

## Driving of the Ball Screw Actuator Using a Global Sliding Mode Control with Bounded Inputs

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**Abstract :** The ball screw actuated by the electric motor is widely used as an essential actuator for driving the mechanical system by virtue of accuracy and force transmission capability. In this paper, a design of the global sliding mode control is presented to drive the ball screw actuator along the minimum time trajectory. In the proposed control scheme, if the ranges of parametric uncertainties and torque limits of the system are specified, the arrival time of the load along the minimum time trajectory can be estimated. Also, the arriving time at the reference input and the maximum acceleration are expressed in a closed form solution. Conversely, the capacity of a ball screw actuator including the motor can be easily designed if the external load and its transportation time are specified. The superior performance of the proposed control scheme and analysis is validated by the computer simulation and experiments comparing with other sliding mode controllers.

**Key words :** Ball screw, Sliding mode control, Bounded inputs.

### 1. Introduction

The ball screw system has been widely used as the drive system of the machine tool such as the numerical control lathe by virtue of accuracy and force transmission capability. A number of researches have been performed on the feed drive system using the ball screw. Kakino and Matsubara modelled the drive system using the ball screw as the fourth order differential equation for tuning and

compared it with the second order model<sup>[1]</sup>. Tsutsumi and Okazaki proposed a mathematical model of the feed drive system using ball screw for computerized numerical control machine tool, where they considered the friction forces occurring at the guider<sup>[2]</sup>.

Since the rapid traveling time in the feed drive system can enhance the productivity, research on the high speed drive system has been one of the major issues in the machine tool industry. One

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way of achieving accurate and high speed drive of the ball screw system with uncertain parameters is to apply an appropriate control algorithm. One of the recommendable controllers with robust performance is a global sliding mode controller(GSMC) following the minimum time trajectory, which is based on the sliding mode control(SMC).

The SMC originated from the variable structure control system was proposed and elaborated in the early 1960's in the Soviet Union by Emelyanov and Ikis<sup>[3,4]</sup>. The SMC has been extensively studied due to invariance properties and the robustness against uncertain system parameters and disturbances. Hung et. al. performed an extensive survey on variable structure control including the SMC by covering fundamental theory, main results, and practical applications<sup>[5]</sup>. Ashchepkov proposed an improved SMC employing an optimal sliding surface to achieve fast trajectory tracking. The optimal sliding surface was determined by minimizing the error performance index for a given initial condition<sup>[6]</sup>.

Despite of the invariance property and the robustness, it was discussed by Slotine and Li that the conventional and previously mentioned SMC had important drawbacks limiting its applicability, such as chattering or large control input torques<sup>[7]</sup>. The condition of the robustness of the conventional SMC is based on the assumption of the unlimited control torques. The obedience of the input torque limit is very important in realistic and practical application. To solve the robustness problem of the conventional

SMC with obedience of the input torque limit, Madani-Esfahani et al<sup>[8]</sup>, proposed a scheme to estimate the region of the asymptotic stability within control input bounds, though not to be applicable due to excessive chatter. To ensure sliding behaviour throughout an entire response, a global sliding mode control(GSMC) scheme was devised by Lu and Chen<sup>[9]</sup>. The GSMC ensures sliding behaviour throughout an entire response. In the GSMC, the range of allowable reference input can be obtained within the control torque limits. However, in this control, the applied control input is dependent on the reference input.

In this paper, we model the ball screw system driven by the electric motor with torque limits as the second order system with parameter uncertainty and uncertain bounded disturbances. To control or design the ball screw system, we propose an improved global sliding mode control scheme. The proposed control scheme is designed to fully exploit the control torques within control torque limit. To do this, the minimum time trajectory function is employed as a forcing function to drive system states. The proposed control algorithm is realistic and practical since the arrival time at the reference input or maximum acceleration are estimated if maximum and minimum bounds of the parameter uncertainties and disturbances are specified within the input torque limits of the system. Especially, the minimum arrival time and the acceleration are expressed in the closed form, which means that calculation time is reduced

and system design can be easily performed.

