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ELASTOKINEMATIC ANALYSIS OF A SUSPENSION SYSTEM WITH LINEAR RECURSIVE FORMULA

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ABSTRACT−This paper presents linear algebraic equations in the form of recursive formula to compute elastokinematic characteristics of a suspension system. Conventional methods of elastokinematic analysis are based on nonlinear kinematic constrant equations and force equilibrium equations for constrained mechanical systems, which require complicated and time-consuming implicit computing methods to obtain the solution. The proposed linearized elastokinematic equations in the form of recursive formula are derived based on the assumption that the displacements of elastokinematic behavior of a constrained mechanical system under external forces are very small. The equations can be easily computerized in codes, and have the advantage of sharing the input data of existing general multibody dynamic analysis codes. The equations can be applied to any form of suspension once the type of kinematic joints and elastic components are identified. The validity of the method has been proved through the comparison of the results from established elastokinematic anlaysis software. Error estimation and analysis due to piecewise linear assumption are also discussed.

KEY WORDS : Elastokinematic analysis, Constraint mechanical system, Recursive formula, Linearized elastokinematic equation, Suspension system

1. INTRODUCTION

Suspension systems are composed of kinematic joints and elastic components such as spring and rubber bushings. The elastokinematic behavior of kinematic and compliance components inherently causes change of wheel attitude due to the forces applied to the tire, which has significant effects on ride and handling characteristics. Thus, when designing a suspension system, the effects of elastokinematic characteristics should be considered carefully in view of handling as well as ride characteristics. Elastokinematic characteristics of suspension systems can be accounted for in the light of the changes of static design factors such as toe angle, camber angle and lateral displacement of the wheel due to the forces and moments applied to the tire-road contact patch. The effects of these characteristics on handling were early investigated (Bundorf, 1976). The elastokinematic effects on ride and handling have been also taken into consideration to design a suspension system (Tsukuda *et al.*, 1988).

Elastokinematic characteristics of a suspension system can be measured through the suspension parameter measuring device (SPMD) (Shimatani *et al.*, 1999; Ellis *et al.*, 1987; Erdogan *et al.*, 1999). Ride and handling performance has been predicted with lumped parameter vehicle model including kinematic and compliance effects (Gorder *et al.*, 2000; Kasprzak *et al.*, 2000). In the early developing stage of a suspension, there should be an analytical means for calculating elastokinematic characteristics. Recently, accurate computation of elastokinematic characteristics has been tried based on the dynamic and algebraic equation neglecting time variant terms of multibody dynamic areas. Orlandea and Chase proposed a computational analysis in conjunction with the numerical method for general multibody systems, which was implemented in ADAMS, a commercial multibody dynamic analysis package (Orlandea *et al.*, 1977). To resolve the drawback that the solution process is sensitive to the initial estimates of the solutions, the method of minimizing the potential energy of the system has been proposed (Wehage *et al.*, 1982).

In this study, based on the assumption that the displacements of multibody systems under the external forces are very small, a linear form of the elastokinematic equations in terms of infinitesimal displacements and joint reaction forces is derived. Then, linear form of recursive formula is suggested for elastokinematic analysis of multibody systems in which the configuration is changed due to continuous quasi-static forces under the

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piecewise linear assumption. The equations can be applied to any form of suspension once the types of kinematic joints and bushings are identified. In this paper, wheel attitude changes with vertical movement are computed for a McPherson strut suspension as an example. The validity of the method has been proved through the comparison of the results from the established elastokinematic analysis software ADAMS.

2. LINEARIZED ELASTOKINEMATIC EQUATIONS

A constrained mechanical system is the interconnection of rigid bodies by kinematic joints and compliance elements such as bushings and springs. The governing equations for a constrained mechanical system are composed of two types of equations: algebraic kinematic constraint equation and force equilibrium equation. The kinematic constraint, which must be satisfied, can be generally represented in algebraic equations as:

$$\Phi(\mathbf{q}) = \mathbf{0} \tag{1}$$

where $\mathbf{q}$ is the generalized coordinates of the system.

