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Application of Information-theoretic Measure (Entropy) to Safety Assessment in Manufacturing Processes

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Abstract : Design of manufacturing process, in general, facilitates the creation of new process that may potentially harm the workers. Design of safety-guaranteed manufacturing process is, therefore, very important since it determines the ultimate outcomes of manufacturing activities involving safety of workers. This study discusses application of information-theoretic measure (entropy) to safety assessment of manufacturing processes. The idea is based on the general principles of design and their applications. Some examples are given.

Key words: Safety, Manufacturing Processes, Information, Entropy

1. Introduction

Safety is considered to be a commonsense approach to removing agents of injury [1]. Safety, as a concept and practice, has shifted to a complex methodology for the reliable control of injury to human beings and damages to property. However, it does lack a theoretical base. As safety is concerned with reducing accidents and controlling or eliminating hazards at the manufacturing processes, accident prevention is a significant step towards safety improvement.

Design of manufacturing process, in general, facilitates the creation of new process that may potentially harm the workers [2, 3]. Design of safety-guaranteed manufacturing process is, therefore, very important since it determines the ultimate outcomes of manufacturing activities involving safety of workers. Safety in manufacturing environment is considered to be a measure of relative freedom from accidents. In order to improve the safety performance, control of accident is essential and the effectiveness of control of accidents needs to be estimated before any new manufacturing process is put into practice. Safety performance criterion, in this case, needs to be defined a priori.

This study discusses application of discrimination information as information-theoretic measure to safety assessment of manufacturing processes. The idea is based on the general principles of design and their applications. Some examples are given.

2. Information (Entropy) Analysis

The concept of entropy was first introduced in statistical thermodynamics by physicist Boltzman to quantify the uncertainty involved in the system [4]. Such uncertainty stems from the randomness of the process.

2.1. Continuous Case

Let *x(n)* be a state of some process that has a set C of possible states. Let Ψ be the set of all possible probability densities q on C such that $q(x \in C) \geq 0$ and

$$\int_C q(x)dx = 1 \qquad (1)$$

The entropy of a process with the probability density q is represented as:

$$E[q] = -\int_C q(x)\log q(x)dx \qquad (2)$$

The entropy is a measure of the amount of information produced by a random process *x(n)*, or a measure of uncertainty in a random process. The larger value of entropy corresponds to more uncertainty in the process.

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2.2 Discrete case

For discrete events, $q_i, q_2, q_3, \ldots, q_i$, the average information content of the discrete events (or entropy) is defined as:

$$I = \sum_i I_i = -\sum_i q_i \log q_i \tag{3}$$

subject to constraints

$$\sum_i q_i = 1, \quad \sum_i c_i q_i = C \tag{4}$$

where $I_i = \log\left(\frac{1}{q_1}\right) = -\log q_i$ for individual event, C is the total energy and c_i is the energy associated with individual components

I is maximum if $q_i=q$ (constant) or every event is equally probable (the state of maximum disorder or maximum uncertainty) [5]. I is minimum (I=0) if one particular event i always happens with the probability of 1 and other events never happen such that $q_i=1$, $q_j=0$, $i \neq j$. For example, drawing a cubic dice gives even probability of 1/6, which implies the equally probable state of having a number 1 to 6. In this case, the total amount of information or uncertainty is $I=0.777$. Suppose uneven probability distribution, however, such that q_i = 1/12, 1/12, 1/6, 1/6, 1/4, 1/4. The uncertainty in this case is $I=0.738$. In the extreme case where every face of cube dice have a number of "1", $q_i=1$, $q_j=0$, $j \neq 1$ and $I=0$ with no uncertainty in the process.