## 2. Model of the ball screw system

The ball screw system driven by the electric motor is composed of the ball screw, support units, and the nuts carrying the loads as shown in Fig. 1, which transforms the rotating motion into the translational motion and is a velocity reducer. The ball screw and the rotor of motors are connected by couplings such that the ball screw system can be defined as the linear second order system as follows:

$$J\ddot{\theta} + B\dot{\theta} = \tau + d(t) \tag{1}$$

where

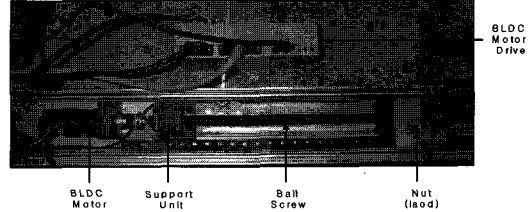
$$J = J_M + J_B + M_L \times \left(\frac{l}{2\pi}\right)^2,$$

$$B = B_M + B_B + \frac{l}{2\pi} B_L$$

where  $J_M$  and  $J_B$  are the moment of inertias of the motor and ball screw, and the mass of the load  $M_L$  along the translational direction is transformed into the rotational load.  $B_M, B_B,$  and  $B_L$  are the friction coefficients of the motor, ball screw, and load, respectively, where the friction coefficient  $B_L$  occurring between the sliding guider and the nut carrying the load is transformed into rotating coefficient.  $\tau$  is the input torque which is equivalent to the applied currents multiplied by the torque coefficient of the motor. The disturbance  $d(t)$  includes disturbance input currents or translational disturbance forces applied on the loads.

In the ball screw system, the moment of

inertia  $J$  and friction coefficients  $B$  are difficult to accurately measure, but their upper and lower bounds can be specified as



**Fig. 1 The Ball screw system**

$$\beta_{\min} \leq J \leq \beta_{\max}$$

$$\alpha_{\min} \leq B \leq \alpha_{\max} \tag{2}$$

$$\max_t |d(t)| \leq D$$

Since the ball screw system carrying ball screw itself and loads is actuated by the motor with input torque limits, the bounds of the input torques are specified as

$$\tau_{\min} \leq \tau \leq \tau_{\max} \tag{3}$$

The ball screw system contains uncertainty but bounded parameters and input disturbances. Within the torque limit, we design an improved GSMC algorithm which drives the nut carrying loads to the reference input along the minimum time trajectory.

## 3. Global sliding mode controller for the ball screw system

### 3.1 Design of the sliding mode control

To control the uncertain ball screw system, the GSMC with the forcing function which drives the sliding line

along the minimum time trajectory is designed as:

$$\tau = -\widehat{\beta}(c\dot{\theta} - \dot{r}) + \widehat{\alpha}\dot{\theta} - \{\Delta\beta|c\dot{\theta} - \dot{r}| + \Delta\alpha|\dot{\theta}| + D\} \operatorname{sgn}(s) \quad (4)$$

where

$$\begin{aligned} \widehat{\beta} &= \frac{\beta_{\max} + \beta_{\min}}{2}, \quad \Delta\beta = \frac{\beta_{\max} - \beta_{\min}}{2} \\ \widehat{\alpha} &= \frac{\alpha_{\max} + \alpha_{\min}}{2}, \quad \Delta\alpha = \frac{\alpha_{\max} - \alpha_{\min}}{2} \\ \operatorname{sgn}(s) &= \begin{cases} 1 & s > 0 \\ -1 & s < 0 \\ 0 & s = 0 \end{cases} \end{aligned}$$

In the GSMC, the conventional sliding surface is driven by the forcing function  $f(t)$  which is referred to the reference<sup>[8]</sup>. The sliding curve has the form

$$s = \dot{e} + ce - f(t) \quad (5)$$

where the error state is  $e = \theta - r$  with reference input  $r > 0$ . The sign of the  $r$  represents the direction. The forcing function of the applied sliding mode drives the system states in any state space to the switching plane directly without reaching phase. In this paper, we propose a new forcing function which drives the system states along the minimum time trajectory. The purpose of the adaptation of the proposed forcing function is to fully exploit the control torques within the control torque limit. In addition to this, if we adapt the forcing function, the estimate of the arrival time at the reference input can be expressed in a closed form.

