

# Optimal Guaranteed Cost Control of Linear Uncertain Systems with Input Constraints

Li Yu, Qing-Long Han, and Ming-Xuan Sun

**Abstract:** The guaranteed cost control problem for a class of linear systems with norm-bounded time-varying parameter uncertainties and input constraints is considered. A sufficient condition for the existence of guaranteed cost state feedback controllers is derived via the linear matrix inequality (LMI) approach, and a design procedure to guaranteed cost controllers is given. Furthermore, a convex optimization problem is formulated to determine the optimal guaranteed cost controller. An example is given to illustrate the effectiveness of the proposed results.

**Keywords:** Guaranteed cost control, input constraints, LMI, uncertain systems.

## 1. INTRODUCTION

The problem of designing robust controllers for systems with model uncertainty has drawn considerable attention in recent control system literature. Much effort has been directed towards finding a controller in order to guarantee robust stability [1-3]. However, when controlling a real plant, it is also desirable to design a controller which not only makes the closed-loop system asymptotically stable but also guarantees an adequate level of performance. One approach to this problem is the so-called guaranteed cost control approach given by Chang and Peng [4]. This approach has the advantage of providing an upper bound on a given performance index and thus the system performance degradation incurred by the model parameter uncertainties is guaranteed to be less than this bound. Based on this idea, many significant results have been proposed [5-10]. In particular, Petersen and McFarlane [6] introduced a notion of quadratic guaranteed cost control which extends the notion of quadratic stabilizability to allow for a quadratic performance index and presented a Riccati equation approach for designing quadratic guaranteed cost controllers. Yu *et al.* [9] presented a linear matrix inequality (LMI) approach for the design of guaranteed

cost controller, and the design problem of optimal guaranteed cost controller, which minimizes the associated guaranteed cost, was formulated as a convex optimization problem with LMI constraints.

On the other hand, all physical control systems have to operate under constraints on the magnitude of the control input due to the physical limitations of actuators. These limitations in terms of input constraints must be considered in the controller design. Otherwise the desired closed-loop system performance cannot be guaranteed and even the closed-loop system will become unstable. Therefore, it is necessary to consider input constraints in the design of the guaranteed cost controllers. However, at the knowledge of the authors, the guaranteed cost control problem for uncertain system subject to input constraints has been received very little attention in literature.

This paper is concerned with the guaranteed cost control problem for a class of uncertain systems subject to input constraints. The model parameter uncertainties are assumed to be time-varying and norm-bounded. Conditions for the existence of state feedback guaranteed cost controllers satisfying the given constraints are derived via the LMI approach. Furthermore, a convex optimization problem with LMI constraints is presented to design the optimal guaranteed cost controller of uncertain systems with input constraints. Finally, an example is given to illustrate the proposed results, and the comparison with the existing results is made.

## 2. PROBLEM AND PRELIMINARIES

Consider the following linear uncertain systems:

$$\begin{aligned} \dot{x}(t) &= [A + \Delta A]x(t) + [B + \Delta B]u(t), \\ x(0) &= x_0, \end{aligned} \quad (1)$$

Manuscript received February 6, 2005; accepted June 24, 2005. Recommended by Editorial Board member Jae Weon Choi under the direction of Editor-in-Chief Myung Jin Chung. This work was supported by the National Natural Science Foundation of P.R.China under grant 60274034.

Li Yu and Ming-Xuan Sun are with the Department of Automation, Zhejiang University of Technology, Hangzhou 310032, P.R.China (e-mails: {lyu, mxsun}@zjut.edu.cn).

Qing-Long Han is with the Faculty of Informatics and Communication, Central Queensland University, Rockhampton, Qld 4702, Australia (e-mail: q.han@cqu.edu.au).

where  $x(t) \in R^n$  is the state vector,  $u(t) \in R^m$  is the control input vector,  $A$  and  $B$  are known constant real matrices of appropriate dimensions,  $\Delta A$  and  $\Delta B$  are real valued matrix functions representing time-varying parameter uncertainties in the system model. The control input  $u$  in system (1) is subjected to the following constraints:

$$-\bar{u}_i \leq u_i \leq \bar{u}_i, \quad i = 1, 2, \dots, m, \quad (2)$$

where  $u_i$  is the  $i$ th element in the control input  $u$ ,  $\bar{u}_i$ ,  $i = 1, 2, \dots, m$  are known constants.

