

Input Constrained Robust Model Predictive Control with Enlarged Stabilizable Region

Young Il Lee

Abstract: The dual-mode strategy has been adopted in many constrained MPC (Model Predictive Control) methods. The size of stabilizable regions of states of MPC methods depends on the size of underlying feasible and positively invariant sets and the number of control moves. The results, however, may perhaps be conservative because the definition of positive invariance does not allow temporal departure of states from the set. In this paper, a concept of periodic invariance is introduced in which states are allowed to leave a set temporarily but return into the set in finite time steps. The periodic invariance can be defined with respect to sets of different state feedback gains. These facts make it possible for the periodically invariant sets to be considerably larger than ordinary invariant sets. The periodic invariance can be defined for systems with polyhedral model uncertainties. We derive a MPC method based on these periodically invariant sets. Some numerical examples are given to show that the use of periodic invariance yields considerably larger stabilizable sets than the case of using ordinary invariance.

Keywords: Input constraints, model uncertainty, periodic invariance, receding horizon control.

1. INTRODUCTION

The 'dual-mode paradigm' is known to be an effective way to handle physical constraints in actuators [1-4]. The basic idea of the dual-mode paradigm is to use feasible control moves to steer the current state into a feasible and invariant set in finite time steps. A constant state feedback control is assumed to be used once the state belongs to the feasible and invariant set.

A feasible and invariant set is defined with respect to a state feedback gain and it requires that the state feedback control satisfies the input constraints for all the states in the set and that states should remain in the set when the state feedback control is applied. This dual-mode strategy has been adopted in many constrained MPC methods. The size of stabilizable regions of states of MPC methods depends on the size of underlying feasible and positively invariant sets and the number of control moves.

The main idea of this paper is to replace the conventional invariant set in constrained model predictive control by a set that has the following extended invariance properties, (i) allows the state to

leave the set temporarily, provided that there are no constraint violations and that the state returns within the set after a finite number of steps and (ii) considers use of multiple state feedback law in the definition of invariance. The concept of quasi-invariant sets was introduced in [3], which allows the state to leave the set temporarily. The approach used in [3] is based on polyhedral type terminal sets. In [3], however, use of a single state feedback gain was assumed in the definition of terminal invariant set and no systematic method of obtaining underlying state feedback gain was given.

In this paper, a concept of periodic invariance is introduced in which states are allowed to leave a set temporarily but return into the set in finite time steps. Moreover, the periodic invariance involves the use of more than one state feedback gain and several ellipsoidal sets. These facts make it possible for the periodically invariant sets to be considerably larger than conventional invariant sets. A computation scheme based on LMIs (Linear Matrix Inequalities) will be proposed so that invariant sets as well as underlying feedback gains can be obtained systematically. The periodic invariance can be defined for systems with polyhedral model uncertainties. We derive a MPC method based on these periodically invariant sets. In the proposed MPC strategy, the convex-hull of periodic invariant sets is used as a target set of the dual-mode approach to yield a large stabilizable set.

In Section 2, the periodic invariance is defined. In

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Section 3, a MPC method that uses the convex hull of the positively invariant sets as a target is developed. A Lyapunov function is defined as a sum of quadratic function and it will be shown that this Lyapunov function can be made to monotonically decrease.

2. PERIODIC INVARIANCE AND FEASIBILITY

Consider the following input constrained linear uncertain system:

$$x(k+1) = \tilde{A}x(k) + \tilde{B}u(k), \quad |u(k)| \leq \bar{u}, \quad (1)$$

where $x(k) \in R^n$, $u(k) \in R^m$ and the matrix functions \tilde{A} and \tilde{B} belong to the polyhedral uncertainty class:

$$\Pi = \left\{ (\tilde{A}, \tilde{B}) \mid (\tilde{A}, \tilde{B}) = \sum_{l=1}^{n_p} \eta_l (A_l, B_l), \right. \\ \left. \eta_l \geq 0, \sum_{l=1}^{n_p} \eta_l = 1 \right\}. \quad (2)$$

