}, \cdots, W'_{uN})$. For V_N, we use $(N^{-1} \sum_{i=1}^{N} W'_{ui} G W_{ui})^{-1}$.

PROPOSITION 3.1. *Under the null of $H_0 : \rho = 0$*

$$t_{\hat{\rho}} = \hat{\sigma}_{\eta} ([\Delta \hat{u}_{-1}]' W_u V_N^{-1} W'_u [\Delta \hat{u}_{-1}])^{-\frac{1}{2}} ([\Delta \hat{u}_{-1}]' W_u V_N^{-1} W'_u \Delta \hat{u}) \sim \mathrm{N}(0,1) \tag{3.5}$$

The proof of asymptotic normality is quite straightforward and is therefore not presented. However, it is worth noting that, unlike the m_2 test, the t-test does not rely on the efficiency of the first-step estimator; *i.e.*, $\hat{\alpha}$. Although '$\widehat{\text{avar}}(\hat{\alpha} - \alpha)$' appears in the estimator of σ_{η} in (3.5), it disappears as $N \to \infty$ because $\sqrt{N}(\hat{\alpha} - \alpha) = O_p(1)$.

3.2. The MA(1) Case

In the previous section, we derived a t-test based on the residuals from IV estimation. In this section, we show that the t-test is valid even if the classical error term in the true disturbances follows an MA(1) process as in (2.5). As is conventional, we use the m_2 test or the Sargan test to detect any serial correlation in the error term. However, the MA(1) error can be converted to an AR(1) error to apply our t-test, as follows:

$$\begin{aligned} \Delta v_{it} &= \Delta\epsilon_{it} + \theta\Delta\epsilon_{i,t-1} \\ &= \theta\Delta v_{i,t-1} + \sum_{j=2}^{\infty} -(-\theta)^j \Delta v_{i,t-j} + \Delta\epsilon_{it} \qquad (3.6) \\ &= \theta\Delta v_{i,t-1} + \Delta\zeta_{it} \qquad (3.7) \end{aligned}$$

where $\Delta\zeta_{it} = \sum_{j=2}^{\infty} -(-\theta)^j \Delta v_{i,t-j} + \Delta\epsilon_{it}$. This equation is similar to the first-differenced AR(1) specification in (3.1). [3] In the MA(1) case, the autocorrelation test case is again a significance test on θ. Hence, the t-test can be applied after (3.7) has been estimated by GMM to test whether θ is significantly different from 0.

PROPOSITION 3.2. *The residual-based GMM t-test is applicable to both forms of serial correlation, AR(1) and MA(1). Hence, under the null of $H_0 : \theta = 0$, $t_{\hat{\theta}} \sim$ N(0.1).*

However, a shortcoming of the test is that it may not be possible to distinguish between the AR(1) and MA(1) structures if the null hypothesis that $\rho = 0$ is rejected. In this case, we suggest a different testing strategy. First, use the t-test to determine whether serial correlation is present. If it is, apply the m_2 or the Sargan test to determine whether the error follows an MA(1) process. If it does not, conclude that the error term has an AR(1) structure. This two-step testing procedure can detect any first-order serial correlation structure in the error term of a dynamic panel data model.

4. MONTE CARLO EXPERIMENTS

To investigate how the three tests, the m_2 test, the Sargan test and the t-test, perform in practice, Monte Carlo simulations were conducted under the

[3] The correlation between $v_{i,t-j}$ and ζ_{it} becomes negligible as j increases.

TABLE 4.1 *Size and power of the three tests (AR(1) error)*

	$\delta = 0.3$			0.5			0.7			0.9		
ρ	m_2	S	t	m_2	S	t	m_2	S	t	m_2	S	t
0.0	0.07	0.01	0.06	0.02	0.01	0.05	0.02	0.01	0.06	0.02	0.03	0.04
0.1	0.19	0.05	0.22	0.14	0.05	0.30	0.13	0.05	0.46	0.18	0.03	0.17
0.2	0.31	0.06	0.67	0.28	0.11	0.77	0.28	0.12	0.84	0.25	0.09	0.68
0.3	0.42	0.31	0.95	0.36	0.16	0.95	0.35	0.18	0.93	0.31	0.20	0.84
0.4	0.43	0.31	0.99	0.53	0.36	0.99	0.52	0.36	0.99	0.46	0.39	0.99
0.5	0.52	0.48	1.00	0.59	0.47	1.00	0.58	0.48	1.00	0.61	0.49	1.00
0.6	0.50	0.54	1.00	0.54	0.61	1.00	0.53	0.62	1.00	0.44	0.44	1.00
0.7	0.48	0.65	1.00	0.38	0.60	1.00	0.37	0.60	1.00	0.36	0.32	1.00
0.8	0.39	0.57	1.00	0.34	0.56	1.00	0.35	0.57	1.00	0.21	0.13	1.00
0.9	0.11	0.40	1.00	0.12	0.35	1.00	0.12	0.29	1.00	0.07	0.02	1.00

