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Application of Discrimination Information (Cross Entropy) as Information-theoretic Measure to Safety Assessment in Manufacturing Processes

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Abstract : Design of manufacturing process, in general, facilitates the creation of new process that may potentially harm the workers. Design of safety-guaranteed manufacturing process is, therefore, very important since it determines the ultimate outcomes of manufacturing activities involving safety of workers. This study discusses application of discrimination information (cross entropy) to safety assessment of manufacturing processes. The idea is based on the general principles of design and their applications. An example of Cartesian robotic movement is given.

Key words: safety, manufacturing processes, information, entropy, discrimination information (Cross Entropy)

1. Introduction

Safety is considered to be a commonsense approach to removing agents of injury[1]. Safety, as a concept and practice, has shifted to a complex methodology for the reliable control of injury to human beings and damages to property. However, it does lack a theoretical base. As safety is concerned with reducing accidents and controlling or eliminating hazards at the manufacturing processes, accident prevention is a significant step towards safety improvement.

Design of manufacturing process, in general, facilitates the creation of new process that may potentially harm the workers[2]. Design of safety-guaranteed manufacturing process is, therefore, very important since it determines the ultimate outcomes of manufacturing activities involving safety of workers. Safety in manufacturing environment is considered to be a measure of relative freedom from accidents. In order to improve the safety performance, control of accident is essential and the effectiveness of control of accidents needs to be estimated before any new manufacturing process is put into practice. Safety performance criterion, in this case, needs to be defined a priori.

This study discusses application of discrimination information (cross entropy) as information-theoretic measure to safety assessment of manufacturing processes. The idea is based on the general principles of design and their applications. An example of Cartesian robotic movement is given.

2. Information (Entropy) Analysis

The concept of entropy was first introduced in statistical thermodynamics by physicist Boltzman to quantify the uncertainty involved in the system [3]. Such uncertainty stems from the randomness of the process.

2.1 Continuous Case

Let $x(n)$ be a state of some process that has a set $\boldsymbol{C}$ of possible states. Let Ψ be the set of all possible probability densities q on $\boldsymbol{C}$ such that $q(x \in C) \geq 0$ and

$$\int_C q(x)dx = 1 \tag{1}$$

The entropy of a process with the probability density q is represented as:

$$E[q] = -\int_C q(x)\log q(x)dx \tag{2}$$

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The entropy is a measure of the amount of information produced by a random process $x(n)$, or a measure of uncertainty in a random process. The larger value of entropy corresponds to more uncertainty in the process. The discrimination information (or cross entropy) is a generalization of entropy when the *prior* density p of $x(n)$ is available, and given by [4]:

$$H[q,p] = \int_C q(x)\log\left(\frac{q(x)}{p(x)}\right)dx \tag{3}$$

Eq(3) states that the total amount of information produced by a process $x(n)$ equals the sum of the amount of information gained by the *posterior* (current) density q and the information already acquired by p. The priors must be strictly positive, i.e.,

$$p(x \in C) > 0 \tag{4}$$

The principle of minimum discrimination information provides a method of inference about a *true* unknown probability density $q^+ \in \boldsymbol{C}$ when there exist a prior estimate of q^+ and new information about $q+$ in the form of constraints on the expected values. $\boldsymbol{I}=(q^+ \in \Phi)$ stands for the newly acquired information and is referred as a *constraint*, and Φ is a *constraint set*. New information $\boldsymbol{I}$ can take the form of equality and inequality constraints such that:

$$\int_C q^+(x)c_k(x)dx = 0 \tag{5}$$

$$\int_C q^+(x)c_k^*(x)dx \geq 0 \tag{6}$$

for known sets of bounded constraint functions $c_k(x)$ and $c_k^*(x)$. Let $p \in \Psi$ be an arbitrary prior estimate of density q^+ prior to learning $\boldsymbol{I}$. $H[q, p]$ is the information-theoretic distortion between densities p and q. It can also be interpreted as the amount of information-theoretic distortion provided by $\boldsymbol{I}$ that is not inherent in p.