We will consider a time-varying state feedback control law as:

$$u(k) = K(k)x(k), \quad (3)$$

which requires

$$|u(k)| = |K(k)x(k)| \leq \bar{u}. \quad (4)$$

Provided that (4) is satisfied, use of $u(k) = K(k)x(k)$, would yield

$$x(k+1) = \tilde{\Phi}(k)x(k), \quad \tilde{\Phi} := \tilde{A} + \tilde{B}K(k). \quad (5)$$

Consider the uncertain linear system described by (1) and (2). A set Ω_0 is defined to be feasible and the **periodic-invariant** with respect to the time varying feedback control $u(k) = K(k)x(k)$, of (3) if there exists a finite positive number ν such that for any initial state $x(k) \in \Omega_0$, the future states $x(k+i)$, ($i=1,2,\dots,\nu$) of the system (5) satisfy the input constraint (4) (feasible) and $x(k+\nu)$ belongs to Ω_0 (periodic-invariant).

Consider an ellipsoidal set defined as:

$$\Omega_0 = \{x \mid x'P_0x \leq 1\}. \quad (6)$$

The periodic-invariance of Ω_0 would be checked by considering propagation of the states in terms of ellipsoidal sets. Assume that the closed-loop dynamics of (5) makes $x(k+1) \in \Omega_1$ for any $x(k) \in \Omega_0$, where

$$\Omega_1 = \{x \mid x'P_1x \leq 1\}. \quad (7)$$

It is easy to see that the following relation:

$$P_0 - \Phi_l(k)' P_1(k) \Phi_l(k) > 0, \quad l=1,2,\dots,n_p \quad (8)$$

guarantees that $x(k+1) \in \Omega_1$ for any $x(k) \in \Omega_0$ and $(\tilde{A}, \tilde{B}) \in \Omega$, where $\Phi_l(k) := A_l + B_l K(k)$. Similarly, an ellipsoidal set Ω_2 can be defined for the ellipsoidal set Ω_1 . Relations

$$P_1 - \Phi_l(k+1)' P_2(k+1) \Phi_l(k+1) > 0, \\ l=1,2,\dots,n_p \quad (9)$$

would guarantee that $x(k+2) \in \Omega_2$ for any $x(k+1) \in \Omega_1$ and $(\tilde{A}, \tilde{B}) \in \Omega$.

The above argument can be applied recursively to yield ellipsoidal sets of states:

$$\Omega_j = \{x \mid x'P_jx \leq 1\}. \quad (10)$$

and relations

$$P_j - \Phi_l(k+j)' P_{j+1}(k+j) \Phi_l(k+j) > 0, \\ l=1,2,\dots,n_p \quad (11)$$

for $j=0,1,2,\dots,\nu-1$. The periodic-invariance of Ω_0 requires that Ω_ν should belong back to Ω_0 . Thus, relation

$$P_\nu - P_0 > 0 \quad (12)$$

would guarantee the periodic-invariance of Ω_0 with respect to the switching control (3). On the other hand, it should be noted that the above arguments hold true for system (1) provided that

$$|K(k+j)x| \leq \bar{u}, \quad \forall x \in \Omega_j, \\ j=0,1,\dots,\nu-1. \quad (13)$$

Conditions (8), (9), (11), (12) and (13) can be transformed into LMIs using the technique proposed in [6] and used in [4] as per the following theorem.

Theorem 1: Consider the constrained uncertain system (1)-(2). An ellipsoidal set:

$$\Omega_0 = \{x \mid x'P_0x \leq 1\} \quad (14)$$

is feasible and periodic-invariant with respect to the time-varying control (3) provided that there exist matrices $Q_j := P_j^{-1}$ ($j=0,1,2,\dots,\nu$), and Y_j, X_j ($j=0,1,2,\dots,\nu-1$) such that $Y_j := K(k+j)Q_j$ and

$$\begin{bmatrix} Q_{j-1} & (A_l Q_{j-1} + B_l Y_{j-1})^T \\ A_l Q_{j-1} + B_l Y_{j-1} & Q_j \end{bmatrix} > 0 \quad (15)$$

