The Synchronization Method for Cooperative Control of Chaotic UAV

Young-Chul Bae
Division of Electronic Communication and Electrical Engineering, Yosu Nat'l University
(*ycbae@yosu.ac.kr*)

In this paper, we propose a method to a synchronization of chaotic UAVs(Unmanned Aerial Vehicle) that have unstable limit cycles in a chaos trajectory surface. We assume all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. The proposed methods are assumed that if one of two chaotic UAVs receives the synchronization command, the other UAV also follows the same trajectory during the chaotic UAVs search on the arbitrary surface.

Key words : Chaos UAV; Synchronization; Unstable Limit cycle

Received: May 2005 Accepted: December 2005 Corresponding Author: Young-Chul Bae

1. Introduction

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1]-[2], chaos synchronization and secure/crypto communication [3]-[7], Chemistry [8], Biology [9] and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot where a mobile robot is equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot are represented by an Arnold equation. They applied obstacles in the chaotic trajectory, but they did not mention obstacle avoidance methods and also there is no try to obstacle avoidance and synchronization methods in the UAVs..

In this paper, we propose a method to a synchronization of chaotic UAVs(Unmanned Aerial Vehicle) that have unstable limit cycles in the chaos trajectory surface. We assume that all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When chaotic UAVs meet obstacles among their arbitrary wandering in the chaos trajectory, which is derived using chaos circuit equations such as the Lorenz equation, Chua's equation, the obstacles reflect the chaotic UAVs.

2. Chaotic Mobile's equations

2.1 Chaotic UAV [24]

We assume that each UAV is equipped with standard autopilots for heading hold and mach hold. In order to focus on the essential issues, we will assume that altitude is held constant. Let $(x,y),\psi,$ and v denote the inertial position, heading angle, and velocity for the UAV respectively. Then the resulting kinematics equations of motion are

$$\begin{aligned} \dot{x} &= v\cos(\psi) \\ \dot{y} &= v\sin(\psi) \\ \dot{\psi} &= \alpha_{\psi}(\xi^{c}-\psi) \\ \dot{v} &= \alpha_{v}(v^{c}-v) \end{aligned} \tag{1}$$

where ψ^c and v^c are the commanded heading angle and velocity to the autopilots, and α_ψ and α_v are positive constraints [22,23].

Assuming that α_v is large compared to α_ψ, Eq. (1) reduces to

$$\begin{aligned} \dot{x} &= v\cos(\psi) \\ \dot{y} &= v\sin(\psi) \\ \dot{\psi} &= \alpha_{\psi}(\xi^{c}-\psi) \end{aligned} \tag{2}$$

Letting $\psi^c = \psi + (1/\alpha_\psi)\omega$ and $v^c \approx v$, Eq. (2) becomes

$$\begin{aligned} \dot{x} &= v\cos(\psi) \\ \dot{y} &= v\sin(\psi) \\ \dot{\psi} &= \omega \end{aligned} \tag{3}$$

Eq.(3) rewritten as follows,

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{4}$$

Eq. (3) is similar to two wheel mobile robot equation (5).

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{5}$$

where (x,y) is the position of the robot and θ is the angle of the robot.

2.2 Chaos equations

In order to generate chaotic motions for the UAVs, we apply chaos equations such as a Chua's and Lorenz equation.

2.2.1 Chua's equation

We define the Chua's equation as follows:

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - g(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \tag{6}$$

where

$$g(x) = m_{2n-1}x + \frac{1}{2}\sum_{k=1}^{2n-1}(m_{k-1}-m_k)(|x+c_k|-|x-c_k|)$$

2.2.2 Lorenz equation

We define the Lorenz equation as follows:

$$\begin{aligned}\dot{x} &= \sigma(y - x)\\ \dot{y} &= \gamma x - y - xz\\ \dot{z} &= xy - bz\end{aligned} \tag{7}$$

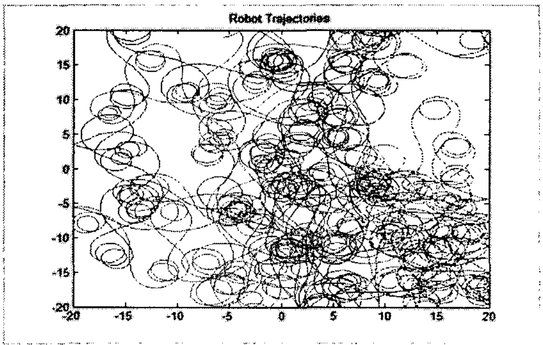
where $\sigma = 10, r = 28, b = 8/3$. The Lorenz equation describes the famous chaotic phenomenon.