In order for the controller to drive the system states to maintain on the sliding surfaces, the conditions of the forcing function  $f(t)$  should be satisfied as

$$f(0) = \dot{e}(0) + ce(0) \quad (6a)$$

$$f(t) \rightarrow 0 \quad \text{as } t \rightarrow \infty \quad (6b)$$

$$\dot{f}(t) \text{ should be bounded} \quad (6c)$$

In the GSMC, the sliding line is designed with the initial state located on it as in Eq. (6) such that the system states are constrained to the sliding surface. Therefore, the sliding mode invariably exist throughout the entire response.

The stability analysis of the proposed controller satisfying the conditions (6) can be shown using the Lyapunov function  $V = \frac{1}{2}Js^2 > 0$ . The negative definite of the time derivative of the  $V$  except for  $s=0$  ensures that the proposed control scheme guarantees the asymptotic stability, which can be easily proved as shown in reference(Lu and Chen 1995).

### 3.2 Design of the forcing function representing the minimum time trajectory

In the stability analysis, the asymptotic stability of the closed loop system is guaranteed if the forcing function satisfies the condition (6). This, eventually, means  $s = 0$ . In this paper, we propose the following function representing minimum time trajectory, which satisfies the condition in (6). The initial and final conditions of the desired performance function is specified as follows:

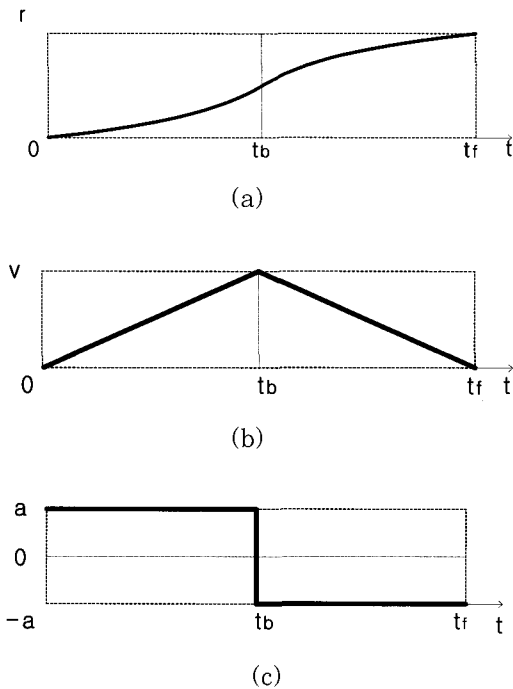
$$\begin{aligned} \text{for } t = 0 : x(0) = 0, \quad \dot{x}(0) = 0 \\ \text{for } t \geq t_f : x(t_f) = r, \quad \dot{x}(t_f) = 0 \end{aligned} \quad (7)$$

where  $t_f$  is the final arrival time. The boundary conditions of the minimum time trajectory are specified as:

$$\begin{aligned} x &= \frac{a}{2} t^2 \\ \dot{x} &= at & \text{for } 0 \leq t < t_b & \quad (8a) \\ \ddot{x} &= a = \frac{v}{t_b} \end{aligned}$$

$$\begin{aligned} x &= r - \frac{a}{2} t_f^2 + at_f t - \frac{a}{2} t^2 \\ \dot{x} &= a(t_f - t) & \text{for } t_b \leq t \leq t_f & \quad (8b) \\ \ddot{x} &= -a \end{aligned}$$

where  $v$  is the maximum velocity,  $a$  is the acceleration, and  $t_b$  is the mid traveling time. The profile of the position, velocity, and acceleration of the trajectory are shown in Fig. 2(a), 2(b), and 2(c), respectively.



