혼합 H_2/H_∞ 제어에 의한 강인한 서보시스템의 설계 -3관성 벤치마크문제의 해법-

Robust Servo System Design by H_2/H_∞ Control -Application to Three Inertia Benchmark Problem-

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요 약

본 논문은 혼합 H_2/H_∞ 제어를 이용한 강인한 서보시스템의 설계법을 제안하고 이것을 3관성 벤치마크문제에 적용하여 이 방법의 유용성을 확인한다. 먼저 내부모델의 원리를 이용하여 혼합 H_2/H_∞ 제어기법을 적용할 수 있는 구조를 제안하고 이 구조의 타당성을 검토한다. 다음에 강인한 서보시스템의 설계조건을 각각 H_2 및 H_∞의 설계사양으로 분리한 뒤 제안된 혼합 H_2/H_∞ 설계법에 적용하여 설계를 LMI이론에 의하여 실행한다. 마지막으로 3관성벤치마크 문제에서 부가된 몇 가지의 사양을 제안된 혼합 H_2/H_∞ 법에 맞도록 수정한 뒤 제어기의 설계를 통한 설계사양의 만족도를 확인한다.

Abstract

The purpose of this paper is to propose an approach to design a robust servo controller based on the mixed H_2/H_∞ theory, and confirm its validity by applying to a benchmark problem. First, the existing H_∞ servo problem is modified to a structure for the mixed H_2/H_∞ control problem by virtue of the internal model principle. By making use of proposed structure, we can divide specifications required in the robust servo system design into H_2 and H_∞ performance criteria, respectively. It is shown that the proposed design approach is quite effective through an application to a three inertia benchmark problem.

Key words : Robust performance, Servo system, mixed H_2/H_∞ Control, LMI

I. Introduction

In the past several years, the H_2 and/or H_∞ control have attracted many researchers' attentions. Also, its effectiveness has been reported in various application fields in these years. The H_∞ control theory provides us a quite powerful tool for shaping the loop gain in the frequency domain or obtaining the robust stability property. Therefore, it is quite useful to obtain satisfactory feedback properties such as low sensitivity. On the other hand, the H_2 optimal control theory has been heavily studied since 1960's as the LQG optimal control problem. The H_2 norm performance measure seems to be suitable for obtaining good command response.

For a servo system design, the following three specifications have been of practical interests: (1) internal stability of the closed loop system which must be guaranteed; (2) desired feedback characteristics such as robust stability, sensitivity reduction and disturbance attenuation; (3) desired transient and steady state properties such as robust tracking to reference inputs.

The H_∞ control is a suitable technique to achieve the first two specifications, because they can be naturally expressed as H_∞ norm constraints. However, since the H_∞ control is based on the maximum singular value of the

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transfer function matrix from disturbance to evaluation signals, it is inevitable that the response be rather conservative. Therefore, it is required to alleviate this phenomenon in order to meet the third specification. Recently, it has been proved that, by introducing H_2 specification into the H_∞ design, we could simultaneously benefit from the H_2 and H_∞ control design [1]. This approach is called a mixed H_2/H_∞ control, and in consequence, a designer can arbitrary determine the trade off between H_2 (e.g. noise rejection) and H_∞ (e.g. robust stability) performance of the closed loop system.

The purpose of this paper is to propose an effective approach to design a robust servo controller based on the mixed H_2/H_∞ control and confirm its effectiveness by application to a benchmark problem. The design objectives such as minimal tracking error and robust performance are first defined in terms of H_2 and H_∞ minimization. These objectives are then converted into linear matrix inequalities (LMIs). That is, the robustness stability criterion is expressed in an H_∞ norm LMI and the tracking performance is expressed in an H_2 norm LMI.

The formulation of the mixed H_2/H_∞ control problem for servo systems is presented in Section II. The main results are given in Section III, where the structure of generalized plant for robust tracking is proposed, and it is shown that there always exists a solution to the problem by virtue of an LMI approach. In Section IV, after designing a robust controller based on the proposed method for the three inertia benchmark problem, we analyze the results of simulation and check the validity of the proposed structure as a robust servo system design method.