2.2 Discrete case

For discrete events, , the average information content of the discrete events (or entropy) is defined as:

$$E[q] = \sum_i E[q_i] = -\sum_i q_i \log q_i \tag{7}$$

subject to constraints

$$\sum_i q_i = 1, \sum_i c_i q_i = C \tag{8}$$

where $E[q_i] = \log\left(\frac{1}{q_i}\right) = -\log q_i$ for individual event, C is the total energy and c_i is the energy associated with individual components. For the principle of minimum cross entropy, suppose a system has a finite set of n states with probabilities q^+. Let p be a prior estimate of q^+ and let new information I be provided in the form

$$\sum_i q_i^+ a_{ki} = 0 \tag{9}$$

or

$$\sum_i q_i^+ c_{ki} \leq 0 \tag{10}$$

for known numbers a_{ki} and c_{ki}. Then, it is clear that there exist problems with continuous states and densities for which the foregoing finite problem is discrete case. It can be proved that the cross entropy functional becomes a function of $2n$ variables and

$$H[q,p] = \sum_i q_i \log(q_i/p_i) \tag{11}$$

subject to the constraints (9) and (10).

3. Uncertainty in a Safety Context

In terms of safety involved in the design of manufacturing process, entropy quantifies the complexity of achieving the safety in the process. The more complex a process is, the more information is required to describe and understand the safety features in the process. It is a measure of knowledge required to satisfy a given level of the safety requirement hierarchy and closely related to the probability of achieving safety requirements involved in the process.

Note that the knowledge required to achieve a task in a safe manner depends on the probability of success. For example, if a task can be achieved safely without prior knowledge or additional knowledge about the potential hazards or no hazards are involved in the task, the probability of success in achieving such task without safety problems is "1" and no requisite information is necessary. Probability of success depends on the complexity of task in guaranteeing the safety involved. Therefore, information is related to complexity. Probability of success in achieving tasks increases as complexity of designed processes decreases. Process design must transmit sufficient knowledge so that probability of achieving task (satisfying safety requirements) is as high as possible. Considering the fact that

$$E[q_i]=\log\left(\frac{1}{q_i}\right)=-\log q_i\geq 0 \quad for\ q_i\leq 1 \tag{12}$$

$$E[q]=\sum_i q_i E[q_i]=q_i\log\left(\frac{1}{q_i}\right)\geq 0 \quad for\ q_i\leq 1 \tag{13}$$

higher I (more information contents or more uncertainty) results as the number of variables increase (having more i's). In general, the minimum information content is achieved by:

1. Choosing designs and tolerance which yield larger q_is
2. Minimizing the number of variables, when other things are nearly equal
3. imposing the maximum number of constraints to the proposed design, which reduces the uncertainty/randomness in process design, thus reduces the safety requirements.

Note that the greater the number of constraints (not much choices that process designer can take), the smaller the entropy or the smaller uncertainty in process. Eq(3) and Eq(11) quantifies the additional uncertainty produced or the additional knowledge required to understand the safety features of the process when the design of process is changed from the *prior* (original) design to the *posterior* (current).

4. Uncertainty in a System Context

Consider in Figure 1 where the performance of the process is quantified in view of the safety. Safety range signifies the tolerance associated with process parameters[5, 6]. System range designates the capability of manufacturing system (in terms of tolerance) and the current performance of designed processes. Common range is the overlap between the safety range and the system range. Figure 1 implies how much of safety requirements are satisfied by the current performance of the designed process (system range).

