Robust Switching-Type Fuzzy-Model-Based Output Tracker

Ho Jae Lee, Jin Bae Park*, and Young Hoon Joo

Abstract: This paper discusses an output-tracking control design method for Takagi–Sugeno fuzzy systems with parametric uncertainties. We first represent the concerned system as a set of uncertain linear systems. The tracking problem is then converted into a stabilization problem thereby leading to a more feasible control design procedure. A sufficient condition for robust practical output tracking is derived in terms of a set of linear matrix inequalities. A numerical example for a flexible-joint robot-arm model has been demonstrated, to convincingly show effectiveness of the proposed system modeling and control design.

Keywords: Generalized Lyapunov function, output-tracking, switching control, Takagi–Sugeno fuzzy systems.

1. INTRODUCTION

A problem of major importance in control task is driving the output of a plant to follow some desired signal generated by an exogenous system. With a practical point of view, remarkable significance of the output-tracking lies in robot control, chemical process control, and aircraft attitude control [11]. However, such applications have often severe nonlinearities and uncertainties, which thus post additional difficulties to the output-tracking control design. So far, variety of techniques in this field has been consistently pursued with tremendous effort by many researchers. One of them is to synergetically merge the linear output-tracking scheme [18,19] and the Takagi–Sugeno (T–S) fuzzy-model-based control technology [1-6,8-10,12-14,16,17], which provides a way to achieve output-tracking for nonlinear control systems [18,19].

Regarding recent progress in the T–S fuzzy-model-based control, it is observed that a number of important works have used a single positive definite matrix [1-5,9-13,16,18,19] or several ones [6,8,15,17] to meet the global Lyapunov stability requirements. The most previous output-tracking strategies via the T–S fuzzy system likewise fall under the former category [18,19].

In [18,19], the output-tracking problem is conducted as a stabilizing one by using an affine transformation [20]. However as is pointed out in [7], the problem reformulation is incorrect because it is very difficult in general, to find such an affine transformation that holds for the entire fuzzy input-space with deliberation of the strong nonlinear interaction among the fuzzy rules. In addition, the uncertainty issue is not treated in their discussions. It is commonly believed that the presence of uncertainty threatens stability of a control system.

The piecewise Lyapunov function approach can be an alternative. Interestingly, the advantage of this approach is the exclusion of the nonlinear interaction between fuzzy rules thereby the ease of switching T–S fuzzy-model-based control synthesis. At this point, we pay attention to the fact that it might admit reemployment of the affine transformation technique to the output-tracking problem that has not yet been fully tackled under this framework. Motivated by the aforementioned observations, this paper aims at studying the robust output-tracking control problem for T–S fuzzy systems in the presence of norm-bounded time-varying uncertainties, in the framework of [15,17]. The main contributions of this paper are twofold: some theoretical analysis of the output-tracking error by means of the piecewise Lyapunov function and the constructive tracker design condition parameterized in terms of linear matrix inequality (LMI).

Specifically to achieve these, an uncertain continuous-time T–S fuzzy system is first represented as a set of uncertain linear systems on the basis of the

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firing strength dominance. Therein the real uncertainty and the fuzzy-rule-interaction are treated in a unified manner. This representation facilitates one to globally exploit the affine transformation on an entire fuzzy-input space, and to properly convert the output-tracking problem into the stabilizing one with bounded impulses.

The switching fuzzy-model-based tracker is employed to seek a control piecewise Lyapunov function that may even not live in C^1. For that reason, the general Lyapunov function concept is imported to manipulate such a nonsmooth Lyapunov function [22]. To verify the existence of the switching T–S fuzzy-model-based tracker, we constitute the tracker design condition as a convex optimization problem in terms of LMIs.

The rest of the paper is organized as follows: Section 2 briefly reviews a continuous-time T–S fuzzy system and its alternative representation. In Section 3, the output-tracking problem for the uncertain T–S fuzzy system is formulated. The control design method for robust output tracking of the uncertain T–S fuzzy system is then proposed in Section 4. Section 5 shows a computer simulation of output-tracking control for a flexible-joint robot arm model subject to the norm-bounded time-varying uncertainties. Finally, conclusions are drawn in Section 6.