II. Problem Formulation

2.1 H_∞ Servo Problem

Consider a unity feedback control system shown in Fig.1, where $G(s)$ and $K(s)$ denote a plant and controller, respectively. Only finite dimensional linear time-invariant (LTI) systems and controller will be considered in this paper. Furthermore, for simplicity, all of the signals are regarded as scalar (SISO). We assume that the class of a reference signal is described as

$$r = G_R(s) r_o \qquad (1)$$

where $G_R(s)$ is a transfer function which does not have stable poles, and r_o is an unknown constant. For example, in the case of step and ramp-type reference, $G_R(s)$ becomes $(1/s)$ and $(1/s^2)$, and r_o represents the magnitude of step and the slant of ramp signal. The purpose of servo system design is to find a feedback compensator $K(s)$ that satisfies the following three specifications: 1) closed loop internal stability, 2) desired feedback characteristics such as robust stability, sensitivity reduction and disturbance attenuation, and 3) robust tracking. Here, the robust tracking means that the output $y(t)$ of the plant must track for any type of reference signal $r(t)$ without steady state error, i.e., $\lim_{t\to\infty} e(t) = 0$, under plant perturbation and/or a step type disturbance input $d(t)$.

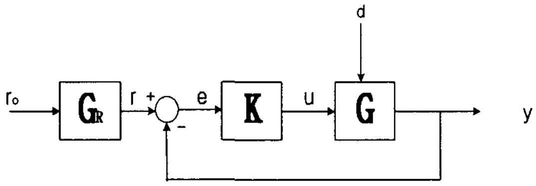


Fig. 1 Feedback Control System

First, we consider the robust servo problem with H_∞ norm bound, called the H_∞ servo problem and defined as follows.

[Definition 1] H_∞ servo problem: Consider a feedback control system depicted in Fig.1. For a given generalized plant G which includes the plant and weighting functions for loop shaping, and a reference signal $r(t)$, find a controller K satisfying the following three specifications:

(S1) K internally stabilizes G

(S2) $\|T_{ed}(s)\|_\infty < \gamma$

(S3) K achieves the robust tracking property for the reference signal.

$T_{ed}(s)$ represents the transfer function from d to e. There have been lots of researches related to the H_∞ servo problem. In [2], sufficient and necessary conditions for the existence of a solution K to the H_∞ servo problem were derived as follows:

[Theorem 1] Consider the feedback control system depicted in Fig.1. It is assumed that the plant $G=ND^{-1}$ and the reference signal generator $G_R=\tilde{D}_R^{-1}\tilde{N}_R$ are described by using left and right coprime factorization, respectively. Then, the necessary and sufficient condition for the existence of a solution K to the H_∞ servo problem is that the following three conditions must be satisfied simultaneously.

(C1) $K \in \Omega(G)$, where Ω represents the set of stabilized compensators to a plant G.

(C2) $(I-NK)\tilde{D}_R^{-1} \in RH_\infty$

(C3) $D_K/\alpha_R \in RH_\infty$, where α_R denotes the largest invariant factor of $\tilde{D}_R$, D_K is a denominator when a solution K is described by right coprime factorization and RH_∞ is the set of all stable real rational functions. □

Hence, H_∞ servo system must have the structure as depicted in Fig.2, where K_r internally stabilizes P/α_R, and z and w denote outputs of interest and exogenous inputs, respectively.

In the case of, for example, step and sinusoid type reference signals, α_R becomes

$$\frac{s}{s+\beta}\ (\text{step}),\ \frac{s^2+\omega_r^2}{(s+\beta)^2}\ (\text{sinusoid}) \tag{2}$$

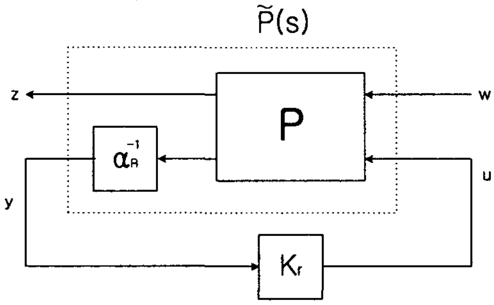


Fig. 2 The Original Problem

where ω_r is the frequency of a reference signal and β is an arbitrary constant.