The external force acting on body i is the sum of all forces such as elastic force and constraint reaction force as shown in Figure 1.

$$\mathbf{f}_i^{ext} = \mathbf{f}_i^{elast} + \mathbf{f}_i^{const} \tag{2}$$

In Equation (2), time variant terms such as inertia force and damping force are neglected by the quasi-static condition, which is assumed to obtain kinematic and compliance characteristics of suspension system.

If the system has nb bodies, then the total external force vector $\mathbf{f}^{ext}$ may be written as:

$$\mathbf{f}^{ext} = [\mathbf{f}_1^{extT} \ \mathbf{f}_2^{extT} \ldots \mathbf{f}_{nb}^{extT}]^T \tag{3}$$

For the linearized elastokinematic equations, it is

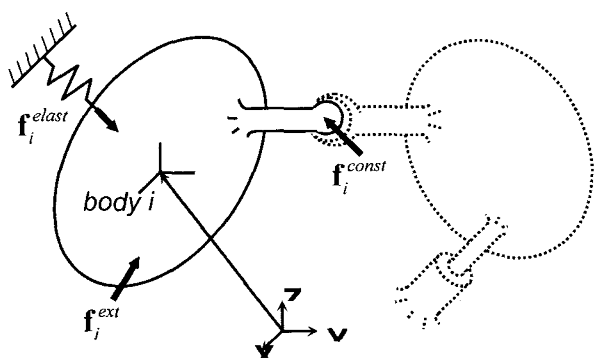

Figure 1. Static force equilibrium.

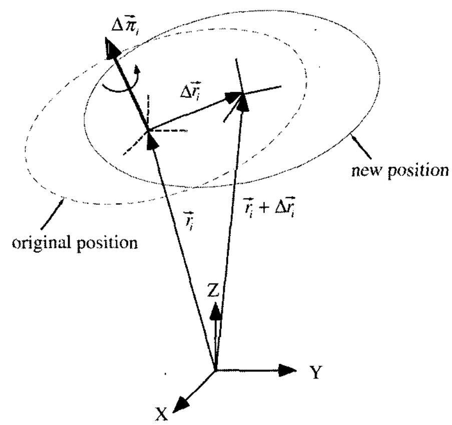

Figure 2. Displacement vector for a body.

assumed that the system undergoes very small displacements from the equilibrium configuration by small change of external forces. As shown in Figure 2, body i has the displacement vector $\Delta\mathbf{q}_i$, which consists of translation $\Delta\mathbf{r}_i = [\Delta x_i \ \Delta y_i \ \Delta z_i]^T$ and rotation $\Delta\pi_i = [\Delta\phi_i \ \Delta\theta_i \ \Delta\psi_i]^T$.

$$\Delta\mathbf{q}_i = [\Delta\mathbf{r}_i^T \ \Delta\pi_i^T]^T \tag{4}$$

where represents Δx_i, Δy_i, Δz_i direction displacement respectively, and $\Delta\phi_i$, $\Delta\theta_i$, $\Delta\psi_i$ is rotations about x, y and z axis respectively. The total displacement vector $\Delta\mathbf{q}$ can be given as follows:

$$\Delta\mathbf{q} = [\Delta\mathbf{q}_1^T \ \Delta\mathbf{q}_2^T \ldots \Delta\mathbf{q}_{nb}^T]^T \tag{5}$$

To obtain the linearized equations for constrained systems, constraint equations are firstly linearized in terms of $\Delta\mathbf{q}$. Under the action of the external forces, the system reaches a new equilibrium. At the new equilibrium, the generalized coordinates change from $\mathbf{q}$ to $\mathbf{q} + \Delta\mathbf{q}$, then the constraint equation can be approximately written as:

$$\Phi(\mathbf{q} + \Delta\mathbf{q}) \approx \Phi(\mathbf{q}) + \Phi_\mathbf{q}\Delta\mathbf{q} = \mathbf{0} \tag{6}$$

where $\Phi_\mathbf{q}$ is the Jacobian of the constraint equation Φ. Typical kinematic constraint equations and Jacobians are summarized by Nikravesh (Nikravesh, 1988). From Equations (1) and (6), constraint equations for small displacement $\Delta\mathbf{q}$ can be given as:

$$\Phi_\mathbf{q}\Delta\mathbf{q} = \mathbf{0} \tag{7}$$

For small displacements, elastic elements can be regarded as linear translational and rotational springs. A general form of the stiffness matrices of the elastic

