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수리 가능한 시스템의 가용도를 위한 최적 교체정책

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Optimal Replacement Policies for the Availability of a Repairable System

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Abstract

In many cases, it is more practical and economical to repair a system than to replace the whole system or to perform a complete overhaul when the system fails. Two basic replacement policies were proposed by Barlow and Hunter(1960) and Morimura(1970), in which the minimal repair times are identically distributed. But, as Lam(1988) pointed out, in many cases of deteriorating system, in view of ageing and cumulative wear, the repair time will tend to be longer and longer. In this note, the two basic replacement policies are considered for a repairable system with linearly increasing repair times. Optimal policies, which maximize the steady state availability of the system, are obtained for the Weibull failure rate case.

1. Introduction

Most repair replacement models assume that the system is replaced when it fails. But it is more practical to perform a repair action other than the replacement of whole system when the cost of replacement is expensive. When a complex system consisting of many components fails, the replacement of only a very small fraction of the system's constituent can return the system to the functioning state. In this case, the failure rate of the system is not altered by the failure and thus the system reliability after a repair is essentially the same as it was immediately before a failure occurred. This repair action is called by minimal repair which is precisely defined by Nakagawa(1983).

Two basic maintenance policies were proposed by Barlow and Hunter(1960) and Morimura(1970). Barlow and Hunter(1960) proposed an age replacement model with minimal repairs between replacements(Policy II). The basic model developed by Barlow and Hunter(1960) has been generalized and modified by many authors to fit more realistic situations. Cleroux et al.(1979) and Boland and Proschan(1982) analyze a model where the minimal repair cost is not fixed. More recently Nakagawa(1984) developed a minimal repair model that combined the age and a failure replacement policy. Bagai and Jain(1994) and Deshpande and Singh(1995) discuss optimal policies for a system undergoing minimal repair and having various types of maintenance and repair cost−structure.

Morimura(1970) and Park(1979) modified entirely the concept of age replacement under minimal repair introduced by Barlow and Hunter(1960). Morimura(1970) and Park(1979), instead of finding a fixed time(age) to replacement as the optimal replacement policy, find the number of failures and minimal repairs before the system is replaced(Policy III).

An important characteristic of a repairable component is availability. Let the state of the component be given by the binary variable

$$X(t) = \begin{cases} 1 & \text{if } \textit{the component is functioning at time } t \\ 0 & \textit{otherwise.} \end{cases}$$

The availability at time t is defined by

$$A(t) = P(X(t) = 1), \tag{1}$$

which is the probability that the component is functioning at time t. Because the study of $A(t)$ is too hard except for a few simple cases, other measures have been proposed, and more attention is being paid to the limiting behavior of this quantity, i.e., engineers are more interested in the extent to which the component will be available after it has been run for a long time. The steady state availability (or limiting availability) of the component is, when the limit exists, defined by

$$A = \lim_{t \to \infty} A(t), \tag{2}$$

which is a significant measure of performance of a repairable component. Some other kinds of availability which are useful in practical applications can be found in Birolini(1985, 1994) and Høyland and Rausand(1994).

In this note, the two basic replacement policies are considered for a repairable system with linearly increasing repair times. Optimal policies, which maximize the steady state availability of the system, are obtained for the Weibull failure rate case.

2. Optimal Replacement Policy of Policy II

Consider a system with failure rate $\lambda(t) = \lambda\beta t^{\beta-1} (\lambda > 0)$ and assume that this system is replaced every time its age reaches at T_0. For each intervening failure only minimal repair is done and assume that the minimal repair times and the times for replacement are not negligible. This replacement policy was called Policy II by Barlow and Hunter(1960). Define N_i' as the number of minimal repairs in ith renewal period and $N_i \equiv N_i' + 1$. Let $X_{i,j}$ be the lifetime of the system which has been renewed $(i-1)$ times and has been minimally repaired $(j-1)$ times after the time of $(i-1)$th renewal, $i = 1, 2, \cdots$, $j = 1, 2, \cdots, N_i$. Define the corresponding repair times $Y_{i,j}$ and assume that $E(Y_{i,j}) = v_0 + j v_1$, for $j = 1, 2, \cdots, N_i - 1$, and $E(Y_{i,j}) = v_2$, for $j = N_i$. Then the steady state availability of the model exists and is given in the following theorem.

Theorem 1. The steady state availability exists and is given by

$$A(T_0)=\frac{T_0}{T_0+\lambda T_0^{\beta}\cdot(v_0+v_1)+(\lambda T_0^{\beta})^2\cdot\frac{v_1}{2}+v_2} \quad (3)$$

proof.