2.2 Mixed H_2/H_∞ Optimal Design Problem

The basic block diagram used in this paper is given in Fig.3, in which the generalized plant $\tilde{P}$ is given by the state space equations

$$\begin{aligned} \tilde{P}: \dot{x} &= Ax + B_1 w + B_2 u \\ z_\infty &= C_\infty x + d_{\infty 1} w + d_{\infty 2} u \\ z_2 &= C_2 x + d_{21} w + d_{22} u \\ y &= C_y x + d_{y1} w \end{aligned} \tag{3}$$

where $x \in R^n$ is the state vector, u is the control input, w is the exogenous input (such as disturbance, sensor noise etc.), y is the measured output and $z = [z_\infty\ z_2]^T$ is the vector of output signal related to the performance of the control system (z_∞ is related to the H_∞ performance and z_2 is related to the H_2 performance).

Let T_{zw} be the closed transfer function from w to z for the system $\tilde{P}$ closed by the output feedback control law $u = K_r y$. Our goal is to compute a dynamical output feedback controller K_r

$$\begin{aligned} K_r\ : \dot{x}_K &= A_K x_K + B_K y \\ u &= C_K x_K + d_K y \end{aligned} \tag{4}$$

that simultaneously meets H_2 and H_∞ performance on the closed loop behavior.

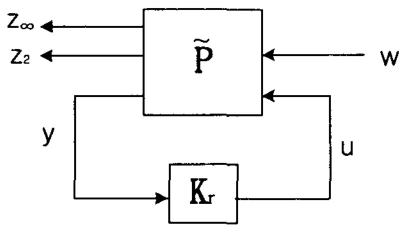


Fig. 3 Block diagram of Mixed H_2/H_∞ control

The closed loop system T_{zw} has the following description

$$\begin{aligned} T_{zw}\ : \dot{x}_d &= A_d x_d + B_d w \\ z_\infty &= C_{d1} x_d + d_{d1} w \\ z_2 &= C_{d2} x_d + d_{d2} w \end{aligned} \tag{5}$$

The problem we concerned with can be summarized as minimizing the H_2 norm of the channel $w \to z_2$ (: T_2), while keeping the bound γ on the H_∞ norm of the channel $w \to z_\infty$ (: T_∞), i.e.

$$\min \|T_2\|_2 \ \text{ subject to : } \|T_\infty\|_\infty < \gamma$$

Since this problem can be reformulated as a convex optimization problem[1], the optimal solution under the given value of γ can be obtained through LMI. As efficient interior point algorithms are now available to solve the generic LMI problems, the mixed H_2/H_∞ problem can be solved without much difficulty in order to find the best trade off between the H_2 and H_∞ minimization.

III. Main Results

In order to find a robust servo controller K which satisfies good reference response (H_2 performance) as well as desired feedback properties (H_∞ performance), we adopt the mixed H_2/H_∞ control system rather than the conventional H_∞ control theory. For the purpose of this, we, first, divide the control objectives into each H_∞ and H_2 performance criterion, then describe the two criteria as one formation. In other words, a new structure for the mixed H_2/H_∞ control is required to deal with two criteria simultaneously. Here, we introduce the following interconnection for robust control system, on which a controller satisfying two criteria is designed.

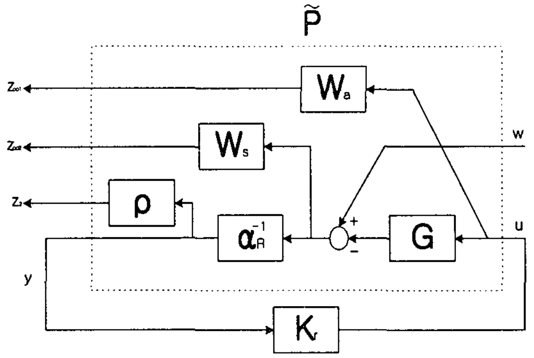


Fig. 4 The Proposed Generalized Plant for Mixed H_2/H_∞ control

A) H_∞ Control Problem : In Fig.4, $W_a(s)$ denotes a weighting function related to the plant uncertainty (this case, additive uncertainty) and $W_S(s)$ is a sensitivity weighting function. We can summarize the robust tracking H_∞ control problem as follows:

(S1) $K_r(s)$ stabilizes $\tilde{P}(s)$.