**Fig. 2 Profiles of the minimum time trajectory:**  
**(a) Position, (b) Velocity, (c) Acceleration**

The forcing function generating the

sliding mode that contains the global characteristics and includes the minimum time trajectory function is specified as:

$$\begin{aligned} f(t) &= \dot{x} + c(x - r) \\ &= at + c\left(-\frac{a}{2} t^2 - r\right) \end{aligned} \quad \text{for } (0 \leq t < t_b) \quad (9a)$$

$$\begin{aligned} f(t) &= a(t_f - t) + c\left(t_f t - \frac{1}{2} t_f^2 - \frac{1}{2} t^2\right) \end{aligned} \quad \text{for } (t_b \leq t \leq t_f) \quad (9b)$$

#### 4. Estimation of the arrival time

The input torques of the system with unknown parameters and uncertain disturbances defined in equation (2) are bounded as specified in equation (3). When the proposed control is applied to the uncertain ball screw system, a scheme to estimate the arrival time to the reference input of the system is proposed. To do this, we represent the sliding mode control law into two parts according to its magnitude as follows:

$$\begin{aligned} \tau_h &= \widehat{\beta}(\dot{f} - c\dot{\theta}) + \widehat{\alpha}\dot{\theta} + \{\Delta\beta|\dot{f} - c\dot{\theta}| + \Delta\alpha|\dot{\theta}| + D\} \end{aligned} \quad \text{for } s < 0 \quad (10a)$$

$$\begin{aligned} \tau_l &= \widehat{\beta}(\dot{f} - c\dot{\theta}) + \widehat{\alpha}\dot{\theta} - \{\Delta\beta|\dot{f} - c\dot{\theta}| + \Delta\alpha|\dot{\theta}| + D\} \end{aligned} \quad \text{for } s > 0 \quad (10b)$$

When controller drives the system states along the desired trajectory,  $s = \dot{s} = 0$  becomes satisfied for  $t \geq 0$ . In the derivative of the sliding mode function,  $\dot{s} = \ddot{\theta} + c\dot{\theta} - f(t)$ ,  $\dot{s}$  is not zero some times, but which is much smaller than  $\ddot{\theta}$  such that we can obtain the estimate

$$\ddot{\theta} = \dot{f} - c\dot{\theta} \quad (11)$$

Rewriting equation (10) using equation

(11) yields

$$\tau_h = \hat{\beta}\ddot{\theta} + \hat{\alpha}\dot{\theta} + \{\Delta\beta|\ddot{\theta}| + \Delta\alpha|\dot{\theta}| + D\} \text{ for } s < 0 \quad (12a)$$

$$\tau_l = \hat{\beta}\ddot{\theta} + \hat{\alpha}\dot{\theta} - \{\Delta\beta|\ddot{\theta}| + \Delta\alpha|\dot{\theta}| + D\} \text{ for } s > 0 \quad (12b)$$

Equation (12) can be rearranged according to the input profiles and trajectory tracking time. Equation (12 a) is expressed as:

$$\begin{aligned} \tau_h &= \beta_{\max}\ddot{\theta} + \alpha_{\max}\dot{\theta} + D \text{ for } 0 \leq t < t_b \\ \tau_h &= \beta_{\min}\ddot{\theta} + \alpha_{\max}\dot{\theta} + D \text{ for } t_b \leq t \leq t_f \end{aligned} \quad (13a)$$

where the input torques are obtained by exploiting the input profiles in Fig. 2. For the time interval  $0 \leq t < t_b$ , we have  $|\ddot{\theta}| = \ddot{\theta} = a$ , and for  $t_b \leq t \leq t_f$ , we have  $|\ddot{\theta}| = -\ddot{\theta} = -a$ . In the same procedure, equation (12 b) is expressed as

$$\begin{aligned} \tau_l &= \beta_{\min}\ddot{\theta} + \alpha_{\min}\dot{\theta} - D \text{ for } 0 \leq t < t_b \\ \tau_l &= \beta_{\max}\ddot{\theta} + \alpha_{\min}\dot{\theta} - D \text{ for } t_b \leq t \leq t_f \end{aligned} \quad (13b)$$