for $l=1,2,\dots,n_p$ and $j=1,2,\dots,\nu$,

$$\begin{bmatrix} Q_v & Q_v \\ Q_v & Q_0 \end{bmatrix} > 0 \quad (16)$$

and

$$\begin{bmatrix} X_j & Y_j \\ Y_j^T & Q_j \end{bmatrix} > 0, \quad X_{j,ii} \leq \bar{u}_i^2 \quad (17)$$

for $i=1,2,\dots,m$ and $j=0,1,2,\dots,v-1$, where $X_{j,ii}$ and \bar{u}_i represent the i^{th} element of X_j and \bar{u}_i , respectively.

Proof: Multiplying $Q_j = P_j^{-1}$ on both sides of (11) yields:

$$\begin{aligned} & Q_j - (A_l Q_j + B_l K(k+j) Q_j)^T Q_{j+1}^{-1} \\ & \cdot (A_l Q_j + B_l K(k+j) Q_j) > 0, \quad (18) \\ & \quad \quad \quad l=1,2,\dots,n_p. \end{aligned}$$

for $j=0,1,2,\dots,v-1$. By the Schur complement, relation (18) can be transformed into LMIs (15) with $Y_j = K(k+j)Q_j$. The LMI (16) can be obtained from (12) in a similar manner.

Denote the i^{th} row of $K(k+j)$ as $K_i(k+j)$, then:

$$\begin{aligned} |u_i(k+j)|^2 &= |K_i(k+j)x(k+j)|^2 \\ &= |K_{ii}(k+j)P_j^{1/2}P_j^{1/2}x(k+j)|^2 \\ &\leq K_i P_j^{-1} K_i^T \cdot x(k+j)^T P_j x(k+j) \quad (19) \\ &\text{(using Cauchy-Schwarz inequality)} \\ &\leq Y_i Q_j Y_i^T. \end{aligned}$$

Thus the existence of a symmetric matrix X_j satisfying (17) guarantees that $|K_i(k+j)x(k+j)| \leq \bar{u}_i$ for $i=1,2,\dots,m$. \square

The relaxation of the definition of invariance through the introduction of periodic invariance allows the state to leave Ω_0 for a period steering it back to Ω_0 after v moves. This in turn allows for the enlargement of the volume of Ω_0 , which can be achieved through convex optimization:

Algorithm 1

$$\begin{aligned} & \min \quad -\log(\det(Q_0)) \\ & Q_j, X_j, Y_j \quad \text{subject to (15)-(17)}. \quad (20) \end{aligned}$$

\square

Algorithm 1 is a Complex Problem described in [6], which is convex and can be solved efficiently in polynomial time. Note that the LMIs (15)-(17) do not depend on the current state, so the Algorithm can be applied offline to obtain a periodic-invariant set of maximum volume.

3. RECEDING HORIZON CONTROL BASED ON PERIODIC INVARIANCE

The optimization of Algorithm 1 was aimed exclusively at the minimization of $-\log(\det(P_0^{-1}))$ with the view to enlarging the volume of Ω_0 . The sizes of accompanied ellipsoids Ω_j , $j=0,1,\dots,v-1$, are expected to be big also.

Consider the convex hull Ξ of the ellipsoids Ω_j , $j=0,1,2,\dots,v-1$. It is clear that Ξ is larger than the union of the ellipsoids Ω_j , $j=0,1,2,\dots,v-1$. Furthermore, Ξ is invariant in the sense that there exists a feasible control input $u(k)$, which makes the current state $x(k) \in \Xi$ remain in Ξ as per the following Lemma.

Lemma 1: Consider the uncertain system (1)-(2) and ellipsoidal sets Ω_j , $j=0,1,2,\dots,v-1$, defined as (10)-(13). Denote the convex hull of Ω_j , $j=0,1,2,\dots,v-1$, as Ξ . If a state $x(k)$ belongs to Ξ , then there exist a feasible control input $u(k)$ that guarantees that $x(k) \in \Xi$.