[Theorem 1] Sum of uncertainty (information): "The sum of uncertainty (information) for a set of events is also uncertainty (information), provided that proper conditional probabilities are used when the events are not statistically independent."

$$q(y) = q(y/x)q(x) \tag{5}$$

if x and y are not independent of each other

[Theorem 2] Uncertainty (information content) of the total system: "If each event is probabilistically independent of each other, the uncertainty (information contents) of the total system is the sum of uncertainty of all individual events."

$$q(x,y) = q(x) \cdot q(y) \tag{6}$$

$$I_i = \log\left(\frac{1}{q(x,y)}\right) = \log\left(\frac{1}{q(x)}\right) + \log\left(\frac{1}{q(y)}\right) \tag{7}$$

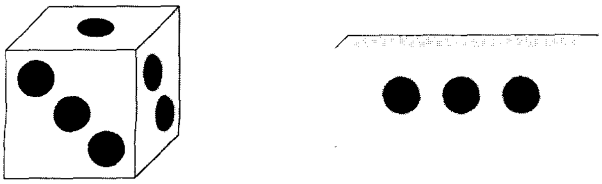

Fig. 1. Dice drawing with even and uneven probability distribution.

3. Uncertainty in a Safety Context

In terms of safety involved in the design of manufacturing process, entropy quantifies the complexity of achieving the safety in the process. The more complex a process is, the more information is required to describe and understand the safety features in the process. It is a measure of knowledge required to satisfy a given level of the safety requirement hierarchy and closely related to the probability of achieving safety requirements involved in the process.

Note that the knowledge required to achieve a task in a safe manner depends on the probability of success. For example, if a task can be achieved safely without prior knowledge or additional knowledge about the potential hazards or no hazards are involved in the task, the probability of success in achieving such task without safety problems is "1" and no requisite information is necessary. Probability of success depends on the complexity of task in guaranteeing the safety involved. Therefore, information is related to complexity. Probability of success in achieving tasks increases as complexity of designed processes decreases. Process design must transmit sufficient knowledge so that probability of achieving task (satisfying safety requirements) is as high as possible.

Note that

$$I_i = \log\left(\frac{1}{q_1}\right) = -\log q_i \geq 0 \quad for \ q_i \leq 1 \tag{8}$$

$$q_i \log\left(\frac{1}{q_1}\right) \geq 0 \quad for \ q_i \leq 1 \tag{9}$$

Then, as the number of variables increase (having more i's), higher I (more information contents or more uncertainty) results. In general, the minimum information content is achieved by:

1. Choosing designs and tolerance which yield larger q_is

2. Minimizing the number of variables, when other things are nearly equal

3. imposing the maximum number of constraints to the proposed design, which reduces the uncertainty/randomness in process design, thus reduces the safety requirements.

Note that the greater the number of constraints (not much choices that process designer can take), the smaller the entropy or the smaller uncertainty in process.

4. Uncertainty in a System Context

Consider in Figure 2 where the performance of the process is quantified in view of the safety. Safety range signifies the tolerance associated with process parameters [6]. System range designates the capability of manufacturing system (in terms of tolerance) and the current performance of designed processes. Common range is the overlap between the safety range and the system range. Figure 2 implies how much of safety requirements are satisfied by the current performance of the designed process (system range).

The probability of achieving the particular safety requirement i and the information content are then defined, respectively, by:

$$q_i = \left(\frac{System\ Range}{Common\ Range}\right)_i \tag{10}$$

$$I_i = \log\left(\frac{1}{\dfrac{Common\ Range}{System\ Range}}\right)_i = \log\left(\frac{System\ Range}{Common\ Range}\right)_i \tag{11}$$

Information content is a measure of the probability of success of achieving the specified safety requirements in manufacturing process or a measure of uncertainty in insuring safety in manufacturing process. It is independent of specific nature of process parameters such as work envelop of a robot motion, noise level in work environment, weight of the load and etc.