(S2) $\|T_\infty(s)\|_\infty = \left\| \begin{matrix} T_{z_{\infty 1}w}(s) \\ T_{z_{\infty 2}w}(s) \end{matrix} \right\|_\infty < \gamma$

where $T_{z_{\infty 1}w}(s)$ denotes the transfer function from w to $z_{\infty 1}$ and is related to the robust stability requirement (for additive uncertainty)

$$\|T_{z_{\infty 1}w}\|_\infty = \|(1+GK)^{-1}KW_a\|_\infty < \gamma \qquad (6)$$
$$K = \alpha_R^{-1}K_r$$

The nominal performance condition is reflected by $\|T_{z_{\infty 2}w}(s)\|_\infty < \gamma$, where $T_{z_{\infty 2}w}(s)$ denotes the transfer function from w to $z_{\infty 2}$. In this case, if (S2) is satisfied under the condition of $\gamma = 0.5$, then robust performance can be guaranteed outright, since (S2) will satisfy the SISO robust performance test for additive uncertainty given by [3]

$$\| \, |T_{z_{\infty 1}w}(s)| + |T_{z_{\infty 2}w}(s)| \, \|_\infty < 1 \qquad (7)$$

The robust performance condition given in (7) is necessary and sufficient, and the left hand side is actually the peak value of μ [4]. In section 4, we will investigate this value as an index to robust performance when designing a control system for the benchmark problem.

By virtue of the Bound Real Lemma the H_∞ norm of $T_\infty(s)$ is smaller than γ if and only if there exists a symmetric positive definite matrix X_∞ with

$$\begin{bmatrix} A_{cl}^T X_\infty + X_\infty A_{cl} & X_{cl} B_{cl} & C_{cl\infty}^T \\ B_{cl}^T X_\infty & -\gamma & d_{cl\infty} \\ C_{cl\infty} & d_{cl\infty} & -\gamma \end{bmatrix} < 0 \qquad (8\text{-a})$$

$$X_\infty > 0 \qquad (8\text{-b})$$

where all the matrices A_{cl}, B_{cl}, C_{cl} and $d_{cl\infty}$ are defined in (4).

B). H_2 control Problem: The traditional H_2 optimization attempts to minimize the energy of the system output when the system is faced with white Gaussian noise input. So, in order to design a controller adept at handling noises, H_2 optimization should be considered. That is, the H_2 norm minimization of the transfer function $T_{z_2 w}(s)$ from w to z_2 in Fig.5 is to be taken as a controller design problem, where ρ is a varying parameter.

It is well known that the upper bound of $\|T_{z_2 w}\|_2^2$ is defined by $\mathrm{tr}(C_{cl2} W_o C_{cl2}^T)$, where W_o solves the Lyapunov equation

$$A_{cl} W_o + W_o A_{cl}^T + B_{cl} B_{cl}^T = 0 \qquad (9)$$

Since $W_o < W$ for any W satisfying

$$A_{cl} W + W A_{cl}^T + B_{cl} B_{cl}^T < 0 \qquad (10)$$

It is readily verified that $\|T_{z_2 w}\|_2^2 < \nu$ if and only if there exists $W > 0$ satisfying (10) and $\mathrm{tr}(C_{cl2} W C_{cl2}^T) < \nu$ [1]. With auxiliary parameter Q, the following analysis result has been known:

[Theorem 2] A_{cl} is stable and $\|T_{z_2 w}\|_2^2 < \nu$ if and only if there exist symmetric $X_2 = W^{-1}$ and Q such that

$$\begin{bmatrix} A_{cl}^T X_2 + X_2 A_{cl} & X_2 B_{cl} \\ B_{cl}^T X_2 & -I \end{bmatrix} < 0 \qquad (11\text{-a})$$

$$\begin{bmatrix} X_2 & C_{cl2}^T \\ C_{cl2} & Q \end{bmatrix} > 0 \qquad (11\text{-b})$$

$$\mathrm{tr}(Q) < \nu \qquad (11\text{-c})$$

□

C. **Mixed H_2/H_∞ Control**: The mixed H_2/H_∞ controller K_r must satisfy both of the following criteria simultaneously

$$\|T_{z_\infty w}\|_\infty < \gamma \qquad (12)$$

$$\|T_{z_2 w}\|_2 < \nu \quad (13)$$

In order to keep the tractability of the constrained optimization problem, the following assumption is considered.

$$X_\infty = X_2 = P \quad (14)$$

Therefore, notice that X can be written as

$$P = \begin{bmatrix} X & * \\ * & * \end{bmatrix} = \begin{bmatrix} Y & * \\ * & * \end{bmatrix}^{-1} \quad (15)$$

where dim(X)=dim(Y)=dim(A_{cl}) and $\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0$ is coupling LMI. The solution $X, Y,$ and Q under the constraints of (12), (13) is dependent on the value of γ and ν, and can be obtained using any available software such as MATLAB LMI toolbox [5].