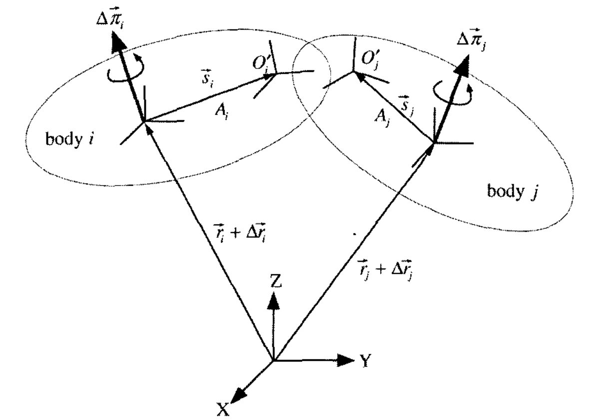

Figure 3. Displacement between two bodies.

element in a local coordinate system can be respectively expressed as:

$$\mathbf{K}_T' = \begin{bmatrix} k_x' & 0 & 0 \\ 0 & k_y' & 0 \\ 0 & 0 & k_z' \end{bmatrix} \tag{8}$$

$$\mathbf{K}_R' = \begin{bmatrix} k_\phi' & 0 & 0 \\ 0 & k_\theta' & 0 \\ 0 & 0 & k_\psi' \end{bmatrix} \tag{9}$$

The stiffness matrix in the local coordinate system can be transformed into the stiffness matrix in the global coordinate system by multiplying the transformation matrix as follows:

$$\mathbf{K}_T = \mathbf{A}_i \mathbf{K}_T' \mathbf{A}_i^T \tag{10}$$

$$\mathbf{K}_R = \mathbf{A}_i \mathbf{K}_R' \mathbf{A}_i^T \tag{11}$$

where $\mathbf{A}_i$ is the transformation matrix from the local coordinate system to the global coordinate system.

Figure 3 shows body i and j, which respectively undergoes small displacements $\Delta\mathbf{q}_i$ and $\Delta\mathbf{q}_j$. Let the $\mathbf{s}_i$ be the position vector of the attachment point O_i' of the elastic element, displacement of the attachment point O_i' can be expressed as:

$$\Delta\mathbf{r}_i' = \Delta\mathbf{r}_i + \Delta\pi_i \mathbf{s}_i = \Delta\mathbf{r}_i - \tilde{\mathbf{s}}_i \Delta\pi_i \tag{12}$$

Similarly, displacement of the attachment point O_j' can be expressed as:

$$\Delta\mathbf{r}_j' = \Delta\mathbf{r}_j - \tilde{\mathbf{s}}_j \Delta\pi_j \tag{13}$$

Force of the elastic element between O_i' and O_j' can be calculated by premultiplying the stiffness matrix $\mathbf{K}_T$ to the relative displacement $\Delta\mathbf{r}_i' - \Delta\mathbf{r}_j'$:

$$\mathbf{F}_i = -\mathbf{K}_T(\Delta\mathbf{r}_i - \Delta\mathbf{r}_j - \tilde{\mathbf{s}}_i\Delta\pi_i + \tilde{\mathbf{s}}_j\Delta\pi_j) \tag{14}$$

where minus sign is taken to treat the tension of the spring as the positive value. The moment about Q_i is the sum of moment due to force $\mathbf{F}_i$ and pure moment by rotational spring.

$$\mathbf{T}_i = \tilde{\mathbf{s}}_i \mathbf{F}_i - \mathbf{K}_R(\Delta\pi_i - \Delta\pi_j) \tag{15}$$