2.3 Chaotic Chua's UAVs

Combination of equation (4) and (6), we define and use the following state variables:

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}\alpha(x_2 - g(x_1))\\ x_1 - x_2 + x_3\\ -\beta x_2\\ v\cos x_3\\ v\sin x_3\end{pmatrix} \tag{8}$$

Using equation (8), we obtain the Chua's chaos UAV trajectories with Chua's equation. [Fig. 1] shows the phase plane of Chua's equation.



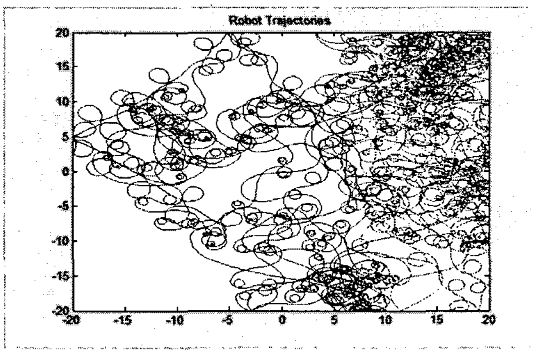
[Fig. 1] Chua's UAVs trajectory

2.4 Chaotic Lorenz UAVs

Combination of equation (4) and (7), we define and use the following state variables:

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}\sigma(y - x)\\ \gamma x - y - xz\\ xy - bz\\ v\cos x_3\\ v\sin x_3\end{pmatrix} \tag{9}$$

Using equation (9), we obtain the Lorenz chaotic UAVs trajectories with Lorenz equation. [Fig. 2] shows the phase plane of Lorenz equation.



[Fig. 2] Lorenz UAVs trajectory

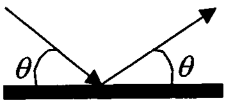
3. Mirror mapping

Equations (8) - (9) assume that the UAVs moves in a smooth state space without boundaries. However, real UAVs move in space with boundaries like walls or surfaces of obstacles. To avoid a boundary or obstacle, we consider mirror mapping when the UAVs approach walls or obstacles using Equation. (10) and (11). Whenever the UAVs approach a wall or obstacle, we calculate the UAVs' new position by using Euation. (10) or (11).

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \quad (10)$$

$$A = 1/1+m \begin{pmatrix} 1-m^2 & 2m \\ 2m & -1+m^2 \end{pmatrix} \quad (11)$$

We can use equation (10) when the slope is infinity, such as θ=90, and use equation (11) when the slope is not infinity.



[Fig. 3] Mirror mapping

4. The UAVs with Van der Pol equation obstacle

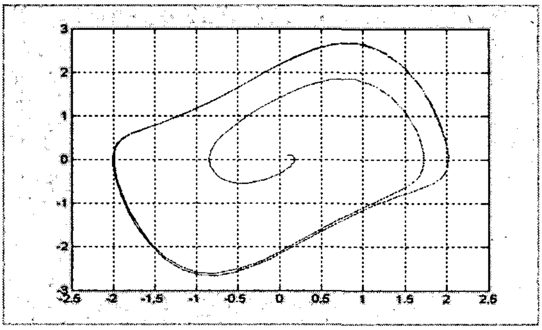
In this section, we will discuss the UAVs's avoidance of Van der Pol(VDP) equation obstacles. We assume the obstacle has a VDP equation with an unstable limit cycle, because in this condition, the UAVs can not move close to the obstacle and the obstacle is avoided.

4.1 VDP equation as a hidden obstacle

In order to represent an obstacle of the UAVs, we employ the VDP, which is written as follows:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= (1-y^2)y - x \end{aligned} \quad (12)$$

From equation (12), we can get the following limit cycle as shown in [Fig. 4].



[Fig. 4] Limit cycle of VDP

4.2 Magnitude of distracting force from the obstacle

We consider the magnitude of distracting force from the obstacle as follows:

$$D = \frac{0.325}{(0.2D_k+1)e^{3(0.2D_k-1)}} \quad (13)$$

where D_k is the distance between each effective obstacle and the UAVs.