IV. Application to The Benchmark Problem

We consider the benchmark problem given in [6]. The model treated in the problem is a coupled three inertia system that reflects the dynamics of mechanical vibrations. A controller, by which robust performance (both in time and frequency domain) conditions must be satisfied, is required in order to solve the problem.

The three inertia benchmark problem is shown in Fig.5, where the meanings of each symbol are as follows:

θ_i $(i=1,2,3)[radian]$= angular displacement

τ= control torque $[Nm]$

τ_{d_i} $(1=1,2,3)$ = torque disturbance

j_i $(i=1,2,3)[kgm^2]$ = moment of inertia

d_i $(i=1,2,3,a,b)[Nms/rad]$ = viscous friction coefficient of motors,

k_i $(i=a,b)$ $[Nm/rad]$ = torsional coefficient of connection part

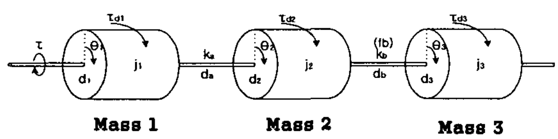


Fig. 5 Coupled three-inertia system

By using these parameters, the equations of motion can be described as

$$j_1\ddot{\theta}_1 = -d_1\dot{\theta}_1 - k_a(\theta_1-\theta_2) - d_a(\dot{\theta}_1-\dot{\theta}_2) + \tau + \tau_{d_1}$$

$$j_2\ddot{\theta}_2 = k_a(\theta_1-\theta_2) + d_a(\dot{\theta}_1-\dot{\theta}_2) - d_2\dot{\theta}_2 - f_b(\theta_2,\theta_3) - d_b(\dot{\theta}_2-\dot{\theta}_3) + \tau_{d_2} \quad (16\text{-a})$$

$$j_3\ddot{\theta}_3 = f_b(\theta_2,\theta_3) + d_b(\dot{\theta}_2-\dot{\theta}_3) - d_3\dot{\theta}_3 + \tau_{d_3}$$

$$f_b(\theta_2,\theta_3) = k_b(\theta_2-\theta_3)$$

It is assumed that the control torque τ is generated by voltage $e[V]$ through a current amplifier, the equation of which is shown below.

$$\dot{\tau} = -a_e\tau + a_e e \quad (16\text{-b})$$

[Problem: Position Control Problem] We want to design a controller by which the six design specifications given in [6] are to be satisfied on condition that all of the 11 parameters are subject to change within the range of variations and there must exist hardware constraints.

4.1 Feedback Controller Design by the Mixed H_2/H_∞ Control

In this problem, there are 11 physical parameters that are assumed to be changing within the given range of variations. If all the variations are reflected in the controller design, the obtained controller may be considerably conservative as well as complex. Therefore, we first find out principal parameters (j_3 and k_a) which strongly affect the resonant frequency of the plant by plotting the frequency response curve, then a robust controller design dependent on these parameters is carried out by making use of the proposed structure for the mixed H_2/H_∞ control. The robustness on variations of the other parameters is evaluated through simulation.

The parameter variations of j_3 and k_a can be described by additive uncertainty such as

$$j_3 = j_{3_o} + W_{j_3}\delta_{j_3} \quad (17\text{-a})$$

$$k_a = k_{a_o} + W_{k_a}\delta_{k_a} \quad (17\text{-b})$$

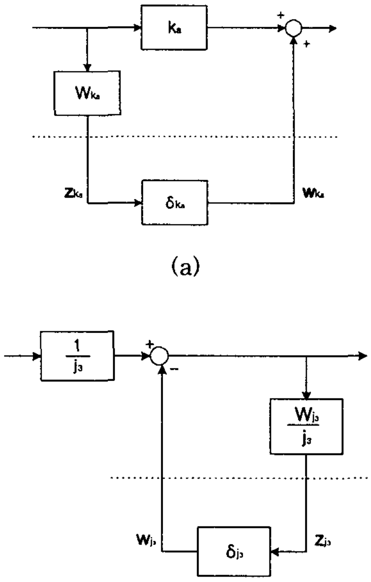
where j_{3_o}, K_{a_o} are nominal values, W_{j_3}, W_{k_a} are constant values representing the range of variations, and