Force and moment exerted on body j can be similarly expressed as:

$$\mathbf{F}_j = -\mathbf{F}_i \tag{16}$$

$$\mathbf{T}_j = \tilde{\mathbf{s}}_j \mathbf{F}_j + \mathbf{K}_R(\Delta\pi_i - \Delta\pi_j) \tag{17}$$

Arranging Equations (14)–(17) into matrix form, elastic forces and displacements relationship can be written as:

$$\begin{Bmatrix} \mathbf{f}_i^{elast} \\ \mathbf{f}_j^{elast} \end{Bmatrix} = \begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ij} \\ \mathbf{K}_{ji} & \mathbf{K}_{jj} \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{q}_i \\ \Delta\mathbf{q}_j \end{Bmatrix} \tag{18}$$

where elastic force vector is $\mathbf{f}_i^{elast} = [\mathbf{F}_i^T \ \mathbf{T}_i^T]^T$ and the stiffness matrix of the elastic element between body i and j is written as:

$$\begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ij} \\ \mathbf{K}_{ji} & \mathbf{K}_{jj} \end{bmatrix} = \left[\begin{array}{cc:cc} -\mathbf{K}_T & \mathbf{K}_T\tilde{\mathbf{s}}_i & \mathbf{K}_T & -\mathbf{K}_T\tilde{\mathbf{s}}_j \\ -\tilde{\mathbf{s}}_i\mathbf{K}_T & \tilde{\mathbf{s}}_i\mathbf{K}_T\tilde{\mathbf{s}}_i - \mathbf{K}_R & \tilde{\mathbf{s}}_i\mathbf{K}_T & -\tilde{\mathbf{s}}_i\mathbf{K}_T\tilde{\mathbf{s}}_j + \mathbf{K}_R \\ \hdashline \mathbf{K}_T & -\mathbf{K}_T\tilde{\mathbf{s}}_i & -\mathbf{K}_T & \mathbf{K}_T\tilde{\mathbf{s}}_j \\ \tilde{\mathbf{s}}_j\mathbf{K}_T & -\tilde{\mathbf{s}}_j\mathbf{K}_T\tilde{\mathbf{s}}_i + \mathbf{K}_R & -\tilde{\mathbf{s}}_j\mathbf{K}_T & \tilde{\mathbf{s}}_j\mathbf{K}_T\tilde{\mathbf{s}}_j - \mathbf{K}_R \end{array}\right] \tag{19}$$

It can be easily shown that stiffness matrix of Equation (19) is symmetric, which is the general feature of linear systems. Elastic forces of the whole system can be constructed by subjoining the stiffness matrices for each spring element.

$$\mathbf{f}^{elast} = \mathbf{K}\Delta\mathbf{q} \tag{20}$$

Constraint forces can be expressed in terms of the constraint equations and Lagrange multipliers. Constraint forces at the new configuration due to the external forces are represented as:

$$\mathbf{f}^{const} = \Phi(\mathbf{q} + \Delta\mathbf{q})_{\mathbf{q}}^T \lambda \tag{21}$$

where $\Phi(\mathbf{q} + \Delta\mathbf{q})_{\mathbf{q}}$ is the Jacobian at the new configuration, and λ is the Lagrange multipliers associated with the constraint forces. In the virtue of linear property with small motion it is assumed that

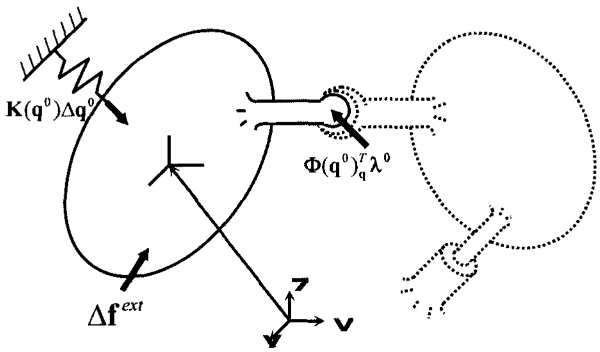

Figure 4. Static force equilibrium at initial step.