See the proof of Theorem 3.4 in Cha(1999). ■

The properties of the existence and the uniqueness of the optimal T_0^* maximizing $A(T_0)$ is stated in the following theorem.

Theorem 2. If the failure rate of the system is IFR($\beta>1$) then there always exists the unique solution T_0^* which maximizes $A(T_0)$ and T_0^* is the value which satisfies ;

$$(v_0+v_1)\cdot\lambda\ T_0^{*\beta}\cdot(\beta-1)+\frac{1}{2}v_1\cdot(\lambda\ T_0^{*\beta})^2\cdot(2\beta-1)=v_2 \quad (4)$$

proof.

Differentiate $A(T_0)$ in (3) with respect to T_0 and set it to zero, which yields (4) and note that the left term as a function of T_0 strictly increases as T_0 increases and, when $T_0=0$ it has its value 0. Therefore there exists the unique T_0^* satisfying the equation (3), which completes the proof. ■

3. Optimal Replacement Policy of Policy III

Consider a system with failure rate $\lambda(t)=\lambda\beta t^{\beta-1}(\lambda>0)$ and assume that this system is replaced on every kth failure from the latest system replacement. But on each intervening failure only minimal repair is done. This replacement policy was called Policy III by Morimura(1970). Assume that the conditions on the repair times are the same as those in Section 2. Then the steady state availability of the model exists and is given in the following theorem.

Theorem 3. The steady state availability exists and is given by

$$A(k)=\frac{\mu}{\mu+(k-1)v_0+\frac{k(k-1)}{2}v_1+v_2}, \tag{5}$$

$$\text{where } \mu=\frac{\Gamma(k+\frac{1}{\beta})}{\Gamma(k)}\cdot\lambda^{-\frac{1}{\beta}}.$$

proof.

See the proof of Theorem 3.2 in Cha(1999). ■

The properties of the optimal k^* maximizing $A(k)$ is stated in the following theorem.

Theorem 4. Let $a=\frac{1}{2}\lambda^{\frac{1}{\beta}}(\frac{1}{\beta}-2)v_1$, $b=\lambda^{\frac{1}{\beta}}[(\frac{1}{\beta}-1)v_0-\frac{1}{2\beta}v_1]$ and $c=\lambda^{\frac{1}{\beta}}(v_2-v_0)\frac{1}{\beta}$, and define k^* as the positive integer which maximizes $A(k)$. Then if the failure rate of the system is IFR($\beta>1$) and

(i) $b^2-4ac\leq 0$, then $k^*=1$,

(ii) $b^2-4ac>0$ and $a+b+c\leq 0$ then $k^*=1$,

(iii) $b^2-4ac>0$ and $a+b+c>0$ then k^* is the value which satisfies;

$$\frac{-b+\sqrt{b^2-4ac}}{2a}<k^*<\frac{-b+\sqrt{b^2-4ac}}{2a}+1, \tag{6}$$

if the inequalities in (6) do not contain any integer,

$$k^*=\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } k^*=\frac{-b+\sqrt{b^2-4ac}}{2a}+1.$$

proof.

Note that maximizing the equation (5) is equivalent to minimizing

$$g(k) \equiv \frac{\Gamma(k)}{\Gamma(k+\frac{1}{\beta})}(k-1)\cdot\lambda^{\frac{1}{\beta}}v_0 + \frac{\Gamma(k)}{\Gamma(k+\frac{1}{\beta})}k(k-1)\cdot\frac{1}{2}\lambda^{\frac{1}{\beta}}v_1 + \frac{\Gamma(k)}{\Gamma(k+\frac{1}{\beta})}\lambda^{\frac{1}{\beta}}v_2.$$

Observe that

$$g(k) - g(k+1) = \frac{\Gamma(k)}{(k+\frac{1}{\beta})\Gamma(k+\frac{1}{\beta})}\{\ ak^2 + bk + c\ \},$$

and note that $a<0$ and $b<0$. Therefore,

(i) $g(k)-g(k+1)<0$, for all $k\geq 1$, if $b^2-4ac\leq 0$,

(ii) $g(k)-g(k+1)<0$, for all $k\geq 1$, if $b^2-4ac>0$ and $g(1)=a+b+c\leq 0$, and

(iii) if $b^2-4ac>0$ and $g(1)=a+b+c>0$, then k^* satisfies

$$g(k^*-1)-g(k^*)>0 \text{ and } g(k^*)-g(k^*+1)<0, \qquad (7)$$

or

when the above inequalities (7) do not contain any integer, $k^*=k_0$ and k_0+1, where k_0 satisfies

$$g(k_0)=g(k_0+1).$$

This completes the proof. ■

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