We can also calculate the VDP obstacle direction vector as follows:

$$\begin{bmatrix} \bar{x}_k \\ \bar{y}_k \end{bmatrix} = \begin{bmatrix} x_o - y \\ 0.5(1-(y_o-y)^2)(y_o-y)-(x_o-x) \end{bmatrix} \quad (14)$$

where (x_o, y_o) are the coordinates of the center point of each obstacle. Then we can calculate the magnitude of the VDP direction vector (L), the magnitude of the moving vector of the virtual UAVs (I) and the enlarged coordinates (I/2L) of the magnitude of the virtual UAVs in VDP($x_k{}'$,

y_k') as follows:

$$\begin{aligned} L &= \sqrt[2]{(\bar{x}_{vdp}^{\,2} + \bar{y}_{vdp}^{\,2})} \\ l &= \sqrt{(x_r^{\,2} + y_r^{\,2})} \\ x_k' &= \frac{\bar{x}_k}{L}\frac{l}{2},\; y_k' = \frac{y_k}{L}\frac{l}{2} \end{aligned} \qquad (15)$$

Finally, we can get the Total Distraction Vector (TDV) as shown by the following equation.

$$\begin{bmatrix} \dfrac{\sum_k^n \left(\left(1 - \frac{D_k}{D_0}\right)\bar{x} + \frac{D_k}{D_0}\bar{x}_k\right)}{n} \\ \dfrac{\sum_k^n \left(\left(1 - \frac{D_k}{D_0}\right)\bar{y} + \frac{D_k}{D_0}\bar{y}_k\right)}{n} \end{bmatrix} \qquad (16)$$

Using equations (13)-(16), we can calculate the avoidance method of the obstacle in the Chua's and Lorenz equation trajectories with one or more VDP obstacles.

5. Chaotic UAV Synchronization

5.1 Chua's UAVs synchronization

In order to apply to coupled-synchronization theory in the Chua's UAVs, we compromised to state equation of Chua's UAVs is written as follows:

The state equation of main UAVs

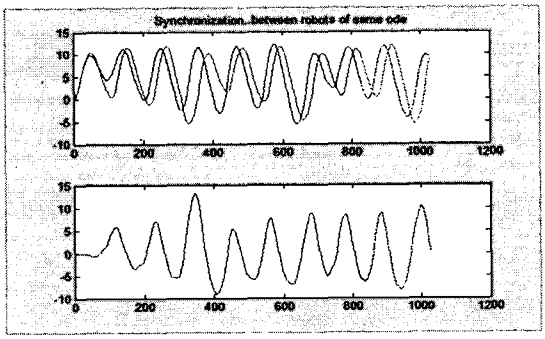
$$\begin{aligned} \dot{x}_1 &= \alpha\,(x_2 - g(x_1)) + k \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= \upsilon \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \qquad (17)$$

The state equation of auxiliary UAVs

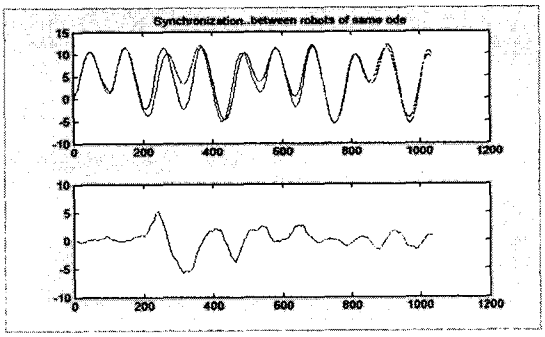
$$\begin{aligned} \dot{x}_1 &= \alpha\,(x_2 - g(x_1)) + k' \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x} &= \upsilon \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \qquad (18)$$

From equation (17) and (18), we apply coupled factors k or k' are 1.0, 2.0. 3.0 at the no obstacle and 3.0 hidden obstacle respectively. The results of Chua's UAVs synchronization are shown in [Fig. 5].

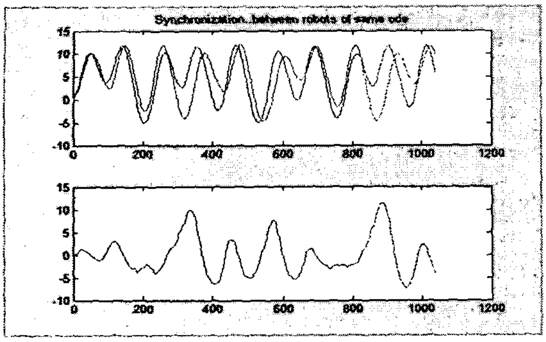
From [Fig. 5] we can recognize synchronizations are generalized synchronization results and according to coupled factor difference, there are different results.