2. T–S FUZZY SYSTEMS

The ith rule of an uncertain T–S fuzzy system we consider has the following form:

R^i : IF x_1 is about Γ_1^i and $\cdots$ and x_n is about Γ_n^i

$$\text{THEN}\begin{cases}\dot{x}=(A_i+\Delta A_i)x+(B_i+\Delta B_i)u \\ y=C_i x,\end{cases} \tag{1}$$

where $x\in\mathbb{R}^n$ state; $u\in\mathbb{R}^m$ control input; $y\in\mathbb{R}^p$ output. Using the center-average deffuzifier, product inference, and singleton fuzzifier the global dynamics of (1) is inferred as

$$\dot{x}=\sum_{i=1}^{r}\theta_i((A_i+\Delta A_i)x+(B_i+\Delta B_i)u), \tag{2}$$

$$y=\sum_{i=1}^{r}\theta_i C_i x, \tag{3}$$

where matrices ΔA_i and ΔB_i are real-valued matrix function representing uncertainties, and

$$\theta_i=\omega_i\Big/\sum_{i=1}^{r}\omega_i,\quad \omega_i(x)=\prod_{j=1}^{n}\Gamma_j^i(x_j)$$

and $\Gamma_j^i:\mathcal{U}_{x_j}\subset\mathbb{R}\to\mathbb{R}_{[0,1]}$ is the membership value of x_j in Γ_j^i, where $\mathcal{U}_{x_j}$ is the universe of discourse of the premise variable x_j.

In order to overcome the nonlinear interaction between the fuzzy rules in this study, the global dynamical behavior (2) is represented as a set of local uncertain linear systems that describe the local dynamical behavior in the corresponding operating sets defined as follows:

$$\Theta_i=\{x\,|\,\theta_i\ge\theta_j,\ j\in\mathcal{I}_r\setminus\{i\}\}_{i\in\mathcal{I}_r}, \tag{4}$$

where $\mathcal{I}_r=\{1,2,\ldots,r\}$. The boundary of Θ_i, denoted by $\mathrm{Bdy}\Theta_i$, on which x drifts from Θ_i to Θ_j, $j\in\mathcal{I}_r\setminus\{i\}$, is defined by

$$\mathrm{Bdy}\Theta_i=\{x\,|\,x^-\in\Theta_i,\ x^+\in\Theta_j,\ j\in\mathcal{I}_r\setminus\{i\}\}_{i\in\mathcal{I}_r}. \tag{5}$$

Then, in the whole space of interest, (2) can be rearranged as follows:

$$\dot{x}=\sum_{i=1}^{r}\kappa_i((A_i+\Delta_{ai})x+(B_i+\Delta_{bi})u), \tag{6}$$

where

$$\Delta_{ai}=\sum_{j=1}^{r}\theta_j(\Delta A_i+\Delta A_{ij})$$

and $\Delta_{bi}=\sum_{j=1}^{r}\theta_j(\Delta B_i+\Delta B_{ij})$ $\Delta A_{ij}=A_j-A_i$ $\Delta B_{ij}=B_j-B_i$, and $\kappa_i:\mathcal{U}_x\times\mathcal{I}_r\to\mathbb{Z}_{[0,1]}$ that characterizes the activation of Θ_i is defined by

$$\kappa_i=\begin{cases}1, & x\in\Theta_i,\\ 0, & x^-\in\mathrm{Bdy}\Theta_i,\ x\notin\Theta_i.\end{cases} \tag{7}$$

Assumption 1: The uncertainties considered here are norm-bounded of the form:

$$[\Delta A_i\ \ \Delta B_i]=H_iF_i[E_{ai}\ \ E_{bi}],$$

where F_i is an unknown matrix function with Lebesgue-measuarable elements and satisfies $F_i^TF_i\le I$.

As is the case of Assumption 1, the following is adopted:

$$[\Delta A_{ij}\ \ \Delta B_{ij}]=H_{ij}^{\mathcal{M}}\left[E_{aij}^{\mathcal{M}}\ \ E_{bij}^{\mathcal{M}}\right].$$

In a set Θ_i, the matrix functions Δ_{ai} and Δ_{bi} can be lumped with ΔA_i and ΔB_i, and decomposed into the specific form

$$[\Delta_{ai}\ \ \Delta_{bi}]=\sum_{j=1}^{r}\theta_j\mathcal{H}_{ij}\mathcal{F}_{ij}[\mathcal{E}_{aij}\ \ \mathcal{E}_{bij}],$$

where

$$\mathcal{H}_{ij}=\left[\mathcal{H}_j\ \ \mathcal{H}_{ij}^{\mathcal{M}}\right],\quad [\mathcal{E}_{aij}\ \ \mathcal{E}_{bij}]=\begin{bmatrix}\mathcal{E}_{aj} & \mathcal{E}_{bj}\\ \mathcal{E}_{aij}^{\mathcal{M}} & \mathcal{E}_{bij}^{\mathcal{M}}\end{bmatrix}$$

and $\mathcal{F}_{ij} = \text{diag}\{F_j, I\}$ satisfies $\mathcal{F}_{ij}^T \mathcal{F}_{ij} \le I$, $j \in \mathcal{I}_r$.