The parameter uncertainties under consideration here are assumed to be norm-bounded and of the form

$$[\Delta A \quad \Delta B] = DF(t)[E_1 \quad E_2], \quad (3)$$

where  $D$ ,  $E_1$ ,  $E_2$  are known constant real matrices of appropriate dimensions, which represent the structure of uncertainties, and  $F(t) \in R^{i \times j}$  is an unknown matrix function with Lebesgue measurable elements and satisfies

$$F^T(t)F(t) \leq I, \quad (4)$$

in which  $I$  denotes the identity matrix of appropriate dimension. The uncertainties is said to be admissible if they satisfy the relations (3) and (4).

Associated with the system (1) is the cost function

$$J = \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt, \quad (5)$$

where  $Q$  and  $R$  are given positive-definite symmetric matrices.

**Definition:** A memoryless state feedback control law  $u(t) = Kx(t)$  is said to be a quadratically guaranteed cost controller of system (1) with cost function (5) if there exists symmetric positive definite matrix  $P \in R^{n \times n}$  such that

$$Q + K^T RK + P[A + BK + DF(E_1 + E_2K)] + [A + BK + DF(E_1 + E_2K)]^T P < 0 \quad (6)$$

for all admissible uncertainties.

**Lemma 1** [6]: If  $u(t) = Kx(t)$  is a quadratically guaranteed cost controller of system (1) with cost function (5), then the closed-loop uncertain system

$$\dot{x}(t) = [A + BK + DF(E_1 + E_2K)]x(t) \quad (7)$$

is quadratically stable, and the cost function value of the closed-loop system is no more than  $J^* = x_0^T P x_0$ , which is said to be a guaranteed cost of system (1).

From the proof of Lemma 1, it follows that the matrix  $P$  is a Lyapunov matrix of the closed-loop system with the controller  $u(t) = Kx(t)$ . Furthermore, a

guaranteed cost of system (1) can be determined in terms of the matrix  $P$  and the initial state. It is clear that such a guaranteed cost depends on the choice of guaranteed cost controllers. In particular, the guaranteed cost controller to minimize the corresponding guaranteed cost is more interesting, such a controller is said to be the optimal guaranteed cost controller.

The objective of this paper is to develop a procedure to designing the optimal guaranteed cost controller for the system (1) subject to input constraints.

### 3. MAIN RESULTS

We first present the following result:

**Theorem 1:** If there exist a positive scalar  $\alpha$ , a matrix  $K$  and symmetric positive definite matrices  $P$  and  $Z$  such that the matrix inequality (6) holds for all admissible uncertainties and

$$x_0^T P x_0 \leq \alpha, \quad (8)$$

$$\begin{bmatrix} Z & K \\ K^T & \alpha^{-1}P \end{bmatrix} \geq 0, \quad (9)$$

$$(Z)_{ii} \leq \bar{u}_i^2, \quad i = 1, 2, \dots, m. \quad (10)$$

Then  $u(t) = Kx(t)$  is a quadratically guaranteed cost controller satisfying the constraint (2) of the system (1).

**Proof:** It follows from the condition of this theorem and Lemma 1 that  $u(t) = Kx(t)$  is a quadratically guaranteed cost controller of the system (1) and the matrix  $P$  is a Lyapunov matrix of the associated closed-loop system. Therefore, the inequality (8) implies that the closed-loop state trajectory  $x(t)$  satisfies  $x^T(t)Px(t) \leq \alpha$ .