To estimate the maximum value of the input torques, we substitute the equation (8) with  $r = \frac{1}{4}at_f^2$  into  $\theta$ ,  $\dot{\theta}$ , and  $r$  of equation (13 a), and arrange it as follows:

$$\begin{aligned} \tau_h &= aW + D \quad (0 \leq t < t_b) \\ &= aX + D \quad (t_b \leq t \leq t_f) \end{aligned} \quad (14)$$

where

$$\begin{aligned} W &= \beta_{\max} + \alpha_{\max}t \\ X &= -\beta_{\min} + \alpha_{\max}(t_f - t) \end{aligned}$$

In the same approach, substituting the functions of the minimum time trajectory

into  $\theta$ ,  $\dot{\theta}$ , and  $r$  of equation (12 b) and arranging it yields

$$\begin{aligned} \tau_l &= aY - D \quad (0 \leq t < t_b) \\ &= aZ - D \quad (t_b \leq t \leq t_f) \end{aligned} \quad (15)$$

where

$$\begin{aligned} Y &= \beta_{\min} + \alpha_{\min}t \\ Z &= -\beta_{\max} + \alpha_{\min}(t_f - t) \end{aligned}$$

The maximum value of  $\tau_h$  in equation (14) and the minimum of  $\tau_l$  in equation (15) exist at a certain time in the tracking time range. Using the maximum or minimum values of the input torques, we can estimate the maximum value of the acceleration of the uncertain system within bounded input torques specified in equation (3). For the realistic and practical application, the input torques should be applied within the physical bounds. The maximum and minimum input torques are bounded as

$$\tau_{\min} \leq \min \tau(t) \leq \max \tau(t) \leq \tau_{\max} \quad (16)$$

To estimate the minimum time of the trajectory tracking, we need to obtain the maximum value of the  $\tau_h$  in equation (14) and the minimum value of the  $\tau_l$  in equation (15). Since the equations are first order with respect to time, we need to only check out the limit value at the initial, mid, and final points. At these times, the values of the control torques are calculated as:

$$\tau_h|_{t=0} = a\beta_{\max} + D \quad (17a)$$

$$\tau_h|_{t=t_{b-0}} = a(\beta_{\max} + \alpha_{\max}t_b) + D \quad (17b)$$

$$\tau_h|_{t=t_{b+0}} = a(-\beta_{\min} + \alpha_{\max} t_b) + D \quad (17c)$$

$$\tau_h|_{t=t_f} = a(-\beta_{\min}) + D \quad (17d)$$

According to the calculated results of the values of the input torques, we can obviously tell that the maximum value exists at  $t=t_{b-0}$ , which is

$$\max \tau(t) = \tau_h|_{t=t_{b-0}} \quad (0 \leq t) \quad (18)$$

The minimum value candidates of  $\tau_l$  can be obtained in the same way as shown in obtaining the maximum value. We can get the minimum candidates at the mid and final points of the tracking time as follows:

$$\tau_l|_{t=t_{b-0}} = a(\beta_{\min} + \alpha_{\min} t_b) - D \quad (19a)$$

$$\tau_l|_{t=t_{b+0}} = a(-\beta_{\max} + \alpha_{\min} t_b) - D \quad (19b)$$

$$\tau_l|_{t=t_f} = a(-\beta_{\max}) - D \quad (19c)$$

By evaluating the minimum candidates in equations (19 a), (19 b), and (19 c), we can tell that  $\tau_l|_{t=t_f}$  is the minimum, which is expressed as:

$$\min \tau(t) = \tau_l|_{t=t_f} \quad (0 \leq t) \quad (20)$$

In this paper, the goal of the proposed control scheme is to achieve tracking along the minimum time trajectory and to estimate the arrival time  $t_f$  at the reference input in the ball screw system with unknown but bounded parameters and disturbances. From the obtained minimum and maximum values, we can decide the arrival time within the input torque limits. By substituting the equation  $a=4r/t_f^2$  transformed from the trajectory function  $r=(1/4)at_f^2$  into equation (17b) and (19c), and arranging

them within the bounded region in equations (16), we obtain the following equations:

$$\frac{4r}{t_f^2} (\beta_{\max} + \alpha_{\max} t_b) + D \leq \tau_{\max} \quad (21)$$

$$\tau_{\min} \leq -\frac{4r}{t_f^2} \beta_{\max} - D \quad (22)$$

Rearranging equation (21) yields the second order inequality as:

$$(\tau_{\max} - D)t_f^2 - 2r\alpha_{\max} t_f - 4r\beta_{\max} \geq 0 \quad (23)$$

From the bounded equation (23), the minimum time candidate to arrive at the reference input  $r$  can be obtained in the closed form as:

$$t_{hmin} = \frac{r\alpha_{\max} + \sqrt{r^2\alpha_{\max}^2 + 4r\beta_{\max}(\tau_{\max} - D)}}{\tau_{\max} - D} \quad (24)$$

In the same way, another minimum time candidate can be obtained by solving the inequality (22) as follows:

$$t_{lmin} = \sqrt{\frac{4r\beta_{\max}}{-\tau_{\min} - D}} \quad (25)$$

An estimate of the arrival time should be any of  $t_{hmin}$  and  $t_{lmin}$  satisfying both torque bounds in (16), which is:

$$t_{\min} = \max(t_{hmin}, t_{lmin}) \quad (26)$$

As shown in equation (24) and (25), the minimum arrival time is expressed in a closed form clearly. Therefore, when we design a mechanical system with a ball screw actuator, only if we know the transporting distance of the load  $r$ , the range of the moment of inertia, the range of the damping coefficient, and the upper bound of the disturbances, we can most nearly estimate the arrival time at  $r$  along the minimum time trajectory.

Conversely, we can easily design or select motors with appropriate torques to drive the mechanical system according to the control specifications.

## 5. Results of the simulation and experiments

### 5.1 Description of the ball screw system

In the experiment, we apply the proposed control scheme to the ball screw system connected with the BLDC motor. The ball screw system has unknown but bounded parameters and disturbances. Arranging the ball screw system dynamics in (1) yields

$$\ddot{\theta} + a \dot{\theta} = b(u + d) \quad (27)$$

where  $a = B/J$ ,  $b = K_t K_c / J$  with the torque coefficient of motor  $K_t$  and PWM inverter currents coefficient  $K_c$ . The control input  $u$  is voltage input. In the experiments, CSM A5 50W BLDC motor made by SAMSUNG Inc. was used where the incremental encoder attached with 1000 pulse per a revolution. The length of the ball screw is 0.82m with 0.015m diameter and 0.02m lead per revolution. The mass of the nut is 0.5kg.

The controller was implemented using the digital I/O data acquisition system. The limit of the control input to the motor driver is  $\pm 5V$  which is transformed by the 8 bit D/A device. The applied input voltage is again transformed into the control torque through the  $K_t$  and  $K_c$ . The control gain is set as  $c = 7.3787$  for the experimental case of the SMC, GSMC, the proposed control, and

simulation. In the GSMC, the same control structure with Eq. (4) is applied with different desired trajectory function. The conventional SMC is constructed as

$$\tau = -\hat{\beta}c\dot{\theta} + \hat{a}\dot{\theta} - \{\Delta\beta|c\dot{\theta}| + \Delta a|\dot{\theta}| + D\} \operatorname{sgn}(s) - K_d s \quad (28)$$

where  $s = \dot{e} + ce$ , and  $k_d = 0.05$ . The parameter value of  $K_t$ ,  $K_c$ ,  $J$ , and  $B$  are given in the catalogue, but specified with about 15% error range in the simulation and experiment. The bounds of the uncertain parameters and disturbances and are specified as follows:

$$\begin{aligned} 5.9293e-5 &\leq J \leq 8.8940e-5 \text{ kgm}^2 \\ 5.0780e-4 &\leq B \leq 7.6170e-4 \text{ Nms/rad} \\ 1.3140e-1 &\leq K_c \leq 1.9710e-1 \text{ As}^2\text{V}^{-1}\text{m}^{-1} \\ 2.9026e-1 &\leq K_t \leq 4.2528e-1 \text{ Nm}^2\text{s}^{-2}\text{A}^{-1} \end{aligned}$$

From previous estimated parameters, the ranges of the parameters of the control torques are specified as follows:

$$\begin{aligned} 6.9095e-4 &\leq \beta \leq 2.3320e-3, \\ 5.9174e-3 &\leq \alpha \leq 1.9971e-2 \\ \hat{\beta} &= 1.5115e-3, \quad \hat{a} = 1.2944e-2, \quad \Delta\beta = 8.2051e-4, \\ \text{and } \Delta\alpha &= 7.0270e-3, \\ D &= 0.03, \text{ Sampling Time} = 0.905\text{ms}. \end{aligned}$$

where  $D$  is the friction disturbance bound.

### 5.2 Results of the experiment

In the simulation and experiments, we set the reference input  $r = 20\pi$ rad which is equivalent to 0.2m translational displacement of the nut in the ball screw. We performed experiments for the proposed control, GSMC, and SMC and one simulation for the proposed control



scheme. For the simulation, we used Runge-Kutta fourth order numerical method and C programming language. Also, using the closed form for the arrival time, we estimated the arrival time at the given reference input, the maximum acceleration, and the maximum control torque using equations (18), (20), and (26), which are shown in Table 1.

We compared the performance of the proposed controller with the GSMC and the conventional SMC, and the simulation results using the same control gains, parameters and uncertainty bounds. Especially, we evaluated the controllers in two aspects: the reference input tracking capability and the obedience of control torque limits. Also, we compared the estimation result of the derived closed form for the arrival time with that of the proposed controller.