3. PROBLEM STATEMENT

Throughout this paper, the reference to be tracked by (3) is assumed to be the output $\upsilon \in \mathbb{R}^p$ of the following exogenous system

$$\dot{\zeta} = \Psi\zeta, \tag{8}$$
$$\upsilon = \Phi\zeta. \tag{9}$$

Assumption 2: To be practical, it is assumed $\|\zeta\|_\infty = \zeta_M$.

Definition 1: Let e be the difference between output of a dynamical system and the reference signal. Given $\rho > 0$, the system of interest is said to be ρ-trackable if there is a control u implying the existence of positive constants ν for all $\eta \in (0, \nu)$, and $T = T(\eta)$ independent of t_0 such that $\|e(t_0)\| < \eta \Rightarrow \|e\| \le \rho$ for all $t \ge t_0 + T$.

Problem 1 (ρ-tracking controller design)**:** Let the output tracking error be $e = y - \upsilon$. The objective is to design a T–S fuzzy-model-based controller such that (3) tracks (9) with the tracking error to be ultimately uniformly bounded (UUB), and the controlled system is robustly stable against the admissible uncertainty.

In order to construct the error dynamics, a new state is defined as

$$\chi := x - T_i\zeta \tag{10}$$

for $\kappa_i = 1$, $i \in \mathcal{I}_r$, where T_i is a solution to the following matrix equations:

$$\begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix}\begin{bmatrix} T_i \\ L_i \end{bmatrix} = \begin{bmatrix} T_i & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} \Psi \\ \Phi \end{bmatrix}.$$

For a detail discussion of the solution to (11), readers may refer [20].

Assumption 3: Assume that

$$\text{rank}\begin{bmatrix} A_i & B_i \\ C_i & 0 \end{bmatrix} = n + p,$$

so that (11) is solvable [20]. It is satisfied if each nominal subsystem in (6) is controllable and the number of outputs is less than or equal to the number of the inputs, i.e., $n \ge p$.

In this study, the following output-tracking controller is employed:

$$u = \sum_{i=1}^{r} \kappa_i (L_i\zeta + \nu),$$

where ν remains to be determined.

Remark 1: It is noted that usage of (10) brings about finite impulses on χ at the instances of beating the closure of the subspaces. Therefore, in the following discussion the dynamical behavior is described by the uncertain linear differential equations with finite jumps.

After some algebraic manipulations using (11), χ defined in (10) satisfies

$$\dot{\chi} = \sum_{i=1}^{r} \kappa_i ((A_i + \Delta_{ai})\chi + (B_i + \Delta_{bi})\nu + \Delta_{di}\zeta) \tag{13}$$

for $x \in \bigcup_{i \in \mathrm{I}_r} \text{Int}\Theta_i$

$$\chi^+ = I\chi + \sum_{k=1}^{r} \theta_k (B_k + \Delta B_k)\nu + \Delta_{ij}\zeta \tag{14}$$

for $x \in \text{Bdy}\Theta_i$, $x^+ \in \Theta_j$, and

$$e = \sum_{i=1}^{r} \theta_i C_i \zeta,$$

where $\Delta_{di} = \Delta_{ai}T_i + \Delta_{bi}L_i$ and $\Delta_{ij} = T_i - T_j$.

Remark 2: Since $\Delta_{di} = \Delta_{ai}T_i + \Delta_{bi}L_i$, this uncertain matrix can also be decomposed as $\Delta_{di} = \mathcal{H}_i\mathcal{F}_i\mathcal{E}_{di}$ for all $t \in [0, \infty)$, where $E_{di} = E_{ai}T_i + E_{bi}L_i$.

It is observed that χ in (13) and (14) is left-continuous but right-discontinuous with respect to the switching instance of Θ_i, thus we may not build the Lyapunov function of which class is C^1. The following relaxed notion allows us to handle such a semicontinuous system.

Definition 2 (Generalized Lyapunov Function)**:** A lower semicontinuous, positive-definite function V: $V: \mathbb{R}^n \to \mathbb{R}_{>0}$, with V being continuous at the origin that satisfies $V(x(t)) \le V(x(\tau))$, for all $t \ge \tau \ge 0$, is called a generalized Lyapunov function [22].