By the Schur complement, it follows that the matrix inequality (9) is equivalent to  $\alpha KP^{-1}K^T \leq Z$ . Denote the  $i$  row of the matrix  $K$  by  $K_i$ , then

$$\begin{aligned} |u_i(t)|^2 &= |K_i x(t)|^2 = \left| K_i P^{-1/2} P^{1/2} x(t) \right|^2 \\ &\leq \left\| K_i P^{-1/2} \right\|^2 \left\| P^{1/2} x(t) \right\|^2 \\ &= K_i P^{-1} K_i^T x^T(t) P x(t) \\ &\leq K_i P^{-1} K_i^T \alpha \leq (Z)_{ii}. \end{aligned}$$

From the inequality (10) we can conclude that the control law  $u(t) = Kx(t)$  satisfies the constraint (2). This completes the proof of the theorem.

The following theorem is the main results of this paper.

**Theorem 2:** If there exist positive scalars  $\alpha$  and  $\varepsilon$ , a matrix  $Y$  and symmetric positive definite matrices  $X$  and  $Z$  such that the following matrix inequalities hold:

$$\begin{bmatrix} \Omega & X & Y^T & (E_1X + E_2Y)^T \\ X & -\alpha Q^{-1} & 0 & 0 \\ Y & 0 & -\alpha R^{-1} & 0 \\ E_1X + E_2Y & 0 & 0 & -\varepsilon I \end{bmatrix} < 0, \quad (11)$$

$$\begin{bmatrix} 1 & x_0^T \\ x_0 & X \end{bmatrix} \geq 0, \quad (12)$$

$$\begin{bmatrix} Z & Y \\ Y^T & X \end{bmatrix} \geq 0, \quad (13)$$

$$(Z)_{ii} \leq \bar{u}_i^2, \quad i = 1, 2, \dots, m, \quad (14)$$

where  $\Omega = AX + BY + (AX + BY)^T + \varepsilon DD^T$ , Then  $u(t) = YX^{-1}x(t)$  is a quadratically guaranteed cost controller satisfying the constraint (2) of the system (1), and the cost function of the corresponding closed-loop system satisfies  $J \leq \alpha$ .

**Proof:** Pre- and post-multiplying the left-hand side of the matrix inequality (9) by matrix  $\text{diag}\{I, \alpha P^{-1}\}$  imply that the matrix inequality (9) is equivalent to

$$\begin{bmatrix} Z & \alpha KP^{-1} \\ \alpha P^{-1}K^T & \alpha P^{-1} \end{bmatrix} \geq 0. \quad (15)$$

By denoting  $X = \alpha P^{-1}$  and  $Y = KX$ , the matrix inequality (13) is immediately obtained from the inequality (15). Pre- and post-multiplying the left-hand side of the matrix inequality (6) by matrix  $\alpha^{1/2}P^{-1}$ , it follows that the matrix inequality (6) is equivalent to

$$\alpha P^{-1}QP^{-1} + \alpha P^{-1}K^TRKP^{-1} + \alpha[A + BK + DF(E_1 + E_2K)]P^{-1} + \alpha P^{-1}[A + BK + DF(E_1 + E_2K)]^T < 0,$$

which can be further written as

$$\alpha^{-1}XQX + \alpha^{-1}Y^TRY + AX + BY + (AX + BY)^T + DF(E_1X + E_2Y) + [DF(E_1X + E_2Y)]^T < 0.$$

By the Schur complement, the above matrix inequality is equivalent to

$$\begin{bmatrix} AX + BY + (AX + BY)^T & X & Y^T \\ X & -\alpha Q^{-1} & 0 \\ Y & 0 & -\alpha R^{-1} \end{bmatrix} + \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} F [E_1X + E_2Y \quad 0 \quad 0] + [E_1X + E_2Y \quad 0 \quad 0]^T F^T \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} < 0.$$

It follows from Lemma 1 in [10] that the above matrix inequality is true for all  $F$  satisfying  $F^TF \leq I$  if and only if there exists a positive scalar  $\varepsilon$  such that

$$\begin{bmatrix} AX + BY + (AX + BY)^T & X & Y^T \\ X & -\alpha Q^{-1} & 0 \\ Y & 0 & -\alpha R^{-1} \end{bmatrix}$$

$$+ \varepsilon \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix}^T$$

$$+ \varepsilon^{-1} [E_1X + E_2Y \quad 0 \quad 0]^T [E_1X + E_2Y \quad 0 \quad 0] < 0.$$

Quoting the Schur complement again, the above matrix inequality is equivalent to the matrix inequality (11). Finally, from the Schur complement and  $X = \alpha P^{-1}$ , it follows that the inequality (8) is equivalent to the matrix inequality (12). Therefore, the results of Theorem 2 can be obtained from Theorem 1, which completes the proof of the theorem.