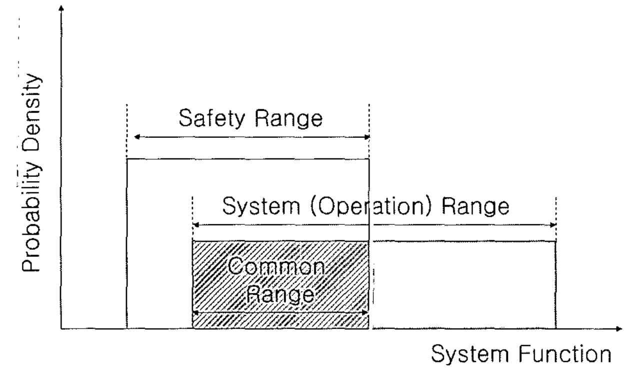

Fig. 2. Probability distribution of a system parameter.

If the safety range does not overlap with the system range (operation range), process design does not reflect the safety requirements. If the safety range covers the entire system range, all the safety requirements are satisfied by the process parameters in the manufacturing processes.

Two ways of reducing uncertainty (information contents) are:

- to reduce the system range so that the process is as simple as possible for safety.
- to increase the common range. This implies that one has to try to satisfy all safety requirements specified by the safety range with process parameters.

5. Examples

Consider the information associated with the dimensional precision of work envelop in robotic assembly process. Here, the process parameters are geometric dimensions of work envelop. The work envelop is usually composed of several components depending on the type of robot. Each component independently influences the safety of workers. Cartesian coordinate robot, for example, has vertical stroke, vertical reach, horizontal stroke, horizontal reach and traverse stroke.

For horizontal reach, the safety range is the "safe" horizontal reach that guarantees the safety of workers and is specified by a process designer, from 1.0m to 1.5 m in Figure 3. The safety range is usually designated as the "Safe Work Area" on floor. This range var-

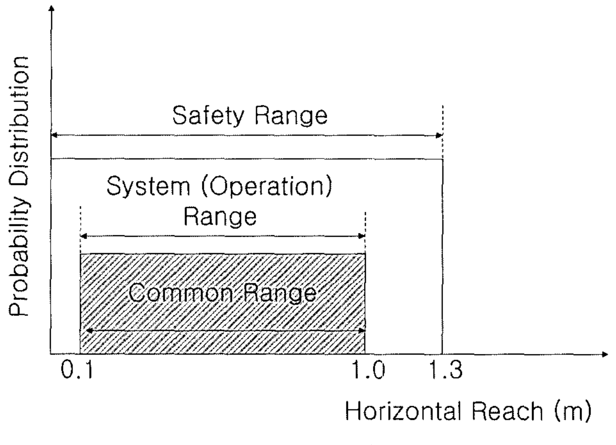

Fig. 3. Probability distribution of horizontal reach that guarantees the safety of workers.

ies depending on the types of the robot and the work involved and can be reduced either intentionally or inadvertently by the work range of workers on floor. The system range is the range of a robot arm to move horizontally and is, say between 0.1m and 0.8m in Figure 3.

When the safety range coincides with the system range, no uncertainty (information contents) in insuring the safety of worker is assumed. However, if either workers break into the safe work area so that the safety range shrinks to between 0.1m and 0.8m as in Figure 4 or the horizontal reach of a robot arm is extended to 1.5m, as in Figure 5, beyond the safety limit that is type of work-specific, the uncertainty exists and the safety of workers is not guaranteed.

The uncertainty in safety in each case is then given by:

$$I = \log\left(\frac{System\ Range}{Common\ Range}\right) = \log\left(\frac{1.0-0.1}{0.8-0.1}\right) = 0.109 \tag{12}$$

$$I = \log\left(\frac{System\ Range}{Common\ Range}\right) = \log\left(\frac{1.5-0.1}{1.3-0.1}\right) = 0.07 \tag{13}$$

