Journal of the Korean Society of Marine Engineering, Vol.29, No.7, pp.779~784, 2005(ISSN 1226-9549)

Laminar Convective Heat Transfer from a Horizontal Flat Plate of Phase Change Material Slurry Flow

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(Manuscript : Received AUG 5, 2005 ; Revised OCT 28, 2005)

Abstract : This paper presents the theory of similarity transformations applied to the momentum and energy equations for laminar, forced, external boundary layer flow over a horizontal flat plate which leads to a set of non-linear, ordinary differential equations of phase change material slurry(PCM Slurry). The momentum and energy equation set numerically to obtain the non-dimensional velocity and temperature profiles in a laminar boundary layer are solved. The heat transfer characteristics of PCM slurry was numerically investigated with similar method. It is clarified that the similar solution method of Newtonian fluid can be used reasonably this type of PCM slurry which has low concentration. The data of local wall heat flux and convective heat transfer coefficient of PCM slurry are higher than those of water more than 150~200%, approximately.

Key words : Similarity transformations, Laminar forced external boundary layer, PCM slurry, Local wall heat flux, Local convective heat transfer coefficient

Nomenclature

c_p : Specific heat, J/kgK
f : Similarity function for momentum
h : Convective heat transfer coefficient, W/m^2K
k : Thermal conductivity, W/mK
L_t : Latent heat, kJ/kg
p : Pressure, Pa
Pr : Prandtl number, ν/α
q_s : Local wall heat flux, W/m^2
T : Temperature, K
T_e : Free stream temperature, K
T_m : Melting point, K
T_w : Wall temperature, K
u : x directional velocity, m/s
u_e : Free stream velocity, m/s
v : y directional velocity, m/s
α : Thermal diffusivity, m^2/s
θ : Non-dimensional temperature, $\frac{T-T_w}{T_e-T_w}$
μ : Dynamic viscosity [kg/ms]
ν : Kinematic viscosity, m^2/s
ρ : Density, kg/m^3

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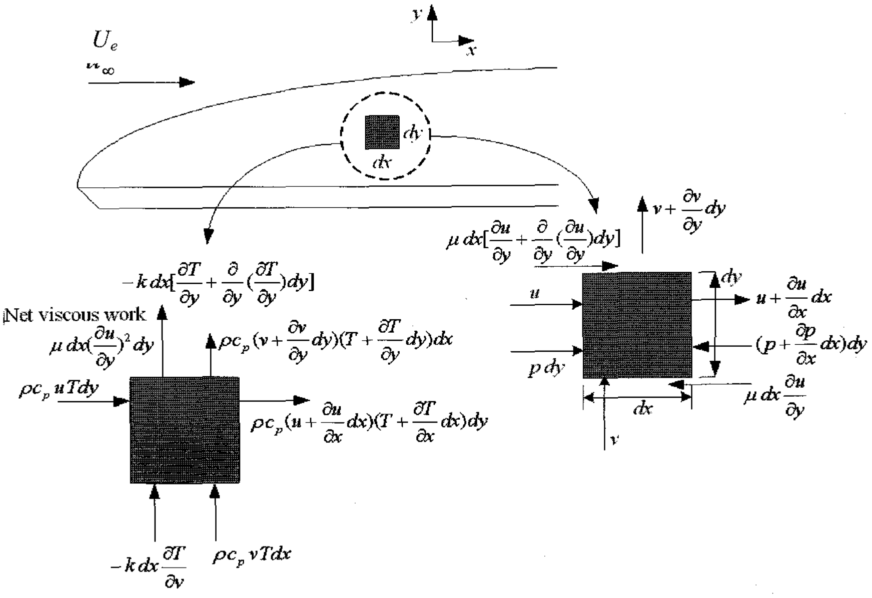

Fig. 1 Velocity and thermal boundary layer

1. Introduction

The concepts and practical usages of phase change material slurry(PCM slurry) have been widely spread as a thermal fluid in recent several decades. Also, the merit of PCM slurry as a thermal fluid was reported[1]. Especially, author's previous papers obtained the measuring results of physical properties of PCM slurry and several predictable equations have been proposed[2], [3].

In the analysis of convective heat transfer, the momentum and energy transfer are mutually connected. So the objective of this study is to confirm the heat transfer mechanism on forced laminar convective boundary layer over a flat plate which leads to a set of non-linear, ordinary differential equations. The momentum and energy equations of this model are transformed using the theory of similarity solutions and these equations are set numerically to obtain the non-dimensional velocity and temperature profiles. Also, the results are arranged as local wall heat flux and convective heat transfer coefficient from the non-dimensional temperature profiles.