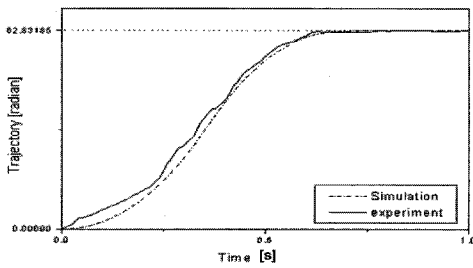


Fig. 3 Target tracking of improved GSMC

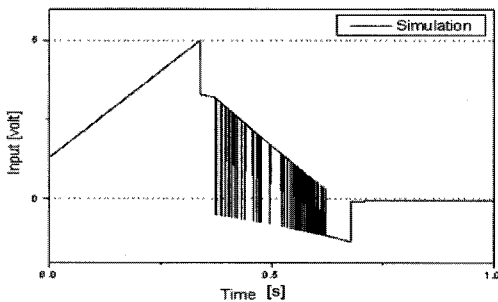


Fig. 4(a) Control inputs of improved GSMC

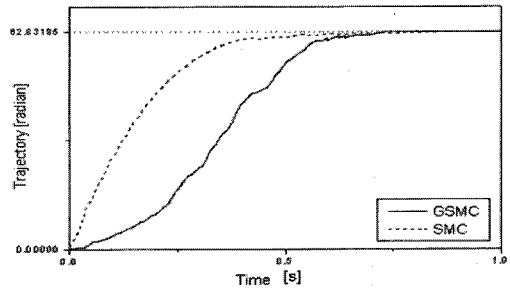


Fig. 4(b) Control inputs of improved GSMC

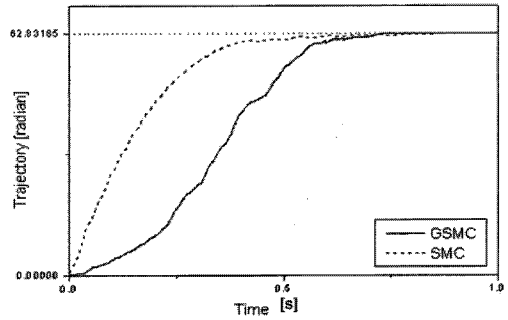


Fig. 5 Target tracking of GSMC and SMC

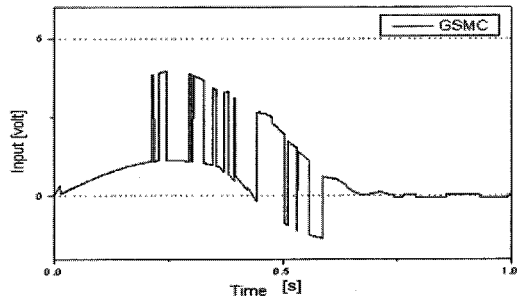


Fig. 6 Control inputs of GSMC

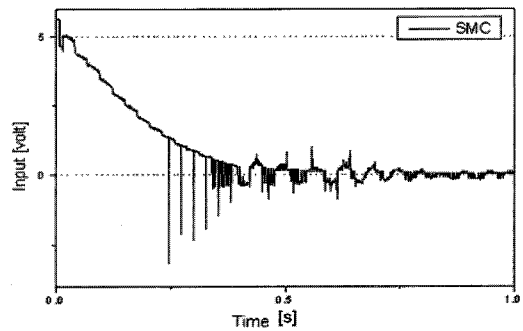


Fig. 7 Control inputs of SMC

**Table 1 Experimental and calculated values of control schemes**

Method	Controller	$t_f$	c	a	Maximum Input
Closed form estimation		0.6769	7.3787	547.3409	5.0
Simulation	Improved GSMC	0.6776	7.3787	547.3409	5.0173
Experiment	Improved GSMC	0.6879	7.3787	547.3409	5.0237
"	GSMC	0.7110	7.3787	547.3409	3.9716
"	SMC	0.8510	7.3787		5.6398

According to the experimental results, the arrival time shown in the proposed control scheme is the shortest out of the three controllers as shown in Table 1, and Fig. 3 and 5. The proposed controller utilizes the control torques as fully as possible within the torque limits but not the other controllers as shown in Fig. 4, 6, and 7. The control input patterns for the simulation and experiment of the proposed controller are shown similar as shown in Fig. 4a and 4b. The SMC has faster rising to the reference, but at near reference input, conversing time is much longer than other controllers despite of applying over-limit control torques as shown in Fig. 7. Though the GSMC does not trespass against the control torque limits, it does not fully exploit the control torques.

One particular result is that the experimental and simulation results of the arrival time  $t_f$ , maximum acceleration, and maximum input are accorded with each other which shown in Fig. 3, and Table 1. Another particular result indicates that the closed form estimation for the arrival time in Eq. (26)

yield quite similar results with the computer simulation and experimental results as shown in Table 1. Hence, if we know the bounds of the unknown parameters and the torque limit of the ball screw system and reference input, by only using the closed form equation, we can estimate the arrival time to the reference input without computer simulation. The estimation scheme of the arrival time in conjunction with the maximum acceleration estimation would be very helpful in designing and application of ball screw actuating systems.

## 6. Conclusion

To drive the mechanical load of the ball screw system along the minimum time trajectory, an improved GSMC was proposed. If ranges of uncertain disturbances and parameters such as inertias of the motor, ball screw, and load are specified, the proposed control scheme coordinates the load to the reference input along the minimum time trajectory within torque limits. The results of the computer simulation on the proposed control scheme are in accordance with those of experiment. According to the experimental results, the proposed control scheme showed superior performance to other sliding mode controllers.

The eminent advantage of the proposed control scheme is that the arrival time at the reference input can be estimated. Especially, the arrival time and the acceleration are expressed in the closed

form such that calculation time is reduced and system design can be easily performed.

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