$$|\delta_{j_3}| \le 1, \ |\delta_{k_a}| \le 1 \quad (18)$$

Furthermore, if we express the reciprocal of j_3 as

$$\frac{1}{j_3} = \frac{1}{j_{3_o}[1+(W_{j_3}/j_{3_o})\delta_{j_3}]} \quad (19)$$

the conservativeness related to the additive uncertainty may be reduced up to a certain point by adopting (19) in place of (17a) in the plant dynamics.



(a)

(b)

Fig. 6 Representation for additive parameter variations

If we define the input and output of variations as

$$z_{\infty_1} = [z_{j_3}\ \ z_{k_a}]^T,\ w_S = [w_{j_3}\ \ w_{k_a}]^T \tag{20}$$

then parameter variation from w_S to z_∞ can be described as a structured perturbation using

$$\Delta = \mathrm{diag}[\delta_{j_3}\ \ \delta_{k_a}] \tag{21}$$

The purpose of a controller is, if possible, to make $z_{\infty_2} = [\theta_1 - \theta_2,\ \theta_2 - \theta_3,\ u]^T$ small in the presence of parameter variations and disturbance. This can be achieved by letting the closed loop transfer function T_{∞_2} from $w_T = [\tau_{d_1}\ \ \tau_{d_2}]^T$ to z_{∞_2} have robust performance. Therefore, if it is possible to design a controller by which we keep the H_∞ norm of the closed loop transfer function T_∞ from $w = [w_{j_3}\ w_{k_a}\ \tau_{d_1}\ \tau_{d_2}]^T$ to $z = [z_{j_3}\ z_{k_a}\ \theta_1 - \theta_2\ \theta_2 - \theta_3\ u]^T$ low, the design specifications can be satisfied. In other words, the output will track the reference signal and torsional vibrations between θ_1 and θ_2, θ_3 and θ_3 will be also suppressed under the condition of torque disturbance and parameter variations.

As a next step, by introducing H_2 specifications into the H_∞ design, we can allow to take noise transmission aspects into account. That is, a controller, by which the H_2 norm of the transfer function T_2 from w (refer to Fig. 7) to z_2 is to be minimized under the condition of the H_∞ norm, can be designed by applying the mixed H_2/H_∞ control to the structure shown in Fig.7.

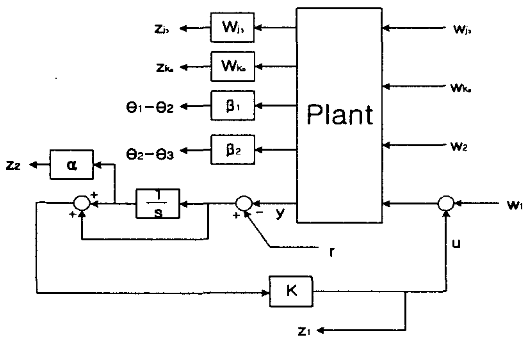


Fig. 7 Structure for Mixed H_2/H_∞ controller design

4.2 Feedforward Controller Design

Since it is impossible to meet all the design specifications related to the output transient response only by a feedback controller, we adopt the structure of the two degree of freedom system to cope with this problem. That is, by adding a feedforward path, we try to improve the output time response. The feedforward controller is designed by using Model Matching Method [2].

Fig.8 shows the basic structure for the two degree of freedom system used, where G denotes the real transfer function from the control input τ to output $y(=\theta_1)$ and G_m denotes an ideal model for design, and F can be arbitrary determined if only $G_m^{-1}F$ is stable and proper. If G and G_m are identical, we can prescribe the output response by making use of the feedforward controller F independent of the feedback controller K_r, because the transfer function from r to y becomes F regardless of K_r. And in case of G not coinciding with G_m, the feedback controller K_r will act as a compensator for the tracking error.