The probability of achieving the particular safety requirement i and the information content are then defined, respectively, by:

$$q_i=\left(\frac{System\ \ Range}{Common\ \ Range}\right)_i \tag{14}$$

$$E[q_i]=\log\left(\frac{1}{\frac{Common\ \ Range}{System\ \ Range}}\right)_i=\log\left(\frac{System\ \ Range}{Common\ \ Range}\right)_i \tag{15}$$

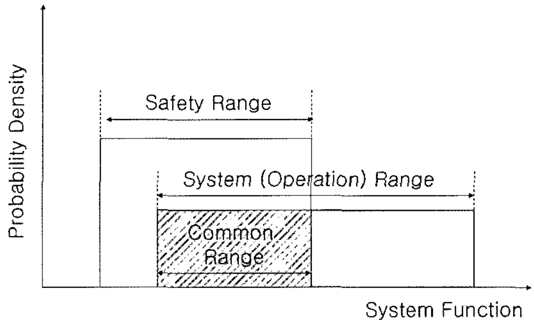

Fig. 1. Probability distribution of a system parameter.

Information content is a measure of the probability of success of achieving the specified safety requirements in manufacturing process or a measure of uncertainty in insuring safety in manufacturing process. It is independent of specific nature of process parameters such as work envelop of a robot motion, noise level in work environment, weight of the load and etc. If the safety range does not overlap with the system range (operation range), process design does not reflect the safety requirements. If the safety range covers the entire system range, all the safety requirements are satisfied by the process parameters in the manufacturing processes.

Two ways of reducing uncertainty (information contents) are:

● to reduce the system range so that the process is as simple as possible for safety.

● to increase the common range. This implies that one has to try to satisfy all safety requirements specified by the safety range with process parameters.

5. Examples

Consider the information associated with the dimensional precision of work envelop in robotic assembly process. Here, the process parameters are geometric dimensions of work envelop. The work envelop is usually composed of several components depending on the type of robot. Each component independently influences the safety of workers. Cartesian coordinate robot, for example, has vertical stroke, vertical reach, horizontal stroke, horizontal reach and traverse stroke. For horizontal reach, the safety range is the "safe" horizontal reach that guarantees the safety of workers and is specified by a process designer, from 1.0 m to 1.5 m in Figure 2. The safety range is usually designated as the "Safe Work Area" on floor. This range varies depending on the types of the robot and the work involved and can be reduced either intentionally or inadvertently by

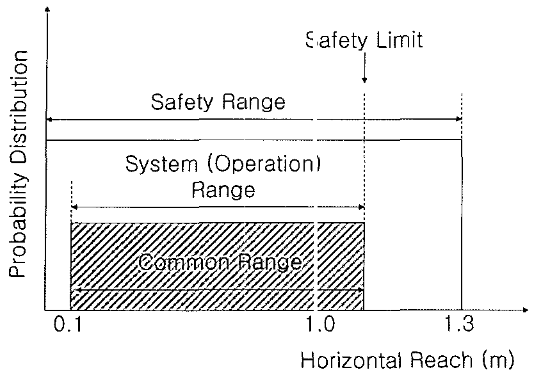

Fig. 2. Probability distribution of horizontal reach that guarantees the safety of workers.

the work range of workers on floor. The system range is the range of a robot arm to move horizontally and is, say between 0.1m and 1.0m in Figure 2. Then, the "Safety Limit" becomes 1.0m, which implies that if a robot arm reaches beyond the safety limit or the system range is reduced below the safety limit, the uncertainty exists and the safety of workers may not be guaranteed.

When the safety range coincides with the system range, no uncertainty (information contents) in insuring the safety of worker is assumed. However, if either workers break into the safe work area so that the safety range shrinks by 0.2 m to between 0.1 m and 0.8 m as in Figure 3 or the horizontal reach of a robot arm is extended by 0.2 m to 1.2 m beyond the safety limit that is work-specific, as in Figure 4, the safety of workers is not guaranteed.