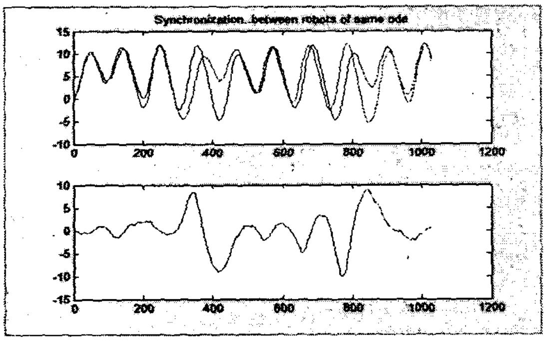
(a) k=1, no obstacle



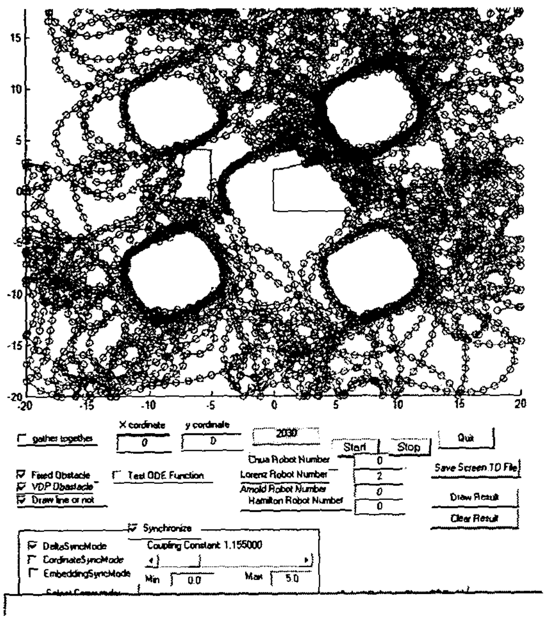
(b) k=2, no obstacle



(c) k=3, no obstacle



(d) k=3, hidden obstacle



(e) Chua's UAVs trajectory

[Fig. 5] Chua's synchronization results

5.2 Lorenz UAVs synchronization

In order to apply to Driven-synchronization theory in the Lorenz's UAVs, we compromised to state equation of Lorenz's UAVs is written as follows:

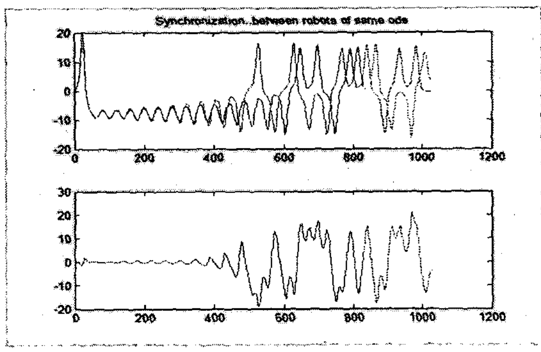
The state equation of main UAVs

$$\begin{aligned} \dot{x}_1 &= \sigma(y - x) \\ \dot{x}_2 &= \gamma x - y + xz \\ \dot{x}_3 &= xy - bz \\ \dot{x} &= \upsilon \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \tag{19}$$

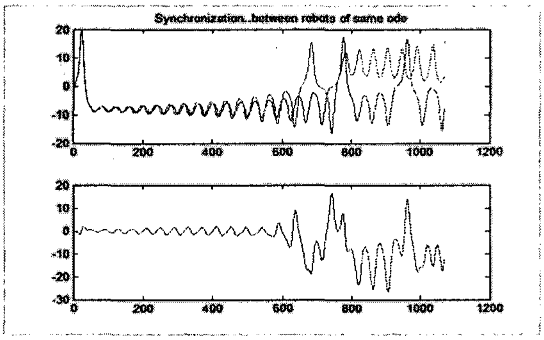
The state equation of auxiliary UAVs

$$\begin{aligned} \dot{x}_1 &= \sigma(y - x) \\ \dot{x}_2 &= \gamma x - y + xz \\ \dot{x} &= \upsilon \cos x_3 \\ \dot{y} &= v \sin x_3 \end{aligned} \tag{20}$$