(11)-(14) is a linear matrix inequality system in  $\varepsilon, \alpha, X, Y, Z$  and defines a convex set of  $(\varepsilon, \alpha, X, Y, Z)$ .

Hence, the existing convex optimization techniques such as interior-point algorithms can be used to test whether this set is nonempty and to generate particular solutions if the LMI system is feasible. Moreover, its solutions parametrize the set of guaranteed cost controllers. This parametrized representation of guaranteed cost controllers can be exploited to design the guaranteed cost controllers with some additional requirements. In particular, we shall use this representation to present a design procedure for the optimal guaranteed cost controller that minimizes the guaranteed cost of the closed-loop uncertain system.

According to the Theorem 2, the design problem of the optimal guaranteed cost controller can be formulated as the following optimization problem:

$$\begin{aligned} \min \quad & \alpha \\ \text{s.t.} \quad & (11), (12), (13), (14). \end{aligned} \quad (16)$$

If the problem (16) has an optimal solution  $\varepsilon, \alpha, X, Y, Z$ , then  $u(t) = YX^{-1}x(t)$  is the optimal guaranteed cost controller satisfying the constraint (2).

It is clear that the problem (16) is a convex optimization problem with LMI constraints. Therefore, the global minimum of the problem can be reached if it is feasible, and it can be easily solved by using the solver mincx in the LMI Toolbox of MATLAB.

#### 4. BLOCK-DIAGONAL PARAMETER UNCERTAINTY

In the section we use the above results to solve the guaranteed cost control problem for systems with

block-diagonal time-varying parameter uncertainties and input constraints.

Consider the uncertain system (1) with the parameter uncertainty described by (3)-(4). Suppose that the uncertain matrix  $F(t)$  is of the following block-diagonal form:

$$F(t) = \text{diag}\{F_1(t), F_2(t), \dots, F_l(t)\}, \quad (17)$$

where  $F_k(t) \in R^{i_k \times j_k}$  and satisfies

$$F_k^T(t)F_k(t) \leq I_{j_k \times j_k}, \quad k=1, 2, \dots, l,$$

in which  $I_{j_k \times j_k}$  denotes  $j_k \times j_k$  identity matrix.

Then, for any constant vector

$$\varepsilon = [\varepsilon_1 \ \varepsilon_2 \ \dots \ \varepsilon_l], \quad \varepsilon_k > 0, \quad k=1, 2, \dots, l.$$

Define

$$\begin{aligned} \tilde{M} &= \text{diag}\{\varepsilon_1 I_{i_1 \times i_1}, \varepsilon_2 I_{i_2 \times i_2}, \dots, \varepsilon_l I_{i_l \times i_l}\}, \\ \tilde{N} &= \text{diag}\{\varepsilon_1^{-1} I_{j_1 \times j_1}, \varepsilon_2^{-1} I_{j_2 \times j_2}, \dots, \varepsilon_l^{-1} I_{j_l \times j_l}\}. \end{aligned}$$

Obviously, we have

$$DF(t)[E_1 \ E_2] = D\tilde{M}F(t)[\tilde{N}E_1 \ \tilde{N}E_2].$$

The following theorem provides a solution to the design problem of the optimal guaranteed cost controller for systems with block-diagonal time-varying parameter uncertainties and input constraints.