Theorem 1: Consider the dynamical system (13) and (14) closed by some ν. If there exist a piecewise C^1 function $V(\chi, \kappa_i): \mathbb{B}_\nu \subset \mathbb{R}^n \times \mathcal{I}_r \to \mathbb{R}$ and class $\mathcal{K}$ functions $\phi_i[0, \nu) \to \mathbb{R}_{>0}$, $i \in \mathbb{Z}_{[1,4]}$, such that

1) $\phi_1(\|\chi\|) \le V(\chi, \kappa_i) \le \phi_2(\|\chi\|)$

2) The left-sided derivative satisfies

$$\frac{d}{dt}V(\chi, \kappa_i) = \lim_{\delta \to 0^-} \inf \left\{ \frac{V(\chi(t+\delta), \kappa_i(x(t+\delta)))}{\delta} - \frac{V(\chi, \kappa_i)}{\delta} \right\}$$
$$\le -\phi_3(\|\chi\|)$$

for all χ outside a hyper-ball $\mathbb{B}_{\mu_1} = \{\chi \mid \|\chi\| \le \mu_1\}$ and $x \in \bigcup_{i \in \mathrm{I}_r} \text{Int}\Theta_i$.

3) The rate of increase of $V(\chi,\kappa_i)$ fulfills

$$\Delta V = V(\chi^+,\kappa_j(x^+)) - V(\chi,\kappa_i) \\ \leq -\phi_4(\|\chi\|)$$

for all χ outside a hyper-ball $\mathbb{B}_{\mu 2} = \{\chi \mid \|\chi\| \leq \mu_2\}$ and $x \in \bigcup \mathrm{Bdy}\Theta_i$, $x^+ \in \Theta_j$.

then any closed-loop solution $\chi(t,t_0,\chi_0)$ for any pair $(t_0,\chi_0) \in \mathbb{R}_{>0} \times \mathbb{B}_\eta$ is UUB with an ultimate bound $\rho_\chi = \phi_1^{-1} \circ \phi_2(\mu)$ where $\mu = \max\{\mu_1,\mu_2\}$ and $\circ$ denotes the composition of functions. If the assumptions hold for $(0,\nu) = \mathbb{R}_{>0}$ and $\phi_1 \in \mathcal{K}_\infty$, then $\chi(t,t_0,\chi_0)$ is globally ultimately uniformly bounded (GUUB).

Proof: From item 2), we know the quantity V strictly decreases on $\bigcup_{i\in\mathcal{I}_r}\mathrm{Int}\Theta_i$. Furthermore, from item 3) V still decreases on $\mathrm{Bdy}\Theta_i$, hence $V(\chi(t)) < V(x(t_0))$ under arbitrary choosing t and t_0 such that $t > t_0$. As a result, there is a class $\mathcal{KL}$ function such that $\phi_5 : [0,\phi_2(\nu)) \times [0,\infty) \to [0,\infty)$ for all χ outside $\mathbb{B}_\mu = \{\chi \mid \|\chi\| \leq \mu\}$. From item 1) we induce that

$$\begin{aligned}\|\chi\| &\leq \phi_1^{-1}(\nu) \\ &\leq \phi_1^{-1} \circ \phi_5(V(\chi(t_0)), t - t_0) \\ &\leq \phi_1^{-1} \circ \phi_5(\phi_2(\|\chi(t_0)\|), t - t_0) \\ &= \phi_6(\|\chi(t_0)\|, t - t_0) \\ &\leq \phi_6(\eta, t - t_0),\end{aligned}$$

where ϕ_6 is in class $\mathcal{KL}$. Once $\phi_6 < \mu$, it is true that $\|\chi\| \leq \phi_1^{-1} \circ \phi_2(\mu)$ for $t \in [T,\infty)$ hence the UUB directly follows. This completes the proof. □

Corollary 1: Suppose that all conditions in Theorem 1 are sufficed with $\phi_1 \in \mathcal{K}_\infty$, then the output-tracking error $e(t,t_0,e_0)$ is GUUB.

Problem 2: Find a control ν for (13) such that the resulting closed-loop system is robustly UUB in the presence of the norm-bounded uncertainties and the disturbance which belongs to $\mathcal{L}_\infty[0,\infty)$. In this case (13) is said to be robustly stabilizable in the presence of structured uncertainties. Furthermore, (2) is said to be robustly ρ-trackable.

4. ROBUST OUTPUT-TRACKING CONTROL DESIGN

Before proceeding, recall the following results which will be used in the proof of our results.