Proof: A state $x(k) \in \Xi$ can be represented as:

$$\begin{aligned} x(k) &= \sum_{j=0}^v \lambda_j x_j(k), \\ \sum_{j=0}^v \lambda_j &= 1, \quad \lambda_j \geq 0, \end{aligned} \quad (21)$$

where $x_j \in \Omega_j$. Consider the control input $u(k) =$

$\sum_{j=0}^{v-1} K_j \lambda_j x_j(k)$, then $x(k+1)$ can be represented as:

$$\begin{aligned} x(k+1) &= \tilde{A}x(k) + \tilde{B}u(k) \\ &= \sum_{j=0}^{v-1} \lambda_j (\tilde{A} + \tilde{B}K_j)x_j(k) \\ &= \sum_{j=0}^{v-1} \lambda_j x_j(k+1). \end{aligned} \quad (22)$$

From the definition of Ω_j , $j=0,1,\dots,v-1$, $x_j(k+1) = (\tilde{A} + \tilde{B}K_j)x_j(k) \in \Omega_{j+1}$. Thus, it is easy to see that $x(k+1) \in \Xi$ also and we can conclude that there always exists a feasible state feedback law that makes $x(k)$ remain in Ξ . \square

Based on the above argument, we would like to propose a MPC strategy using Ξ as a target set. Assume that Ω_0 , corresponding ellipsoids Ω_j ,

$j = 0, 1, 2, \dots, v-1$, and their convex hull Ξ were obtained by solving (20). Our control strategy is to steer the current state into Ξ using a feasible control move $u(k)$. According to the uncertainties (2) that reside in the system, $x(k+1)$ would belong to the polyhedral set of states defined as:

$$\mathfrak{S} := \left\{ x \in R^n \mid x = \sum_{l=1}^{n_p} \eta_l (A_l x(k) + B_l u(k)) \right. \\ \left. (\eta_l \geq 0), \sum_{l=1}^{n_p} \eta_l = 1 \right\}. \quad (23)$$

It is easy to see that $\mathfrak{S} \subset \Xi$ is guaranteed if and only if all the vertices of \mathfrak{S} i.e. $A_l x(k) + B_l u(k)$ belong to Ξ . If $x_l(k+1) = A_l x(k) + B_l u(k) \in \Xi$, then $x_l(k+1)$ can be represented as:

$$x_l(k+1) = \sum_{j=0}^{v-1} \lambda_{l,j} x_{l,j}(k+1) \quad (24)$$

$$(\lambda_{l,j} \geq 0, \sum_{j=0}^{v-1} \lambda_{l,j} = 1),$$

where $x_{l,j}(k+1) \in \Omega_j$. If we denote $\lambda_{l,j} x_{l,j}$ as $\hat{x}_{l,j}$ then the conditions (24) and $x_{l,j} \in \Omega_j$ can be rewritten as:

$$A_l x(k) + B_l u(k) = \sum_{j=0}^{v-1} \hat{x}_{l,j}(k+1), \quad (25)$$

$(|u(k)| \leq \bar{u})$ and

$$\hat{x}_{l,j}^T \frac{P_j}{\lambda_{l,j}} \hat{x}_{l,j} \leq \lambda_{l,j}, \quad (26)$$

respectively, for $l = 1, 2, \dots, n_p$. Thus, the existence of vectors $x_{l,j}(k+1)$ and scalar values $\lambda_{l,j}$ for $l = 1, 2, \dots, n_p$ and $j = 0, 1, 2, \dots, v-1$ satisfying (25)-(26) guarantees that $x(k+1) \in \Xi$.