**Theorem 3:** If the following convex optimization problem

$$\begin{aligned} & \min_{\varepsilon, \alpha, X, Y, Z} \alpha \quad (18) \\ \text{s.t. (i)} & \begin{bmatrix} \Omega_1 & X & Y^T & \Omega_2^T & D\tilde{M} \\ X & -\alpha Q^{-1} & 0 & 0 & 0 \\ Y & 0 & -\alpha R^{-1} & 0 & 0 \\ \Omega_2 & 0 & 0 & -\tilde{N} & 0 \\ \tilde{M}D^T & 0 & 0 & 0 & -\tilde{M} \end{bmatrix} < 0 \\ \text{(ii)} & \quad (12), (13), (14) \end{aligned}$$

has a solution  $\varepsilon, \alpha, X, Y, Z$ , where

$$\begin{aligned} \Omega_1 &= AX + BY + (AX + BY)^T, \\ \Omega_2 &= E_1 X + E_2 Y. \end{aligned}$$

Then  $u(t) = YX^{-1}x(t)$  is the optimal guaranteed cost controller satisfying the constraint (2) of the system (1) with block-diagonal time-varying parameter uncertainties (17). Where

$$\begin{aligned} \varepsilon &= [\varepsilon_1 \ \varepsilon_2 \ \dots \ \varepsilon_l] \\ \tilde{M} &= \text{diag}\{\varepsilon_1 I_{i_1 \times i_1}, \varepsilon_2 I_{i_2 \times i_2}, \dots, \varepsilon_l I_{i_l \times i_l}\} \\ \tilde{N} &= \text{diag}\{\varepsilon_1 I_{j_1 \times j_1}, \varepsilon_2 I_{j_2 \times j_2}, \dots, \varepsilon_l I_{j_l \times j_l}\} \end{aligned}$$

Although the problem (16) can be also used to solve the guaranteed cost control problem for systems with block-diagonal parameter uncertainties, Theorem 3 will give a less conservative results due to the introduction of free parameters  $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_l$ .

## 5. ILLUSTRATIVE EXAMPLES

Consider the same example as in [11]. This example represents an uncertain model of the dynamics of a helicopter in a vertical plane. The uncertain dynamical model is as follows:

$$\dot{x} = (A + r_1 A_1 + r_2 A_2)x + (B + s_1 B_1)u, \quad x(0) = x_0, \quad (19)$$

where

$$\begin{aligned} A &= \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.2855 & -0.707 & 1.3229 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.4422 & 0.1711 \\ 3.0447 & -7.5922 \\ -5.52 & 4.99 \\ 0 & 0 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.2192 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.2031 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ 1.0673 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \\ -1 \leq r_1 \leq 1, \quad -1 \leq r_2 \leq 1, \quad -1 \leq s_1 \leq 1. \end{aligned}$$

The control input  $u$  in system (19) is subjected to the following constraints:

$$-1 \leq u_i \leq 1, \quad i=1, 2$$

the associated performance index is

$$J = \int_0^\infty (x^T Q x + u^T R u) dt,$$

where

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Define

$$F = \text{diag}\{r_1, r_2, s_1\}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 & 0.2192 & 0 & 0 \\ 0 & 0 & 0 & 1.2031 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1.0673 & 0 \end{bmatrix}.$$

Then (19) can be rewritten as

$$\dot{x}(t) = (A + DFE_1)x(t) + (B + DFE_2)u(t).$$

This is a system with block-diagonal time-varying parameter uncertainties. By applying Theorem 3 and solving the corresponding optimization problem (18), we obtain the optimal guaranteed cost controller

$$u(t) = \begin{bmatrix} -0.0885 & 0.2062 & 0.3237 & 0.3287 \\ 0.0515 & 0.1619 & -0.0686 & -0.5525 \end{bmatrix} x(t) \tag{20}$$

and the guaranteed cost of the uncertain closed-loop system is  $J^* = 6.2041$ .

If we do not consider the input constraints, Yu *et al.* (1999) gave the optimal guaranteed cost controller

$$u(t) = \begin{bmatrix} -0.6721 & 0.1921 & 0.8604 & 1.1844 \\ 0.3406 & 1.0065 & -0.2986 & -1.2138 \end{bmatrix} x(t) \tag{21}$$