Lemma 1: For any vectors $x, y \in \mathbb{R}^n$ and for any compatible constant matrices H_j, E_j, F_j satisfying $F_j^T F_j \leq I$ and some scalar $\varsigma \in \mathbb{R}_{>0}$, the following holds:

$$2x^T\left(\sum_{j=1}^{r}\theta_j H_j F_j E_j\right)y \leq \frac{1}{\varsigma}x^T HH^T x + \varsigma y E^T E y,$$

where $HH^T = \mathcal{M}(H_j,\mathcal{I}_r)$ and

$$\mathcal{M}(H_j, I_L) := \arg\min_{X_H, j\in\mathcal{I}_r} (\alpha_H + \mathrm{trace}(X_H))$$

subject to

$$-X_H + H_j H_j^T \prec 0$$

$$\begin{bmatrix} -\alpha_H I & (\bullet)^T \\ X_H & -\alpha_H I \end{bmatrix} \prec 0,$$

where $(\bullet)^T$ denotes the transposed element in the symmetric position. Matrix $E^T E = \mathcal{M}(E_j^T,\mathcal{I}_r)$ is also determined in a same manner.

Lemma 2: Given constant matrices $\mathcal{H}_j$, $\mathcal{E}_j$, and $\mathcal{S} = \mathcal{S}^T$, the following inequalities are equivalent:

1) $\mathcal{S} + \sum_{j=1}^{r}\theta_j(\mathcal{H}_j\mathcal{F}_j\mathcal{E}_j + \mathcal{E}_j^T\mathcal{F}_j^T\mathcal{H}_j^T) \prec 0$

2) $\mathcal{S} + \varepsilon\mathcal{H}\mathcal{H}^T + \varepsilon^{-1}\mathcal{E}^T\mathcal{E} \prec 0$

where $\mathcal{F}_j$ satisfies $\mathcal{F}_j^T\mathcal{F}_j \prec I$ and ε is some positive scalar, and $\mathcal{H}\mathcal{H}^T = \mathcal{M}(\mathcal{H}_j,\mathcal{I}_r)$, $\mathcal{E}^T\mathcal{E} = \mathcal{M}(\mathcal{E}_j^T,\mathcal{I}_r)$.

Proof: $(\Leftarrow)$ For any $x \neq 0$ by Lemma 1,

$$\begin{aligned} 0 &> x^T(\mathcal{S} + \varepsilon^{-1}\mathcal{E}^T\mathcal{E} + \varepsilon\mathcal{H}\mathcal{H}^T)x \\ &\geq x^T\left(\mathcal{S} + \sum_{j=1}^{r}\theta_j(\mathcal{H}_j\mathcal{F}_j\mathcal{E}_j + \mathcal{E}_j^T\mathcal{F}_j^T\mathcal{H}_j^T)\right)x.\end{aligned}$$

$(\Rightarrow)$ For all $x \neq 0$

$$x^T\mathcal{S}x < -2\max\left\{x^T\left(\sum_{j=1}^{r}\theta_j\mathcal{H}_j\mathcal{F}_j\mathcal{E}_j\right)x\right\}.$$

The above implies

$$(x^T\mathcal{S}x)^2 > 4\max\left\{\left(\sum_{j=1}^{r}\theta_j(x^T\mathcal{H}_j\mathcal{F}_j\epsilon_j x)\right)^2\right\}.$$

By using the Jensen's inequality and the Schwarz inequality, it is seen that

$$\left\{\sum_{j=1}^{r}\theta_j(x^T\mathcal{H}_j\mathcal{F}_j\mathcal{E}_jx)\right\}^2 \le \sum_{j=1}^{r}\theta_j(x^T\mathcal{H}_j\mathcal{F}_j\mathcal{E}_jx)^2$$
$$\le \sum_{j=1}^{r}\theta_j(x^T\mathcal{H}_j\mathcal{H}_j^T xx^T\mathcal{E}_j^T\mathcal{E}_jx)$$
$$\le \sum_{j=1}^{r}\theta_j(x^T\mathcal{H}\mathcal{H}^T xx^T\mathcal{E}^T\mathcal{E}x)$$
$$= x^T\mathcal{H}\mathcal{H}^T xx^T\mathcal{E}^T\mathcal{E}x.$$

From [23,Lemma 2] we know the existence of $\varepsilon \in \mathbb{R}_{>0}$ such that $\varepsilon^2\mathcal{H}\mathcal{H}^T + \varepsilon\mathcal{S} + \mathcal{E}^T\mathcal{E} \prec 0$.