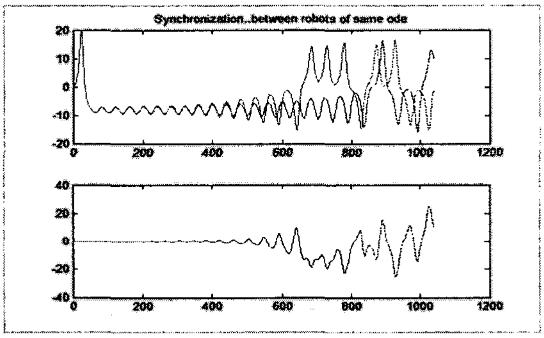
From equation (19) and (20), we can get the results of Lorenz UAVs synchronization are shown in [Fig. 6]



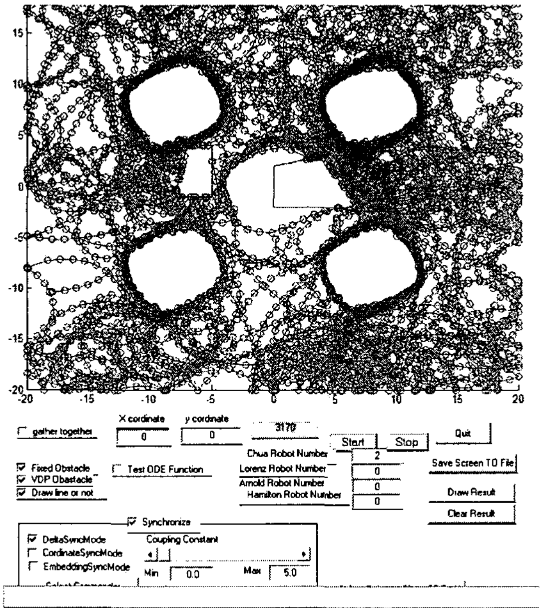
(a) No obstacle synchronization



(b) Fixed obstacle synchronization



(c) Hidden obstacle synchronization



(d) Lorenz UAVs trajectory

[Fig. 6] Lorenz synchronization results

From [Fig. 6] we can recognize synchronization result is generalized synchronization and also we can see that there are different synchronization results according to different obstacle such as fixed and hidden obstacles.

6. Concluding remark

In this paper, we proposed a chaotic UAVs, which employs a UAVs with Chua's equation and Lorenz equation trajectories, and also proposed a UAVs synchronization methods in which coupled-synchronization and driven synchronization.

We designed chaotic UAVs trajectories such that the total dynamics of the UAVs was characterized by a Chua's equation or Lorenz equation, and we also designed the chaotic UAVs trajectories to include an obstacle avoidance method. As a result, we realized that the result of synchronization is generalized synchronization.

Reference

[1] E. Ott, C.Grebogi, and J.A York," Controlling Chaos", Phys. Rev.Lett., vol. 64, no.1196-1199, 1990.

[2] T. Shinbrot, C.Grebogi, E.Ott, and J.A.Yorke, " Using small perturbations to control chaos", Nature, vol. 363, pp. 411-417, 1993.

[3] M. Itoh, H. Murakami and L. O. Chua, "Communication System Via Chaotic Modulations" IEICE. Trans. Fundmentals.

vol.E77-A, no. , pp.1000-1005, 1994.

[4] L. O. Chua, M. Itoh, L. Kocarev, and K. Eckert, "Chaos Synchronization in Chua's Circuit" J. Circuit. Systems and computers, vol. 3, no. 1, pp. 93-108, 1993.

[5] M. Itoh, K. Komeyama, A. Ikeda and L. O. Chua, " Chaos Synchronization in Coupled Chua Circuits", IEICE. NLP. 92-51. pp. 33-40. 1992.

[6] K. M. Short, "Unmasking a modulated chaotic communications scheme", Int. J. Bifurcation and Chaos, vol. 6, no. 2, pp. 367-375, 1996.

[7] L. Kocarev, " Chaos-based cryptography: A brief overview," IEEE, Vol. pp. 7-21. 2001.

[8] M. Bertram and A. S. Mikhailov, "Pattern formation on the edge of chaos: Mathematical modeling of CO oxidation on a Pt(110) surface under global delayed feedback", Phys. Rev. E 67, pp. 036208, 2003.