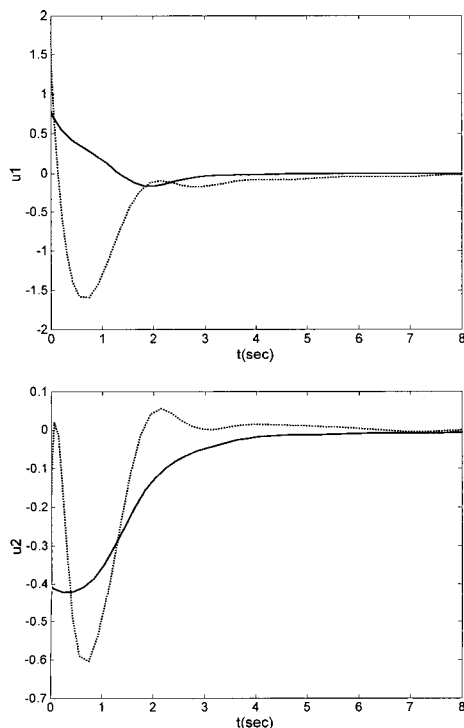


Fig. 1. Control law.

and the guaranteed cost of the uncertain closed-loop system is  $J^* = 5.3124$ .

To compare the effect of the controllers (20) and (21) by simulation, we assume that  $s_1 = \sin t$ ,  $r_1 = \sin 2t$ ,  $r_2 = \sin 3t$ . The control law (20) (solid line) and (21) (dot line) are shown in Fig. 1. The state variables of corresponding closed-loop systems are shown in Fig. 2. Where the solid lines stand for the

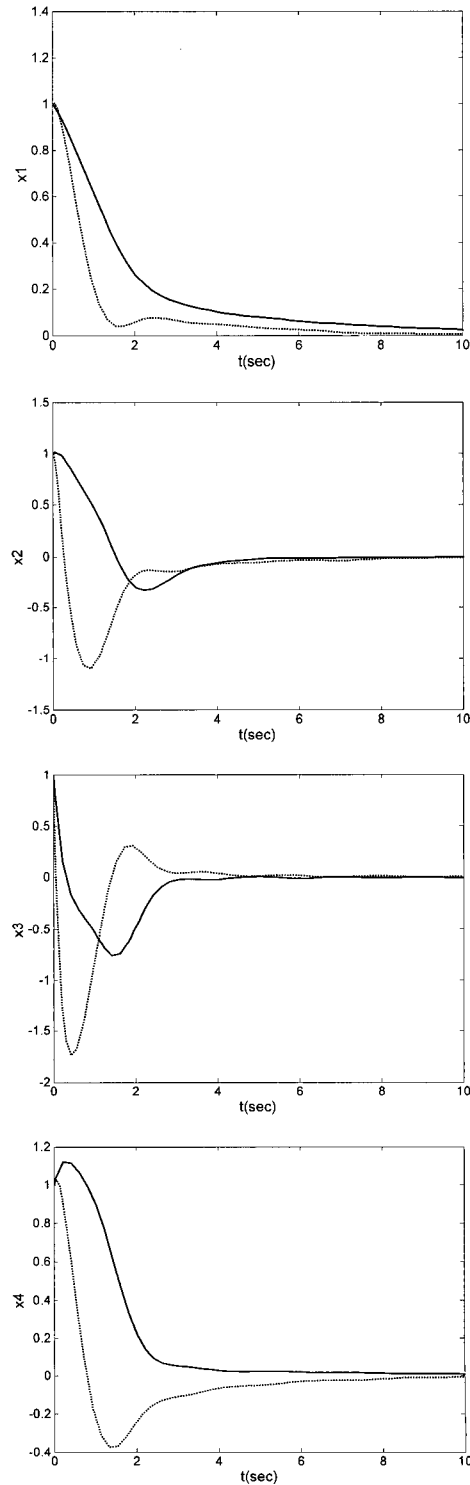


Fig. 2. The closed-loop state variables.

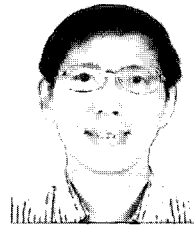
state variables of the closed-loop system resulted from controller (20), and the dot lines denote ones of the closed-loop system resulted from the controller (21). It can be seen from Fig.1 that the magnitudes of the input variables  $u_1$  and  $u_2$  in the controller (20) are significantly reduced due to considering the input constraints (2) in the design.