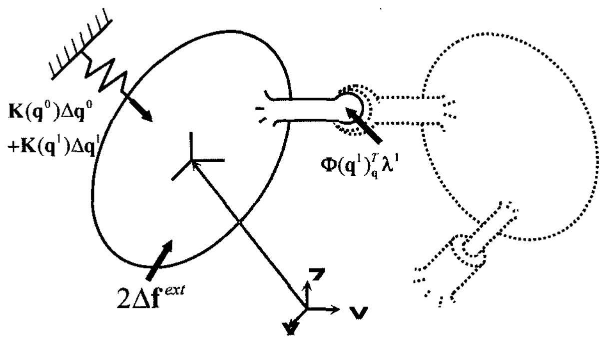

Figure 5. Static force equilibrium at second step.

$\Phi(\mathbf{q}+\Delta\mathbf{q})_{\mathbf{q}} \approx \Phi(\mathbf{q})_{\mathbf{q}}$, then Equation (21) is expressed as:

$$\mathbf{f}^{const} \approx \Phi(\mathbf{q})_{\mathbf{q}}^{T}\lambda \tag{22}$$

Substituting Equations (20) and (22) into Equations (2) and (3), linearized static equations for the constrained system with small displacement are obtained as:

$$\mathbf{K}\Delta\mathbf{q}+\Phi_{\mathbf{q}}^{T}\lambda = \mathbf{f}^{ext} \tag{23}$$

Equations (1) and (23) can be combined to a matrix form.

$$\begin{bmatrix} \mathbf{K} & \Phi_{\mathbf{q}}^{T} \\ \Phi_{\mathbf{q}} & 0 \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{q} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^{ext} \\ 0 \end{Bmatrix} \tag{24}$$

Linear elastokinematic equations for constrained mechanical system have been achieved as Equation (24). Once types of kinematic joints, elastic elements and external forces are defined, the displacements $\Delta\mathbf{q}$ and constraint reaction forces λ can be easily obtained by linear algebraic matrix methods.

Based on the linear elstokinematic equations of Equation (24), linear form of recursive formula is derived for elastokinematic analysis of multibody systems in which the configuration is changed due to continuous quasi-static forces under the piecewise linear assumption. For stepwise derivation, we consider the system as shown in Figure 4, in which initial configuration $\mathbf{q}^0$ is deflected by the incremental force $\Delta\mathbf{f}^{ext}$. In Figure 4, elastic force and constraint force of the system are denoted as $\mathbf{K}(\mathbf{q}^0)\Delta\mathbf{q}^0$ and $\Phi(\mathbf{q}^0)_{\mathbf{q}}^{T}\lambda^0$ from Equations (20) and (22), respectively. Equation (24) can be rewritten as:

$$\begin{bmatrix} \mathbf{K}(\mathbf{q}^0) & \Phi(\mathbf{q}^0)_{\mathbf{q}}^{T} \\ \Phi(\mathbf{q}^0)_{\mathbf{q}} & 0 \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{q} \\ \lambda \end{Bmatrix}^{0} = \begin{Bmatrix} \Delta\mathbf{f}^{ext} \\ 0 \end{Bmatrix} \tag{25}$$

Solution of initial step, $\Delta\mathbf{q}^0$ can be obtained from Equation (25).

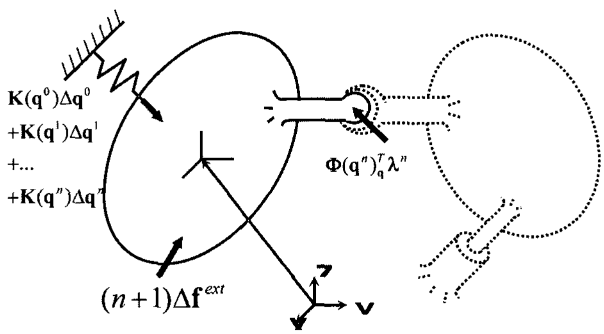

Figure 6. Static force equilibrium at (n+1)th step.