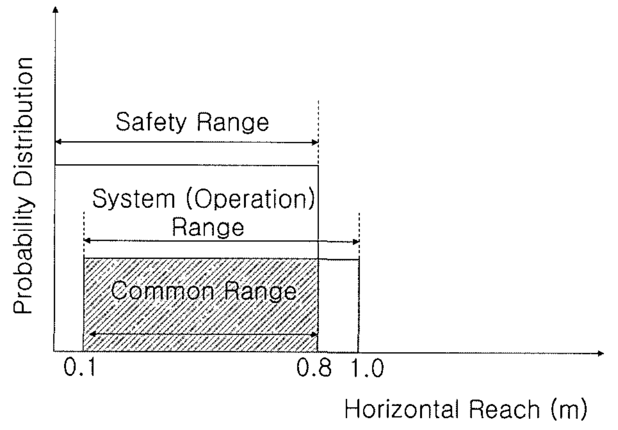

Fig. 4. Probability distribution of horizontal reach that does not guarantee the safety of workers due to shrinkage of safety range.

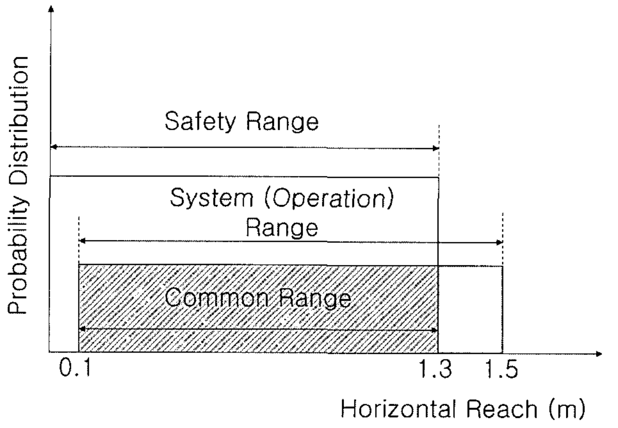

Fig. 5. Probability distribution of horizontal reach that does not guarantee the safety of workers due to extension of system range.

Next, Consider the uncertainty in safety associated with the noise level in work place. Process parameter in this case is the noise level in dB in Figure 6. Suppose that the noise level must be kept below 65dB in order to minimize the adverse effect on the health of workers. The safety range specified by a process designer in this case is from 0dB to 65dB. For the case where the system range is from 47dB to 79dB, the uncertainty in safety is:

$$I = \log\left(\frac{1}{\dfrac{Common\ Range}{System\ Range}}\right) = \log\left(\frac{79-47}{65-47}\right) = 0.250 \tag{14}$$

Consider the uncertainty in safety for degree of comfort that workers feel in moving the heavy load in a particular task. Process parameters are the physical entities related to the weight and size of the load, and the moving distance and height in the work place.

The maximum allowable weight that a worker can carry without any adverse effect on his safety and health varies from worker to worker. In order to deal with the uncertainty involved in quantifying the safety range and the system range in terms of given process parameters (physical quantities), the degree of discom-

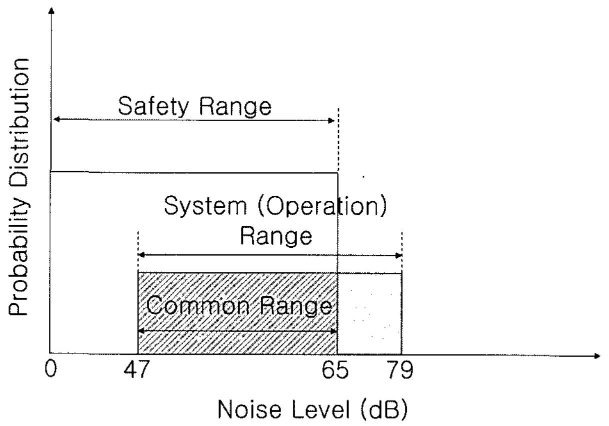

Fig. 6. Probability distribution of noise level that does not guarantee the safety and health of workers.

fort function was introduced. In defining the safety range for the weight of load, it is difficult to say precisely what the safety range should be. One may say that, "if the weight of load is below 5kg (say, in the range of 1kg to 5kg), I feel completely comfortable," or "I feel completely uncomfortable if the weight is over 9kg (say, in the range of 9kg to 13kg)." Between these limits, the degree of discomfort varies from 0% to 100% as shown in Figure 7. This relationship between the degree of discomfort and the weight of the load can take any form and needs to be experimentally evaluated. In this study, however, a linear relationship was assumed for simplicity.