[9] K. Krantz, f. H. Yousse, R.W , Newcomb, "Medical usage of an expert system for recognizing chaos", Engineering in Medicine and Biology Society, 1988. Proceedings of the Annual International Conference of the IEEE, 4-7, pp. 1303-1304, 1988.

[10] Nakamura, A., Sekiguchi. " The chaotic mobile robot", IEEE Transactions on Robotics and Automation, Volume: 17 Issue: 6, pp. 898-904, 2001.

[11] J.A.K. Suykens, "N-Double Scroll Hypercubes in 1-D CNNs" Int. J. Bifurcation and Chaos, vol. 7, no. 8, pp. 1873-1885, 1997.

[12] P.Arena, P.Baglio, F.Fortuna & G.Manganaro, "Generation of n-double scrolls via cellular neural networks," Int. J. Circuit Theory Appl, 24, 241-252, 1996.

[13] P. Arena, S. Baglio, L. Fortuna and G..Maganaro, "Chua's circuit can be generated by CNN cell", IEEE Trans. Circuit and Systems I, CAS-42, pp. 123-125. 1995.

[14] H. Okamoto and H. Fujii, Nonlinear Dynamics, Iwanami Lectures of Applied Mathematics, Iwanami, Tokyo, 1995, vol. 14.

[15] Y. Bae, J. Kim, Y. Kim, "The obstacle collision avoidance methods in the chaotic mobile robot", 2003 ISIS, pp.591-594, 2003.

[16] Y. Bae, J. Kim, Y.Kim, "Chaotic behavior analysis in the mobile robot: the case of Chua's equation", Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp.5-8, 2003.

[17] Y. Bae, J. Kim, Y.Kim, "Chaotic behavior analysis in the mobile robot: the case of Arnold equation", Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp110-113, 2003.

[18] Y. C. Bae, J.W. Kim, Y.G. Kim, "Chaotic behavior analysis in the mobile robot: the case of Arnold equation", Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp110-113, 2003

[19] Y. C. Bae, J.W. Kim, N.S. Choi , The Collision Avoidance Method in the Chaotic Robot with Hyperchaos Path", KIMICS Conference 2003 Fall, vol. 7, no. 2, pp.584-588, 2003.

[20] Y. C. Bae, J.W. Kim, N.S. Choi, "The Analysis of Chaotic Behaviour in the Chaotic Robot with Hyperchaos Path ov Van der Pol(VDP) Obstacle", KIMICS Conference 2003 Fall, vol. 7, no. 2, pp.589-593, 2003.

[21] Y. C. Bae, J.W. Kim, Y.I, Kim, Chaotic Behaviour Analysis in the Mobile of Embedding some Chaotic Equation with Obstacle", J.ournal of Fuzzy Logic and Intelligent Systems, vol. 13, no.6, pp.729-736,

2003.

[22] Y. C. Bae, J. W. Kim, Y.I, Kim, Obstacle Avoidance Methods in the Chaotic Mobile Robot with Integrated some Chaotic Equation", International Journal of Fuzzy Logic and Intelligent System, vol. 3, no. 2. pp. 206-214, 2003.

[23] Y. C. Bae, J.W. Kim, Y.I, Kim, "The Obstacle Collision Avoidance Methods in the Chaotic Mobile Robots", ISIS 2003 Proceeding of the 4th International symposium on Advanced Intelligent System, pp. 591-594, 2003.

[24] A.Proud, M. Pachter, and J.J. D'Azzo, "Close formation Flight Control" in proceeding of the AIAA Guidance, Navigation and Control Conference and Exhibit,(Portland OR), pp. 1231-1246, American Institute of Aeronautics and Astronautics, Aug, 1999.

요약

카오스 소형 무인 비행체의 협조 제어를 위한 동기화 기법

배영철*

본 논문에서는 카오스 궤적 표면에서 불안정한 리미트 사이클을 가지는 카오스 무인 비행체의 동기화 기법을 제안하였다. 카오스 궤적 표면에서의 모든 장애물들은 불안정한 리미트 사이클을 가지는 반데어 폴(Van der Pol) 방정식을 가지고 있다고 가정하였다. 제안한 기법들은 만약 2대의 카오스 무인 비행체 중 한대가 동기화 명령을 받으면 나머지 한대의 무인 비행체는 임의의 표면상에 카오스적인 탐색을 하는 동안 동일한 궤적을 가지고 추종하게 됨을 확인하였다.

* 여수대학교 공과대학 전자통신전기공학부