Generalized coordinates of changed configuration of the system are $\mathbf{q}^1=\mathbf{q}^0+\Delta\mathbf{q}^0$. In the second step, incremental force $2\Delta\mathbf{f}^{ext}$ is applied to the new configuration as shown in Figure 5. Equation (24) can be rewritten as:

$$\begin{bmatrix} \mathbf{K}(\mathbf{q}^1) & \Phi(\mathbf{q}^1)_{\mathbf{q}}^{T} \\ \Phi(\mathbf{q}^1)_{\mathbf{q}} & 0 \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{q} \\ \lambda \end{Bmatrix}^{1} = \begin{Bmatrix} 2\Delta\mathbf{f}^{ext}-\mathbf{K}(\mathbf{q}^0)\Delta\mathbf{q}^0 \\ 0 \end{Bmatrix} \tag{26}$$

Since $\Delta\mathbf{f}^{ext}-\mathbf{K}(\mathbf{q}^0)\Delta\mathbf{q}^0=\Phi(\mathbf{q}^0)_{\mathbf{q}}^{T}\lambda^0$ from Equation (25), Equation (26) is rewritten as:

$$\begin{bmatrix} \mathbf{K}(\mathbf{q}^1) & \Phi(\mathbf{q}^1)_{\mathbf{q}}^{T} \\ \Phi(\mathbf{q}^1)_{\mathbf{q}} & 0 \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{q} \\ \lambda \end{Bmatrix}^{1} = \begin{Bmatrix} \Delta\mathbf{f}^{ext} \\ 0 \end{Bmatrix} + \begin{Bmatrix} \Phi(\mathbf{q}^0)_{\mathbf{q}}^{T}\lambda^0 \\ 0 \end{Bmatrix} \tag{27}$$

In the same manner, equation of (n+1)th step can be derived as above procedure considering Figure 6. For the incremental force $(n+1)\Delta\mathbf{f}^{ext}$ of (n+1)th step, linear elastokinematic equation of constrained system is written as:

$$\begin{bmatrix} \mathbf{K}(\mathbf{q}^n) & \Phi(\mathbf{q}^n)_{\mathbf{q}}^{T} \\ \Phi(\mathbf{q}^n)_{\mathbf{q}} & 0 \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{q} \\ \lambda \end{Bmatrix}^{n} \tag{28}$$

$$= \begin{Bmatrix} (n+1)\Delta\mathbf{f}^{ext}-\mathbf{K}(\mathbf{q}^0)\Delta\mathbf{q}^0-\mathbf{K}(\mathbf{q}^1)\Delta\mathbf{q}^1-\ldots-\mathbf{K}(\mathbf{q}^{n-1})\Delta\mathbf{q}^{n-1} \\ 0 \end{Bmatrix}$$

Since $n\Delta \mathbf{f}^{ext} - \mathbf{K}(\mathbf{q}^0)\Delta\mathbf{q}^0 - \mathbf{K}(\mathbf{q}^0)\Delta\mathbf{q}^0 - \mathbf{K}(\mathbf{q}^1)\Delta\mathbf{q}^1 - \ldots - \mathbf{K}(\mathbf{q}^{n-1})\Delta\mathbf{q}^{n-1} = \Phi(\mathbf{q}^{n-1})_{\mathbf{q}}^T\lambda^{n-1}$ from previous step, Equation (28) is rewritten as:

$$\begin{bmatrix} \mathbf{K}(\mathbf{q}^n) & \Phi(\mathbf{q}^n)_{\mathbf{q}}^T \\ \Phi(\mathbf{q}^n)_{\mathbf{q}} & 0 \end{bmatrix} \begin{Bmatrix} \Delta\mathbf{q} \\ \lambda \end{Bmatrix}^n = \begin{Bmatrix} \Delta\mathbf{f}^{ext} \\ 0 \end{Bmatrix} + \begin{Bmatrix} \Phi(\mathbf{q}^{n-1})_{\mathbf{q}}^T\lambda^{n-1} \\ 0 \end{Bmatrix} \tag{29}$$

In the end, linear recursive formula of constrained mechanical system is obtained as Equation (29). One can notice that initial variable $\mathbf{q}^0$ means initial coordinates and λ^{-1} is 0 from Equation (25). The system equation is derived as simple and straightforward form easily used to compute displacements of the system undergoing quasi-static motion.