## 6. CONCLUSIONS

In this paper, we have presented an LMI based approach to the optimal guaranteed cost control problem via state feedback control laws for a class of uncertain systems. Contrast to the Riccati equation based approach, this approach has the advantage that no tuning of parameters and/or matrices is involved, and some additional requirements and constraints can be effectively treated.

## REFERENCES

- [1] B. R. Barmish, "Stabilization of uncertain systems via linear control," *IEEE Trans. on Automatic Control*, vol. 28, no. 3, pp. 848-850, March 1983.
- [2] P. P. Khargonekar, I. R. Petersen, and K. Zhou, "Robust stabilization of uncertain linear systems: Quadratic stabilizability and  $H_\infty$  control theory," *IEEE Trans. on Automatic Control*, vol. 35, no. 3, pp. 356-361, March 1990.
- [3] I. R. Petersen, "A stabilization algorithm for a class of uncertain systems," *Systems & Control Letters*, vol. 8, no. 3, pp. 181-188, 1987.
- [4] S. S. L. Chang and T. K. C. Peng, "Adaptive guaranteed cost control of systems with uncertain parameters," *IEEE Trans. on Automatic Control*, vol. 17, no. 4, pp. 474-483, April 1972.
- [5] D. S. Bernstein and W. M. Haddad, "Robust stability and performance via fixed-order dynamic compensation with guaranteed cost bounds," *Math. Contr. Signals and Systems*, vol. 3, no. 2, pp. 139-163, June 1990.
- [6] I. R. Petersen and D. C. McFarlane, "Optimal guaranteed cost control and filtering for uncertain linear systems," *IEEE Trans. on Automatic Control*, vol. 39, no. 9, pp. 1971-1977, September 1994.
- [7] I. R. Petersen, D. C. McFarlane, and M. A. Rotea, "Optimal guaranteed cost control of discrete-time uncertain linear systems," *International Journal of Robust & Nonlinear Control*, vol. 8, no. 7, pp. 649-657, July 1998.
- [8] L. Yu and J. Chu, "An LMI approach to guaranteed cost control of linear uncertain time-delay systems," *Automatica*, vol. 35, no. 6, pp. 1155-1159, June 1999.
- [9] L. Yu, G. Chen, and M. Yang, "Optimal guaranteed cost control of linear uncertain systems: LMI approach," *Proc. of the 14th IFAC World Congress*, vol. G, pp. 541-546, 1999.
- [10] L. Yu and F. Gao, "Optimal guaranteed cost control of discrete-time uncertain systems with both state and input delays," *Journal of the Franklin Institute*, vol. 338, no. 1, pp. 101-110, January 2001.
- [11] A. Fishman, J. M. Dion, L. Dugard, and A. T. Neto, "A linear matrix inequality approach for guaranteed cost control," *Proc. of the 13th IFAC World Congress*, pp. 197-202, 1996.



decentralized control.

**Li Yu** received the B.S. degree in Control Theory from Nankai University in 1982, and the M.S. and Ph.D. degrees from Zhejiang University, Hangzhou, China. He is currently a Professor in the College of Information Engineering, Zhejiang University of Technology, China. His research interests include robust control, time-delay systems,



**Qing-Long Han** received the B.S. degree in Mathematics from the Shandong Normal University, Jinan, China, in 1983, and the M.E. and Ph.D. degrees in Information Science from the East China University of Science and Technology, Shanghai, China, in 1992 and 1997, respectively. From September 1997 to December 1998, he was a Post-Doctoral Researcher Fellow at LAII-ESIP, Université de Poitiers, France. From January 1999 to August 2001, he was a Research Assistant Professor in the Department of Mechanical and Industrial Engineering, Southern Illinois University at Edwardsville, USA. In September 2001 he joined the Faculty of Informatics and Communication, Central Queensland University, Australia, where he is currently a Senior Lecturer. His research interests include time-delay systems, robust control, networked control systems, complex systems and software development processes.



**Ming-Xuan Sun** received the Ph.D. degree from Nanyang Technological University, Singapore, in 2002. He is currently a Professor in the College of Information Engineering, Zhejiang University of Technology, China. His main research interests include iterative learning control and optimal control.