The control input $u(k)$ satisfying (25)-(26) would not be unique. Thus, we need certain criteria to choose a particular $u(k)$ that is optimal in some sense. Consider the state decomposition (25) and define a quadratic function:

$$V(x_l(k+1|k)) := \sum_{j=0}^{v-1} \hat{x}_{l,j}(k+1)^T P_j \hat{x}_{l,j}(k+1). \quad (27)$$

We would like to use an upper bound on $V(x_l(k+1|k))$ as our cost index i.e.

$$\alpha \geq \sum_{j=0}^{v-1} \alpha_j \quad (28)$$

$$\alpha_j \geq \hat{x}_{l,j}(k+1)^T P_j \hat{x}_{l,j}(k+1). \quad (29)$$

According to the relations (11)-(12), $V(x_l(k+1|k))$ and in turn α can be made to be monotonically decreasing, which will be shown later. Note that relations (25)-(26) and (28)-(29) can be rewritten as the following LMIs:

$$\text{diag}(A_l x(k) + B_l u(k) - \sum_{j=0}^{v-1} \hat{x}_{l,j}(k+1)) = 0 \quad (30)$$

$$\begin{bmatrix} \lambda_{l,j} & \hat{x}_{l,j}(k+1)^T \\ \hat{x}_{l,j}(k+1) & \lambda_{l,j} Q_j \end{bmatrix} > 0, \quad (31)$$

$$j = 0, 1, \dots, v-1, l = 1, 2, \dots, n_p$$

$$\text{diag}(\bar{u} - u(k)) \geq 0 \quad (32)$$

$$\alpha \geq \sum_{j=0}^{v-1} \alpha_j \quad (33)$$

$$\begin{bmatrix} \alpha_j & \hat{x}_{l,j}(k+1)^T \\ \hat{x}_{l,j}(k+1) & Q_j \end{bmatrix} > 0, \quad (34)$$

$$j = 0, 1, \dots, v-1, l = 1, 2, \dots, n_p.$$

Now the receding horizon control method based on the above argument can be described as follows:

Algorithm 2

Step 1: (off-line) Obtain matrices $P_j, j = 0, 1, 2, \dots, v$ and corresponding ellipsoidal sets $\Omega_j, j = 0, 1, \dots, v-1$ according to Algorithm 1.

Step 2: (on-line) For a given current state $x(k)$ compute the optimal control $u(k)$ as:

$$u^*(k) = \arg \left\{ \min_{u(k)} \max_{l, \hat{x}_{l,j}, \alpha_j} \alpha \right\} \quad (35)$$

subject to (30)-(34).

Apply the optimal $u(k)$ to the system and repeat this on-line procedure on the next time steps. \square

The closed-loop stability of Algorithm 2 can be established as per the following theorem:

Theorem 2: Consider the uncertain system (1)-(2). Assume that matrices $P_{j,j}=0, 1, 2, \dots, v$ and corresponding ellipsoidal sets $\Omega_j, j = 0, 1, 2, \dots, v-1$ were obtained as Step 1 of Algorithm 2 and that Step 2 was feasible at the initial time step, then Step 2 of Algorithm 2 remains feasible and the use of the optimal control $u(k)$ obtained at each time step guarantees the asymptotic stability of the closed-loop system.

Proof: Feasibility: Conditions (25)-(26) guarantee that $x(k+1) \in \Xi$ for a given $x(k)$. Once the state is steered into Ξ , (25)-(26) would have feasible solutions for all the subsequent time steps since Ξ is invariant as Lemma 1.

Stability: Assume that $x(k+1) = \sum_{j=0}^{v-1} \lambda_j x_j(k+1)$ and $x_j \in \Omega_j$ for $j=0,1,2,\dots,v-1$. Consider the case in which the control input $u(k) = \sum_{j=0}^{v-1} K_j \lambda_j x_j(k)$ is applied to the system, then $x(k+2)$ can be represented as:

$$\begin{aligned} x(k+2) &= \tilde{A}x(k+1) + \tilde{B}u(k) \\ &= \sum_{j=0}^{v-1} (\tilde{A} + \tilde{B}K_j) \hat{x}_j(k+1) \\ &= \sum_{j=0}^{v-1} \hat{x}_{j+1}(k+1), \end{aligned} \tag{36}$$

where $\hat{x}_{j+1}(k+2) := (\tilde{A} + \tilde{B}K_j) \hat{x}_j(k+1)$. Then, from relations (11)-(12) we have:

$$\hat{x}_j(k+1)^T P_j \hat{x}_j(k+1) > \hat{x}_{j+1}(k+2)^T P_{j+1} \hat{x}_{j+1}(k+2) \tag{37}$$

for $j=0,1,2,\dots,v-1$ with $\hat{x}_v(k+2) := \hat{x}_0(k+2)$. Denote the upper bounds as:

$$\alpha_j(k+2) \geq \hat{x}_j(k+2)^T P_j \hat{x}_j(k+2), \tag{38}$$

$$\alpha(k+2) \geq \sum_{j=0}^{v-1} \alpha_j(k+2). \tag{39}$$

Then from (37), we could have $\alpha(k+2)$ smaller than $\alpha(k+1)$. This argument can be applied recursively to conclude that $\alpha(k+i)$ can be made to be monotonically decreasing and in turn the states approach to the origin. \square

4. NUMERICAL EXAMPLE

Consider the uncertain system (1) with polyhedral set Π defined by (2) with $\bar{u} = 1$ and matrices

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.9347 & 0.5194 \\ 0.3835 & 0.8310 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.0591 & 0.2641 \\ 1.7971 & 0.8717 \end{bmatrix}, \\ B &= \begin{bmatrix} -1.4462 \\ -0.7012 \end{bmatrix}. \end{aligned} \tag{40}$$

This system is unstable and has uncertainties in the system matrix A . We apply Algorithm 2 to this system. Its feasibility depends on the current state $x(k)$.

Once a feasible solution has been obtained, the state can be steered to the origin as it was shown in Theorem 2. The set of states for which Algorithm 2 is feasible would become the stabilizable region of states.

Fig. 1 shows stabilizable region of states with $v=3, 5$ and 9 . This figure shows that by increasing v , we can obtain a considerable increase of volume for a stabilizable set. Note that $v=1$ is equivalent to using ordinary feasible and invariant sets. The stabilizable regions are bigger than those of earlier works [2] and [4], which are based on ordinary invariant sets.

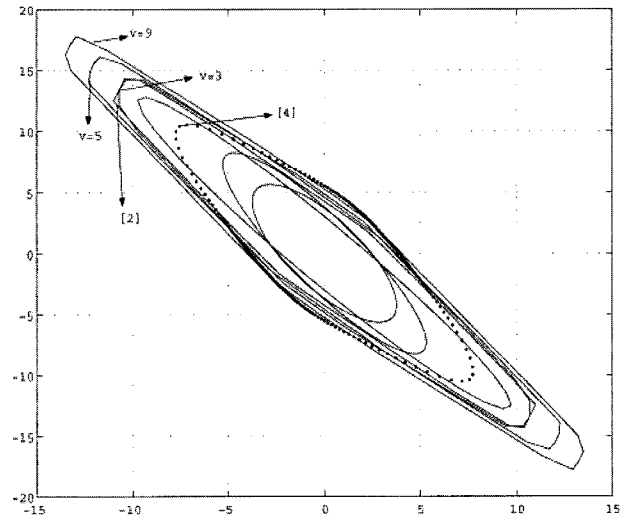


Fig. 1. Regions of states for which Algorithm 2 has feasible solutions with $v=3$ (inner line), 5 and 9 (outer line), which are larger than those of earlier works [4] (dotted line).

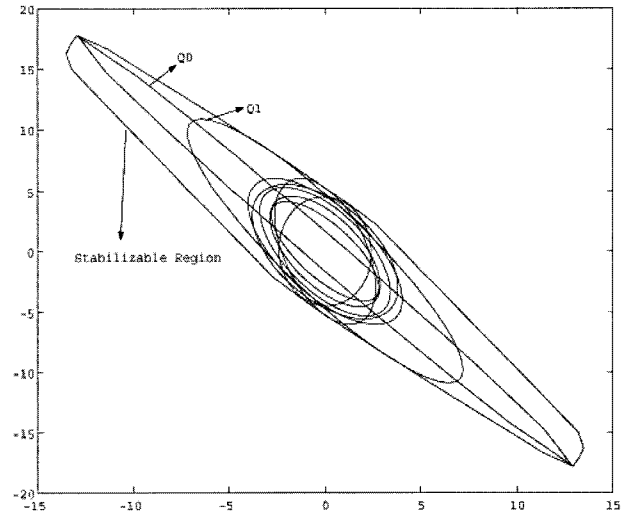


Fig. 2. The region of state for which Algorithm 2 has feasible solutions with $v=9$ along with corresponding ellipsoidal solutions $\Omega_j, j=0,1,2,\dots,8$.