3. EXAMPLE

Suspension systems can be considered as interconnection of rigid bodies by kinematic joints and elastic elements. Thus, each component of the suspension system has inherent motion characteristics when they are subject to external forces.

A McPherson strut type suspension system is shown in Figure 7, which consists of three rigid bodies of lower control arm, wheel assembly, and strut. Wheel assembly is connected to the strut with a linear one-axis spring ($\mathbf{K}^1$), and the strut is connected to the car body by a strut mount rubber bushing with three-axis spring ($\mathbf{K}^2$). Also lower control arm is connected to the car body with two bushings ($\mathbf{K}^3$, $\mathbf{K}^4$). A cylindrical constraint between wheel assembly and strut, a distance constraint between wheel assembly and car body, a spherical constraint

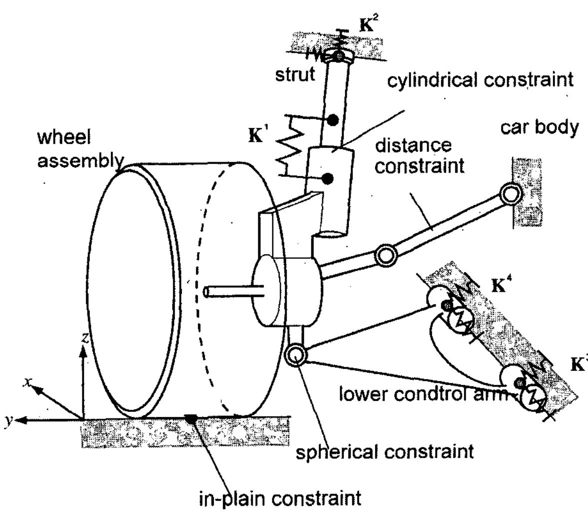

Figure 7. Modeling of a McPherson strut suspension system.

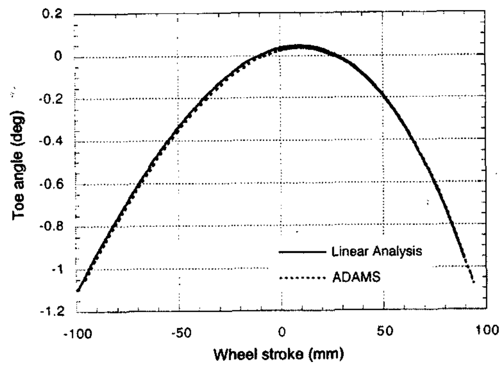

Figure 8. Toe angle change vs wheel stroke.

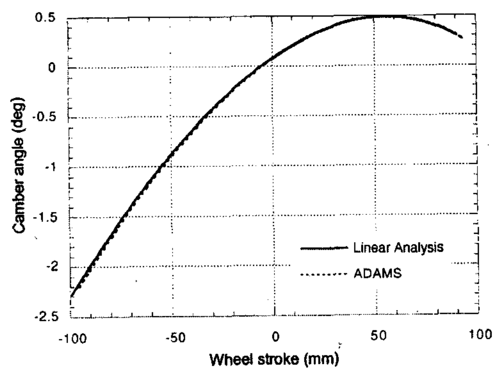

Figure 9. Camber angle change vs wheel stroke.

between lower control arm and wheel assembly, and an in-plain constraint between wheel assembly and ground are respectively imposed. There are 9 constraints, 4 for cylindrical constraint, 1 for distance constraint, 3 for spherical constraint and 1 for in-plain constraint. Since the number of generalized coordinates is 18 and the number of constraints is 9, there are 27 system variables.