NOTE :$T = 7, N = 100$. *S and t denote the Sargan test and the t-test, respectively. Size-corrected powers in the AR(1) case for* $T = 7$ *and* $T = 11$ *are available from the author on request. The results remained unchanged.*

null hypotheses, $\rho = 0$ and $\theta = 0$. Following Nerlove (1971a) and Sevestre and Tronogon (1991), we assume that the data-generating process is given by

$$\begin{aligned} y_{it} &= \delta y_{i,t-1} + \beta x_{it} + u_{it} \\ x_{it} &= \phi x_{i,t-1} + \omega_{it} \qquad \omega_{it} \sim \mathrm{U}(-1/2, 1/2) \\ u_{it} &= \mu_i + v_{it} \qquad \mu_i \sim N(0,1) \end{aligned}$$

The classical error term, v_{it}, is generated either by the AR(1) process (2.4) or by the MA(1) process in (2.5). For x_{i1}, we used ω_{i1}, and for y_{i1}, we generate $\beta x_{i1}/(1-\delta) + \mu_i/(1-\delta) + v_{i1}/(\sqrt{1-\delta^2})$. The testing procedures were repeated five thousand times for each set of parameter values. The parameter δ takes the values $0.3, 0.5, 0.7$ and 0.9 while $\beta = 2, \phi = 0.4$ remain fixed. The parameters in the error process, ρ or θ, take the values $0, 0.1, \cdots 0.9$.

First, the three tests were applied to AR(1) errors. Table 4.1 shows the size and power of the three test statistics when there is an AR(1) error process . The empirical sizes of the m_2 test, the Sargan test and the t-test are reported in the first row for $\rho = 0$. The tests have reasonable size properties except that the Sargan test rarely rejects the null. Theoretically, the m_2 test and the Sargan test are maximized at around $\rho = 0.5$ and $\rho = 0.7$, respectively. This makes the conventional autocorrelation test difficult to apply as ρ approaches unity because of the increased likelihood of a Type II error. The bias in two tests implies that the presence of serially correlated errors invalidates the use of lagged values of y

TABLE 4.2 *Size and power of the three tests (MA(1) error)*

	$\delta = 0.3$			0.5			0.7			0.9		
θ	m_2	S	t	m_2	S	t	m_2	S	t	m_2	S	t
0.0	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.1	0.13	0.06	0.26	0.15	0.08	0.29	0.17	0.06	0.35	0.19	0.05	0.39
0.2	0.35	0.11	0.67	0.42	0.16	0.78	0.53	0.14	0.85	0.48	0.05	0.87
0.3	0.73	0.29	0.94	0.83	0.49	0.98	0.88	0.30	0.99	0.76	0.06	0.99
0.4	0.97	0.63	0.99	0.99	0.84	1.00	0.98	0.43	1.00	0.88	0.06	1.00
0.5	1.00	0.91	1.00	1.00	0.99	1.00	0.99	0.52	1.00	0.90	0.07	1.00
0.6	1.00	0.99	1.00	1.00	1.00	1.00	1.00	0.59	1.00	0.87	0.06	1.00
0.7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.67	1.00	0.82	0.07	0.99
0.8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.74	1.00	0.77	0.07	0.99
0.9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.78	1.00	0.73	0.07	0.99

NOTE :*Sizes were corrected previously.*

as instruments. If follows that, both the standard GMM estimation and two tests are biased when we use lagged ys as instruments. However, the t-test is unbiased and consistent because it uses consistently estimated residuals from the first-step IV estimation, which does not use lagged ys as instruments.

We also applied the same three tests to MA(1) errors. To apply the t-test, the MA(1) error process is approximated by an AR(1) process. Table 4.2 shows the size and power of the three test statistics. Although there is no maximum value of the power, unlike in the case of the AR(1) alternative, the m_2 and Sargan tests have lower power than the t-test. Note also that the size of the Sargan test becomes distorted as T increases. The use of too many moment conditions dramatically reduces the size and power of the Sargan test. This result confirms previous work by Bowsher (2002). Consequently, the t-test is a useful alternative to the standard m_2 and Sargan tests because of its size and power and its performance when T is large.

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