The uncertainty in safety in each case is then given by:

$$E[p_i] = \log\left(\frac{System\ \ Range}{Common\ \ Range}\right) = \log\left(\frac{1.0-0.1}{0.8-0.1}\right) = 0.109 \tag{16}$$

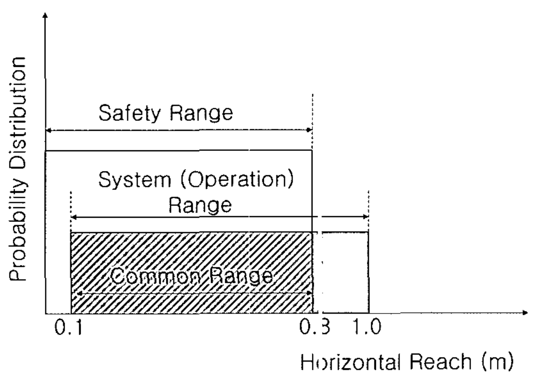

Fig. 3. Probability distribution of horizontal reach that does not guarantee the safety of workers due to shrinkage of safety range.

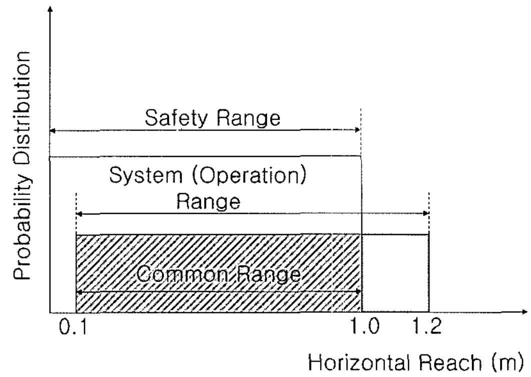

Fig. 4. Probability distribution of horizontal reach that does not guarantee the safety of workers due to extension of system range.

$$E[p_2] = \log\left(\frac{System\ \ Range}{Common\ \ Range}\right) = \log\left(\frac{1.2-0.1}{1.0-0.1}\right) = 0.087 \tag{17}$$

where p_1 and p_2 designate the event of shrinking the safety range by 0.2 m and the event of extending the reach of a robot arm by 0.2 m, respectively. The result in Eq(16) and (17) suggest that breaking into the safety limit by workers causes more uncertainty in terms of safety than extending the reach of a robot arm beyond the safety limit. In general, reducing the common range (by reducing the safety range) causes more uncertainty than extending the system range. Therefore, uncertainty measure must be taken into account in adjusting the safety limit so as to minimize the increase of uncertainty involved in the process. If the safety range shrinks by 0.2 m and the system range increases by 0.2 m at the same time, the uncertainty in safety is given by:

$$E[q] = \log\left(\frac{System\ \ Range}{Common\ \ Range}\right) = \log\left(\frac{1.2-0.1}{0.8-0.1}\right) = 0.196 \tag{18}$$

where q designates the event of both shrinking the safety range by 0.2 m and extending the reach of a robot arm by 0.2 m. If one specifies the increase of uncertainty using the cross entropy, one can have:

$$E[q,p_1] = \log(q/p_1) = \log\left(\frac{1.2-0.1}{0.8-0.1}\right) - \log\left(\frac{1.0-0.1}{0.8-0.1}\right) = 0.087 \tag{19}$$

$$E[q,p_2] = \log(q/p_2) = \log\left(\frac{1.2-0.1}{0.8-0.1}\right) - \log\left(\frac{1.2-0.1}{1.0-0.1}\right) = 0.109 \quad (20)$$

where $H[q,p_1]$ specifies the uncertainty increased by extending the reach of a robot arm by 0.2 m. Similarly, $H[q,p_2]$ specifies the uncertainty increased by shrinking the safety range by 0.2 m.

5. Conclusions

In this study, application of entropy and cross entropy as information-theoretic measure to safety assessment of manufacturing processes was suggested. The idea is based on the general principles of design, design axioms and their applications. An example of Cartesian robotic movement was given in which the entropy and the cross entropy proved to be effective in determining the ultimate outcomes of manufacturing activities involving safety of workers.

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