As an additional control, the following is introduced:

$$\nu = \begin{cases} \sum_{i=1}^{r}\kappa_i k_i \chi, & \text{for } x \in \text{Int}\,\Theta_i \\ k_{ij}\chi, & \text{for } x \in \text{Bdy}\,\Theta_i,\ x^+ \in \Theta_j. \end{cases} \quad (16)$$

Theorem 2: Given constants $\gamma_1 \in \mathbb{R}_{>0}$ and $\gamma_2 \in \mathbb{R}_{(0,1)}$, if there exist some symmetric and positive definite matrices P_i, constant matrices k_i, k_{ij}, and positive scalars $\varepsilon_i, \varepsilon_{ij}$ such that the following LMIs have solutions, the output-tracking error e of (15) is GUUB by using (12) and (16) in the presence of norm-bounded uncertainties:

$$\begin{bmatrix} \begin{pmatrix} \gamma_1 Q_i \\ +Q_iA_i^T + A_iQ_i \\ +M_i^TB_i^T + B_iM_i \\ +\varepsilon_i\mathcal{H}_i\mathcal{H}_i^T \end{pmatrix} & (\bullet)^T \\ \begin{bmatrix} Q_i \\ M_i \end{bmatrix} & -\varepsilon_i\left(\begin{bmatrix}\mathcal{E}_{ai}^T \\ \mathcal{E}_{bi}^T\end{bmatrix}\begin{bmatrix}\mathcal{E}_{ai} & \mathcal{E}_{bi}\end{bmatrix}\right)^{-1} \end{bmatrix} \prec 0 \quad (17)$$

$$\begin{bmatrix} -(1-\gamma_2)Q_i & (\bullet)^T & (\bullet)^T \\ Q_i + B_{ij}M_{ij} & -Q_j + \varepsilon_{ij}H_iH_i^T & (\bullet)^T \\ M_{ij} & 0 & -\varepsilon_{ij}(E_{bij}^TE_{bij})^{-1} \end{bmatrix} \prec 0 \quad (18)$$

for all $i \in \mathcal{I}_r$ and for all $(i,j) \in \mathcal{I}_r \times (\mathcal{I}_r / \{i\})$, where $H_{ij}H_{ij}^T = \mathcal{M}(H_h, \mathcal{I}_r)$, $E_{bij}^TE_{bij} = \mathcal{M}(E_h^T, \mathcal{I}_r)$, $\mathcal{H}_i\mathcal{H}_i^T = \mathcal{M}(\mathcal{H}_{ij}, \mathcal{I}_r)$, $\begin{bmatrix}\mathcal{E}_{ai}^T \\ \mathcal{E}_{bi}^T\end{bmatrix}\begin{bmatrix}\mathcal{E}_{ai} & \mathcal{E}_{bi}\end{bmatrix} = \mathcal{M}\left(\begin{bmatrix}\mathcal{E}_{aij} & \mathcal{E}_{bij}\end{bmatrix}^T, \mathcal{I}_r\right)$, $Q_i = P_i^{-1}$, $M_{ij} = k_{ij}P_i^{-1}$.

Proof: Consider a generalized Lyapunov function candidate: $V(\chi, \kappa_i(x)) = \sum_{i=1}^{r}\kappa_i\chi^TP_i\chi$ where each $P_i = P_i^T \succ 0$. By some algebraic manipulation, i.e., sequentially applying the congruence transformation with $\text{diag}\{P, I\}$, Schur complement, Lemma 2, and Assumption 2 to (17), we can easily know

$$D\_V(\chi, \kappa_i) \le -\nu_1\|\chi\|^2 + \nu_2\|\zeta\|^2$$
$$\le -\nu_1(1-\pi_1)\|\chi\|^2 - \nu_1\pi_1\|\chi\|^2 + \nu_2\zeta_M^2$$
$$\le -\nu_1(1-\pi_1)\|\chi\|^2$$

for all $\|\chi\| \ge \sqrt{\nu_2\zeta_M^2/\nu_1\pi_1}$ and $\chi \in \bigcup_{i\in\mathcal{I}_r}\text{Int}\,\Theta_i$, where $\mathcal{H}_i\mathcal{H}_i^T = \mathcal{M}(\mathcal{H}_{ij}, \mathcal{I}_r)$, $E_{di}^TE_{di} = \mathcal{M}(E_{dij}^T, \mathcal{I}_r)$, and

$$\nu_1 = \min_{i\in\mathcal{I}_r}\left(\lambda_{\min}\left(\gamma_1P_i - \frac{1}{\varsigma_{1i}}P_i\mathcal{H}_i\mathcal{H}_i^TP_i\right)\right),$$

$$\nu_2 = \max_{i\in\mathcal{I}_r}(\lambda_{\max}(\zeta_{1i}\mathcal{E}_{di}^T\mathcal{E}_{di})),$$

and $\pi_1 \in \mathbb{R}_{(0,1)}$ is an arbitrary number close to one, and ς_{1i} is arbitrarily chosen such that ν_1 is positive. Similarly (18) implies

$$\Delta V < -\nu_3\|\chi\|^2 + \nu_4\|\zeta\|^2$$
$$\le -\nu_3(1-\pi_2)\|\chi\|^2 - \nu_3\pi_2\|\chi\|^2 + \nu_4\zeta_M^2$$
$$\le -\nu_3(1-\pi_2)\|\chi\|^2$$