If the weight of load is 6kg, for example, the degree of discomfort ranges from 15% to 55%. Now, one can define the safety range to be the degree of discomfort that workers feel, ranging from 0% to 65%. For the case where the system range of the work environment ranges from 40% to 80% in Figure 8, the uncertainty in safety is estimated to be:

$$I = \log\left(\frac{System\ Range}{Common\ Range}\right) = \log\left(\frac{80-40}{65-40}\right) = 0.204 \tag{15}$$

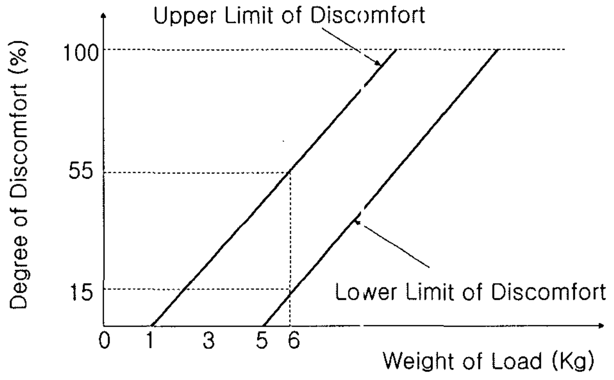

Fig. 7. Degree of discomfort as a function of the weight of the load.

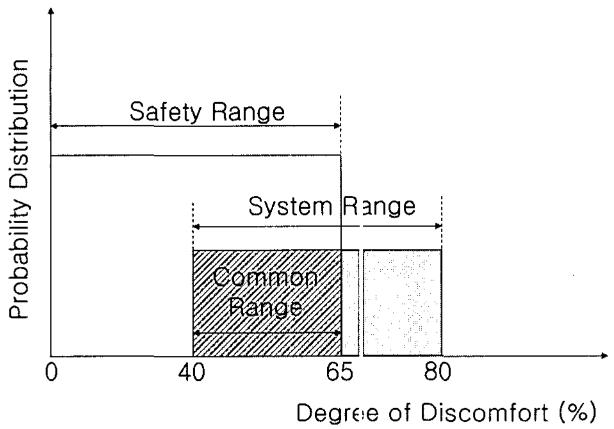

Fig. 8. Probability distribution of degree discomfort that does not guarantee the safety and health of workers.

In case that the size causes another type of discomfort and potentially harm the health of workers in moving the load, a similar relationship between the degree of discomfort and the size of the load can be defined.

According to *Theorem 2*, the total safety-related uncertainty in the process is given by Eq(16), provided that each event is probabilistically independent of each other:

$$\begin{aligned} I_i &= \log\left(\frac{1}{q(x,y,z)}\right) = \log\left(\frac{1}{q(x)}\right) + \log\left(\frac{1}{q(y)}\right) + \log\left(\frac{1}{q(z)}\right) \\ &= 0.109 + 0.250 + 0.204 = 0.554 \end{aligned} \tag{16}$$

The process must be designed in such a way that the total uncertainty in safety given in Eq(16) is minimized.

6. Conclusions

In this study, application of entropy as information-theoretic measure to safety assessment of manufacturing processes was suggested. The idea is based on the general principles of design, design axioms and their applications. Design of any manufacturing process, in general, has to facilitate the creation of new process that may potentially harm the workers. Design of safety-guaranteed manufacturing process is, therefore, very important since it determines the ultimate outcomes of manufacturing activities involving safety of workers. Examples of Cartesian robotic movement, noise level in work place and the loading comfort of heavy weight load were given.

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