It is assumed that G_m is an ideal model that has no viscous friction and torsion, that is, $\theta_1 = \theta_2 = \theta_3$. Therefore, we define

$$G_m^{-1} = (j_1 + j_2 + j_3)s^2 \tag{22}$$

$$F = \frac{1}{(T_1^2 s^2 + 2\zeta T_1 s + 1)(T_2 s + 1)} \tag{23}$$

where $T_1 = 0.023,\ T_2 = 0.03,\ \zeta = 0.9$, by which the output response can sufficiently satisfy the design specifications when a step reference is added.

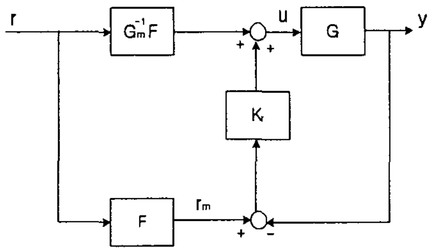


Fig. 8 Two-Degree-of-Freedom System

4.3 Simulation Results

After designing a robust controller based on Fig.7, we check the six design specifications through simulation. MATLAB is used for computation.

In Table 1, the parameter values used in the simulation are shown, where 'nominal' means an ideal case without parameter variations, 'Casel' represents that the moments of inertia have their minimum values and the torsional and viscous friction coefficients are varied maximally within the range of variation, 'Case2', on the contrary, represents that the moments of inertia have their maximum values and the torsional and viscous friction coefficients are varied minimally, and the case that all of the parameters have their minimal value is represented by 'Case3'.

The values of W_{j_3}, W_{k_a} which express the magnitude of variations on j_3, k_a are given as 0.004 (20% variation) and 8 (10% variation), respectively. And constants α, β_1, β_2 are determined as 10, 0.08, 0.05, respectively through several trial and errors.

Although six specifications were originally given in [6], we will show two representative simulation results on account. of space considerations.

Table 1. Nominal and varied parameters

parameters	Nominal	Case 1	Case 2	Case 3
j_1	0.001	0.0009	0.0011	0.0009
j_2	0.001	0.0009	0.0011	0.0011
j_3	0.002	0.001	0.003	0.003
k_a	920	1012	828	828
k_b	80	88	72	72
d_1	0.005	0.055	0.045	0.045
d_2	0.001	0.0011	0.0009	0.0009
d_3	0.007	0.035	0.0014	0.0014
d_a	0.001	0.01	0.0002	0.0002
d_b	0.001	0.0011	0.0009	0.0009
a_e	5000	5000	5000	4500

(1) Tracking ability - specification no. 1

The reference tracking ability is shown in Fig. 9, where θ_3 and τ represent the plant output and control input, and each line represents 4 cases of the simulation results. The results of simulation are arranged in the Table 2, from which we know that the design specifications were sufficiently satisfied in the presence of the parameter variations and disturbance.

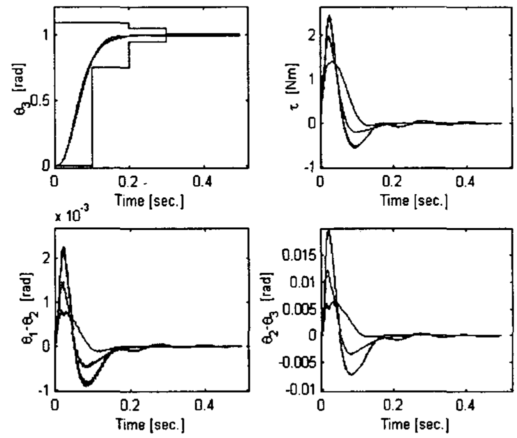


Fig. 9 Step Responses to the reference input

(2) Complementary Sensitivity Function - Specification no. 4

The gain plots of the complementary sensitivity function are shown in Fig.10. We know that, although the condition the gain must be under 20[dB] over all the frequencies considered - is satisfied despite of variations, the other one - the gain must be under -20[dB] above 300[rad/sec] frequency - cannot be met in any cases. Actually, since it was already known that specification no. 4 and the others had a reciprocal relationship each other, it is impossible to meet all the specifications simultaneously. Numerical results are arranged in Table 3.