for all $\|\chi\| \ge \sqrt{\nu_4\zeta_M^2/\nu_3\pi_2}$ and $x \in \text{Bdy}\,\Theta_i$, $x^+ \in \Theta_j$, where

$$\nu_3 = \min_{(i,j)\in\mathcal{I}_r\times(\mathcal{I}_r/\{i\})}\left\{\lambda_{\min}\left(\gamma_2P_i - \frac{1}{\zeta_{2ij}}(I + B_{ij}k_{ij})^T \times (I + B_{ij}k_{ij}) - \frac{1}{\zeta_{3ij}}k_{ij}^TE_b^TE_bk_{ij}\right)\right\},$$

$$\nu_4 = \max_{(i,j)\in\mathcal{I}_r\times(\mathcal{I}_r/\{i\})}\left\{\lambda_{\max}\left(\zeta_{2ij}\Delta_{tij}^TP_jP_j\Delta_{tij} + \zeta_{3ij}\Delta_{tij}^TP_jH_{ij}H_{ij}^TP_j\Delta_{tij} + \Delta_{tij}^TP_j\Delta_{tij}\right)\right\},$$

and ζ_{2ij} and ζ_{3ij} are arbitrary chosen such that ν_3 is positive.

Now, define two hyper-balls as

$$\Sigma_1 := \left\{\chi \in R^n \,\middle|\, \|\chi\| \le \sqrt{\frac{\nu_2}{\nu_1\pi_1}}\zeta_M\right\},$$

$$\Sigma_2 := \left\{\chi \in R^n \,\middle|\, \|\chi\| \le \sqrt{\frac{\nu_4}{\nu_3\pi_2}}\zeta_M\right\}.$$

According to Theorem 1, we know that $V(\chi(t)) < V(\chi(\tau))$ for all $t > \tau$ as long as χ is outside the union of Σ_1 and Σ_2. According to a standard Lyapunov extension, this demonstrates all χ thereby e re GUUB.

5. AN EXAMPLE

This section presents an illustrative example, to show the effectiveness of the proposed tracking controller design technique. More precisely, the output tracking problem for a flexible-joint robot-arm model in Fig. 1 is considered, which is modeled as follows:

$$R^1: \text{ IF } x_1 \text{ is about } \Gamma_1^1 \text{ THEN} \begin{cases} \dot{x} = A_1 x + B_1 u \\ y = C_1 x, \end{cases}$$

$$R^2: \text{ IF } x_1 \text{ is about } \Gamma_1^2 \text{ THEN} \begin{cases} \dot{x} = A_2 x + B_2 u \\ y = C_2 x, \end{cases}$$

where

$$A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ (2,1)_i & 0 & \frac{k}{I} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J} & 0 & -\frac{k}{J} & 0 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J} \end{bmatrix}^T,$$

$$(2,1)_1 = -\frac{Mgl + k}{I} \text{ and } (2,1)_2 = -\frac{\alpha Mgl + k}{I}.$$

Let the output matrices be $C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$, which satisfy Assumption 3. The membership functions —solid line for R^1 and dashed one for R^2—are shown in Fig. 2. The exogenous system parameters are chosen as

$$\Psi = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

with initial value $\zeta_0 = \zeta(0) = \begin{bmatrix} 3 & 0 \end{bmatrix}^T$, which satisfy Assumption 2. We assume that the total mass M is unknown but bounded within 30% of its nominal value due to time-varying load. By using our main result, we compute

$$T_1 = \begin{bmatrix} 1 & 0 \\ 0 & -4 \\ 1.07 & 0 \\ 0 & -4.28 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 1 & 0 \\ 0 & -4 \\ 0.94 & 0 \\ 0 & -3.94 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 2.08 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.51 & 0 \end{bmatrix},$$

$$P_1 = 1e^5 \begin{bmatrix} 2.1981 & 0.0786 & 0.8743 & 0.0023 \\ 0.0786 & 0.0051 & 0.0751 & 0.0003 \\ 0.8743 & 0.0751 & 1.2793 & 0.0066 \\ 0.0023 & 0.0003 & 0.0066 & 0.0001 \end{bmatrix},$$

$$P_2 = 1e^5 \begin{bmatrix} 2.2250 & 0.0811 & 0.9224 & 0.0027 \\ 0.0811 & 0.0053 & 0.0784 & 0.0004 \\ 0.9224 & 0.0784 & 1.3415 & 0.0071 \\ 0.0027 & 0.0004 & 0.0071 & 0.0001 \end{bmatrix},$$

$$k_1 = \begin{bmatrix} -80.11 & -5.55 & -79.01 & -0.75 \end{bmatrix},$$

$$k_2 = \begin{bmatrix} -78.19 & -5.25 & -74.08 & -0.75 \end{bmatrix},$$

$$k_{12} = \begin{bmatrix} -0.19 & -0.03 & -0.59 & -0.01 \end{bmatrix},$$

$$k_{21} = \begin{bmatrix} -0.28 & -0.03 & -0.69 & -0.01 \end{bmatrix},$$

when $\gamma_1 = 63$ and $\gamma_2 = 0.01$. The simulation results are reported in Fig. 3: the controlled output of the T–S