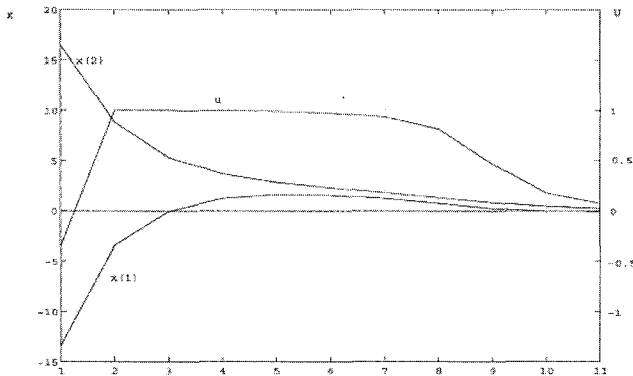


Fig. 3. State and input trajectories when Algorithm 2 is applied to system (40) with initial state $x(0) = [-13.4 \ 16.5]^T$ and $v=9$.

The stabilizable region of states with $v=9$ is shown in Fig. 2 along with corresponding ellipsoidal sets Ω_j , $j=0,1,2,\dots,v-1$. In Fig. 3, both state and input trajectory when Algorithm 2 was applied to the system with initial state $x(0) = [-13.4 \ 16.5]^T$ are shown. The control input is almost saturated for the time steps 2 to 6.

5. CONCLUSIONS

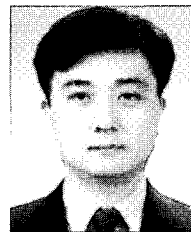
A receding horizon control strategy was developed for input constrained linear uncertain systems based on periodically invariant sets. The definition of the periodically invariant set allows the state to leave the set temporarily. An ellipsoidal set is said to be periodically invariant if there is a series of feedback gains such that the use of these gains guarantees that all the states in the set return to the set in finite time steps. The convex hull of these periodically invariant sets can be shown to be positively invariant in the sense that there exists a feasible input that makes the states remain in the convex hull.

A receding horizon control strategy in which the current state is steered into the convex hull of periodically invariant sets was proposed. A Lyapunov function is defined as a sum of quadratic functions and it was shown that this Lyapunov function can be made to be monotonically decreasing by using a nonlinear control law based on the partitioning of the current state and applying different feedback gains for the partitioned states.

The invariant set used in this paper contains the ellipsoidal invariant sets from earlier works, which were based on ordinary invariant sets, as a special case. It will provide a larger invariant set and a larger stabilizable set in turn.

REFERENCES

- [1] Y. I. Lee and B. Kouvaritakis, "Robust receding horizon predictive control for systems with uncertain dynamics and input saturation," *Automatica*, vol. 36, pp. 1497-1504, 2000.
- [2] Y. I. Lee and B. Kouvaritakis, "A linear programming approach to constrained robust predictive control," *IEEE Trans. on Automatic Control*, vol. 45, no. 9, pp. 1765-1770, 2000.
- [3] Y. I. Lee, "Receding horizon control for input constrained linear parameter varying systems," *IEE Proceedings-Control Theory and Application*, vol. 151, no. 5, pp. 547-553, September 2004.
- [4] M. V. Kothare, V. Balakrishnan, and M. Morari, "Robust constrained model predictive control using linear matrix inequalities," *Automatica*, vol. 32, no. 10, pp. 1361-1379, 1996.
- [5] Y. Lu and Y. Arkun, "A scheduling quasi-minmax MPC for LPV systems," *Proc. of American Control Conference*, pp. 2272-2276, 1999.
- [6] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*, SIAM, Philadelphia, 1994.



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