2. Mathematical formulation and thermo-physical properties

Fig. 1 shows a thin viscous flow region near the surface of a plate in forced convection. Within this very thin viscous layer the fluid velocity increases from its value at the surface $y=0$(zero velocity for a no slip condition) to the free stream velocity, u. The concept of the boundary layer was first proposed by L. Prandtl at the turn of the 20th century[6].

The continuity and momentum equations for laminar, steady, two-dimension, incompressible, forced convection boundary layer flow are given by equations (1) and (2), respectively. When the first law of thermodynamics is applied to the small control volume within the boundary layer, the differential energy equation (3) is obtained.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

where, the components of velocity in the x and y directions are indicated by u and v, respectively.

$$u\cdot\frac{\partial u}{\partial x}+v\cdot\frac{\partial u}{\partial y}=-\left(\frac{1}{\rho}\right)\cdot\frac{\partial p}{\partial x}+\nu\cdot\frac{\partial^2 u}{\partial y^2} \tag{2}$$

where, the momentum equation is based upon Newton's second law applied to a small boundary layer control volume.

For a flat plate, $\frac{\partial p}{\partial x}=0$. In equation (3), the thermal diffusivity, specific heat and viscosity are treated as constants at melting point of PCM particle(T_m= 278.9K).

$$u\cdot\frac{\partial T}{\partial x}+v\cdot\frac{\partial T}{\partial y}=\alpha\cdot\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho\cdot c_p}\cdot\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The same similarity transformation can be used to change the energy equation into an ordinary differential equation[7]. At the same time, the dependent variable is expressed in terms of a non-dimensional temperature given by $\theta=\frac{T-T_w}{T_e-T_w}$, where T_w is the temperature at the surface(y = 0) and T_e is the constant temperature in the free stream. And the result is equation (4).

$$\theta''+\mathrm{Pr}\cdot f\cdot\theta'=\mathrm{Pr}\cdot Ec\cdot x^{2\cdot m}\cdot f'^2 \tag{4}$$

In equation (4), Ec is the Eckert number defined by $Ec=\frac{u_e^2}{c_p\cdot(T_w-T_e)}$. For an inviscid free stream and flat plate, the exponent $m$is zero. Also, the primes indicate derivative with respect to $\eta=y\cdot\sqrt{\frac{u_e}{\nu\cdot x}}$.

When the flow velocity is low enough to neglect frictional heating(viscous dissipation), Ec = 0 and equation (4) simplifies to equation (5).

$$\theta''+\mathrm{Pr}\cdot f\cdot\theta'=0 \tag{5}$$

Transformed momentum equation for a flat plate from the Blasius equation[8] is derived as equation (6). And the boundary conditions are as follows.

$$f'''+f\cdot f''=0 \tag{6}$$

Table 1 Thermo-physical properties of PCM slurry(at the melting point, T_m)

Properties	Values	Units
Density, ρ	960.000	kg/m^3
Specific heat, c_p	32.809×10^3	J/[kg·K]
Dynamic viscosity, μ	2.941×10^{-3}	kg/[m·s]
Thermal conductivity, k	521.898×10^{-3}	W/[m·K]

$$f(0) = 0,\ f'(0) = 0,\ f'(\infty) = 1$$
$$\theta(0) = 0,\ \theta(\infty) = 1 \qquad (7)$$

The thermo-physical properties of PCM slurry are summarized in Table 1. These values of calculation and experimental results are both derived from author's previous reports[2], [3]. The values in Table 1 indicate that the concentration of micro-PCM is 20mass% and the core material of micro-PCM is tetradecane ($C_{14}H_{30}$, T_m = 278.9K, L_t = 163.9kJ/kg).

3. Numerical solution to the ordinary, non-linear differential equations

Fig. 2 shows the calculated results of non-dimensional temperature profile and non-dimensional velocity profile of PCM slurry and water according to similarity variable, η .