Incremental force $\Delta\mathbf{f}^{ext} = 10\text{N}$ is applied to the wheel in vertical direction to compute quasi-static solution of a suspension system. Step size is 600 and external forces are varied from −3000N to 3000N that push the wheel almost from −100 mm to 100 mm. Figure 8 shows toe change curve due to the wheel stroke. Figure 9 shows camber change curve due to the wheel stroke. Figure 10 shows lateral movement due to vertical movement of the wheel. Results of piecewise linear approach of this paper are very consistent to those of ADAMS. It can be shown that the linear recursive formula presented in this paper seems to well predict elastokinematic characteristics of a suspension system.

The solution of piecewise linear approach is expected to be sensitive to step size. Therefore, it needs to discuss aspect of errors. Error norm can be defined as constrained equations as:

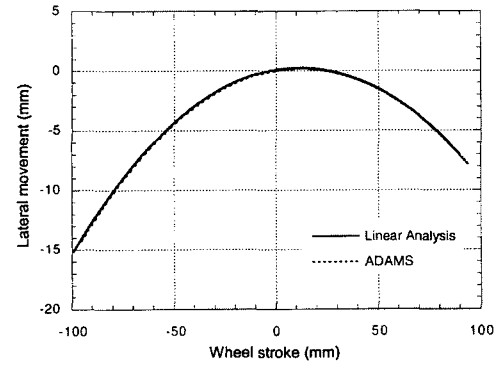

Figure 10. Lateral movement vs wheel stroke.

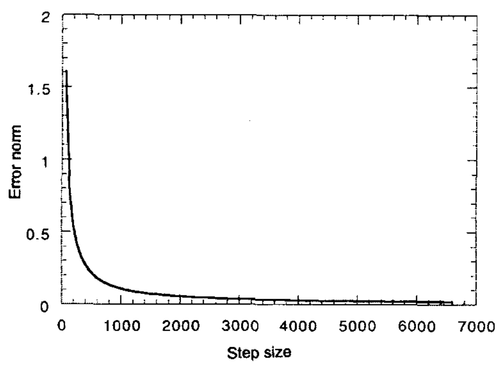

Figure 11. Error vs step size.

$$err = |\Phi(\mathbf{q}) - \Phi(\mathbf{q}^*)| \quad (30)$$

Where, $\mathbf{q}$ is true solution and $\mathbf{q}^*$ is computed solution. Since $\Phi(\mathbf{q})=0$, error norm is redefined as:

$$err = |\Phi(\mathbf{q}^*)| \quad (31)$$

Figure 11 shows the relationship between step size and error norm of last step in the above example. Error is shown to be rapidly decreased with step size increased. Though large step size warrants good approximation, computation time may be increased. So we can see that selection of proper step size is important.

As shown in the above example, elastokinematic behavior of a suspension system can be easily predicted with the linear recursive formula presented in this paper. Conventional methods of elastokinematic analysis are based on nonlinear kinematic constraint equations and force equilibrium equations for constrained mechanical systems. A proper estimate of initial solutions for generalized coordinates and Lagrangian multipliers of these nonlinear algebraic equations is difficult. In order to circumvent the difficulty of finding reasonable estimates, the complex computing methods such as the method of minimizing the potential energy of the system or fictitious damping method has been proposed. On the other hand, the proposed linearized equations can be directly computerized in codes by virtue of recursive formula. In addition, since the solutions for linear matrix form of equations are offered by simple linear algebraic methods, the proposed methods have the advantage of obtaining solutions by simple and fast way.

4. CONCLUSION

Kinematic and compliance characteristics of suspension systems have direct influence on ride and handling of a vehicle. In this study, linear recursive formula has been derived to compute elastokinematic characteristics of a suspension systems. The analytical linearization procedure based on force equilibrium approach is straightforward and simple. Moreover, they can be accurately solved using linear algebraic methods, instead of complicated and time-consuming implicit methods. Derived linearized form of recursive formula can be easily computerized in codes and solved by simple matrix methods. It is shown that these equations can be applied to any suspension system and are verified through the comparison ADAMS results.

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