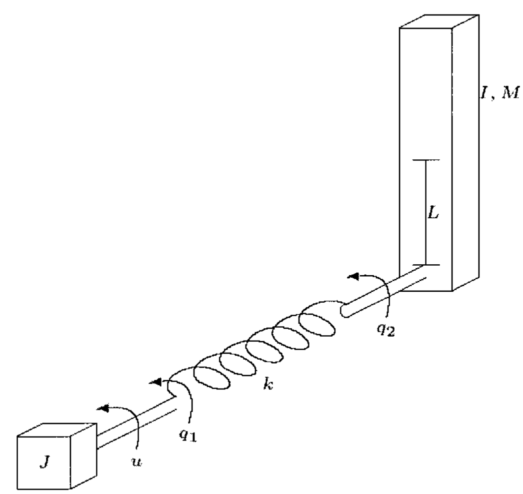

Fig. 1. A flexible-joint robot-arm.

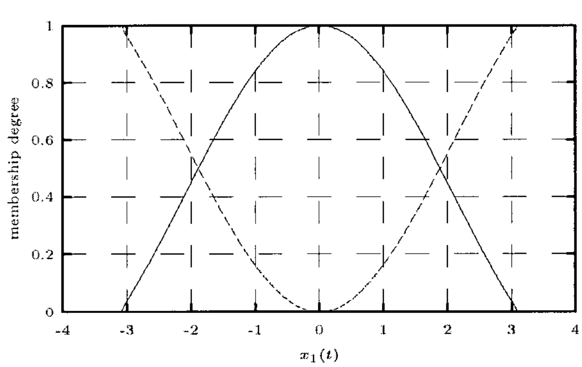

Fig. 2. Membership functions.

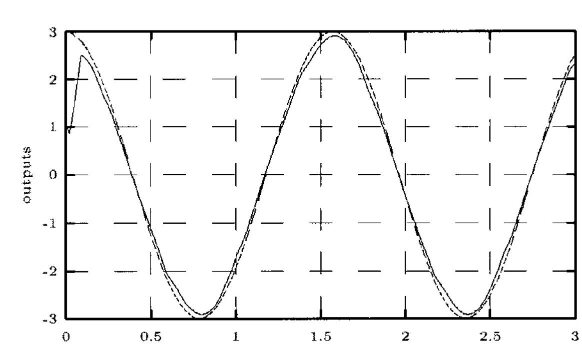

Fig. 3. Outputs of the controlled system.

fuzzy system (solid line) and the output of the exogenous system (dashed line). For the purpose of a clearer comparison, the control input is activated at t = 0.5s. Before the control input is activated, the system output x_1 (solid line) does not follow the output of the exogenous signal system (dashed line). After t = 0.5s, the output of the controlled system is quickly guided to the output of the exogenous system. Indeed, from the simulation results, one can see that the T–S fuzzy-model-based controller has a good tracking performance as well as a strong robustness against the admissible norm-bounded uncertainties.

6. CONCLUSIONS

In this paper, a new and systematic design procedure has been investigated for the robust output-tracking control of an uncertain continuous-time T–S fuzzy system in the setting of [6,15,17]. By means of the generalized Lyapunov function concept, our method has thoroughly proven the GUUB of the output-tracking error. Some constructive conditions of the tracker design have also been provided in terms of LMIs. Here, it is worth mentioning that the presented method should be distinguished from the earlier works [18,19] in that the affine transformation scheme is refined and the norm-bounded uncertainty is taken into account. Consequently, it is easily applicable to general uncertain T–S fuzzy systems. Simulation result on a flexible-joint robot arm model, with uncertainties, has convincingly demonstrated the feasibility of the developed design technique. It indicates the potential of the new method for future industrial applications.

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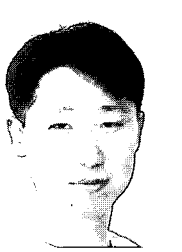

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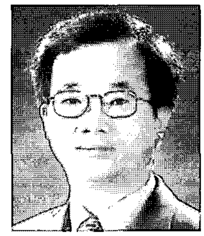

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