From the calculated data, the local wall heat flux is given by Fourier's law as defined in Eq.(8). And also, the local convective heat transfer coefficient of h_x is expressed as Eq.(9)

$$\begin{aligned} q_s &= -k \cdot \left.\frac{\partial T}{\partial y}\right|_0 \\ &= -k \cdot \left.\frac{\partial T}{\partial \eta}\right|_0 \frac{\partial \eta}{\partial y} \\ &= -k \cdot (T_w - T_e) \cdot \theta'(0) \cdot \left(\frac{u_e}{2 \cdot \nu \cdot x}\right)^{1/2} \end{aligned} \qquad (8)$$

$$\begin{aligned} h_x &= \frac{q_s}{(T_w - T_e)} \\ &= -\frac{T_e - T_w}{T_w - T_e} \cdot k \cdot \theta'(0) \\ &= k \cdot \left(\frac{u_e}{\nu \cdot x}\right)^{1/2} \cdot \theta'(0) \end{aligned} \qquad (9)$$

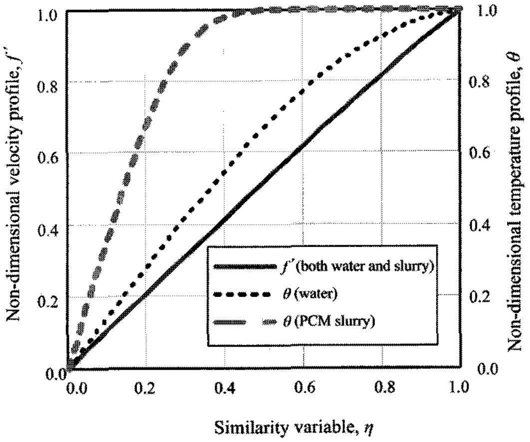

Fig. 2 Velocity and temperature profiles in laminar boundary layer

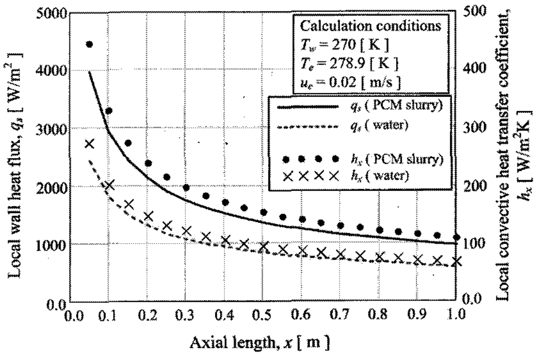

Fig. 3 Local wall heat fluxes and heat transfer coefficients of PCM slurry and water

From Fig. 3, it is cleared that the data of local wall heat flux and convective heat transfer coefficient of PCM slurry are higher than the values of water. In the case of constant wall temperature condition, the higher wall heat flux generation means that the heat transfer coefficient becomes higher. Consequently, in this calculation, the data of PCM slurry are higher approximately two times. These calculated tendencies of data are well agreed with the author's previous results[4], [5] of co-axial double pipe as shown in Fig. 4.

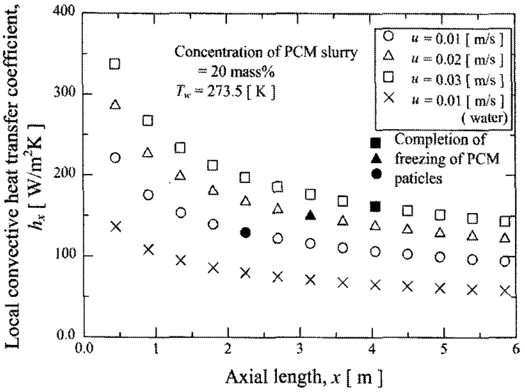

Fig. 4 Experimental result of author's previous result[5] of co-axial double pipe

4. Conclusions

The heat transfer characteristics of PCM slurry was numerically investigated with similar solution method. It is clarified that the similar solution method of Newtonian fluid can be used reasonably this type of PCM slurry which has low concentration. The main conclusions and the results of investigation are summarized as follows :

- The equations of momentum and energy on laminar boundary layer over a horizontal flat plate which lead to a set of non-linear, ordinary differential equations were numerically calculated.
- The values of local wall heat flux and convective heat transfer coefficient of PCM slurry were higher than those of water more than 150~200%, approximately.
- The tendencies of calculated results were well agreed with the experimental results of co-axial double pipe.

Acknowledgement

This work was financially supported by the Kunsan National University's academic support program in the year 2003.

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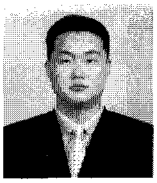
Author Profile



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