Table 2 Results for spec. no. 1

	Requirements	Nominal	Worst Case
$\max_{t\geq 0} \lvert\tau\rvert$	≤ 3	1.9637	2.4508 (Case 2)
$\max_{t\geq 0} \lvert\theta_1-\theta_2\rvert$	≤ 0.02	0.0015	0.0023 (Case 3)
$\max_{t\geq 0} \lvert\theta_2-\theta_3\rvert$	≤ 0.02	0.0121	0.0197 (Case 2)

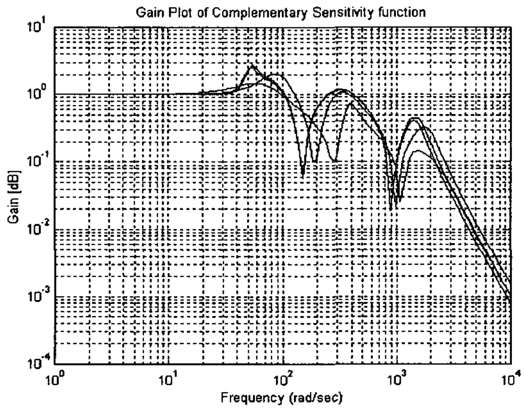


Fig. 10 Gain plots of complementary sensitivity function

Table 3 Results for spec. no. 4

	requirements	Nominal	Worst Case
$\max_{\omega} \lvert G(j\omega) \rvert$	≤ 20	6.3577	8.7150 (Case 2)
$\max_{\omega \geq 300} \lvert G(j\omega) \rvert$	≤ -20	0.7484	1.4813 (Case 2)

(3) Robust stability by μ analysis

After closing the plant with the mixed H_2/H_∞ controller, we take the additive uncertainty of the parameters j_3, k_a as an input and output of the closed loop system (see Fig.6). Then the system is robustly stable for all structured $\Delta(s)$ satisfying $\|\Delta\|_\infty < 1$ if and only if the interconnected system in Fig.11 is stable. This can be done by checking

$$\mu_m := \sup_{\omega} \mu_\Delta(\tilde{P}(j\omega)) < 1 \tag{24}$$

That is, we can check robust stability of Fig.11 by evaluating (24). The μ value is shown in Fig.12, when we let the class of model error as $\Delta \in \text{diag}[C\ C]$, where C denotes the set of complex numbers. Since the maximum μ value is about 0.28 at ω=300[rad/sec], we can confirm that the robust stability is satisfied.

V. Conclusion

In this paper, we have proposed a generalized plant structure for the mixed H_2/H_∞ control in order to design a robust servo controller achieving good robust performance. And the effectiveness of the proposed structure was confirmed by applying to a coupled three inertia benchmark problem. For the purpose of designing a robust controller, the design objectives such as better transient response and robust tracking are first defined in terms of H_2 and H_∞ optimization theory, then the generalized plant for the mixed H_2/H_∞ control is determined and solved by using a LMI algorithm. Practical computation to get a controller is now quite easy thanks to some excellent software such as MATLAB.

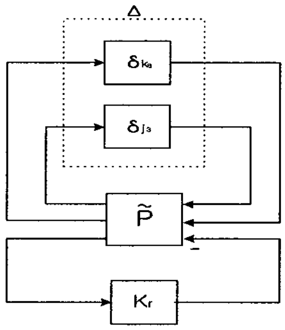


Fig. 11 Representation for the structured singular value

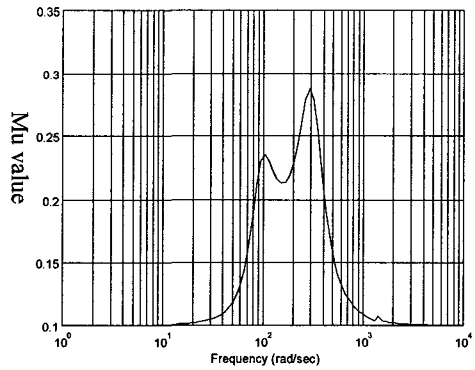


Fig. 12 Structured Singular value

It is thought that the difficulty in selecting weighting functions, that is essential to the general H_∞ control theory, can be avoided to some degree if we use the proposed structure for controller design. While the LMI based approach is computationally more involved for large problems, it has the merit of eliminating the regularity restrictions attached to the Riccati based solution. For example, the problems caused by adding an integrator in the loop for servo system design can be easily handled through the generalized plant proposed here.

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