Jour. of the KGS, Vol. 21, No. 7. September 2005, pp. 73~80

Prediction of Layer Rutting on Pavement Foundations Based on Stress Dependency

응력의존성을 고려한 도로기초의 층변형 예측

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요 지

도로기초에서 입상재료의 응력의존 특성을 반영하면 응력의존 탄성계수와 응력 의존 포와송 비 모두를 동시에 고려할 수 있다. 이 방식은 기존 연속체 역학에 기초한 해석 방식과는 달리 층모형 내에서 인장력의 발생 대신 압축력의 구현이 가능하여 재료의 강성과 연성에 대한 거동을 동시에 반영하여 입상재료층의 변형 예측에 많은 영향을 주고 있다. 따라서 본 논문에서는 도로기초를 대상으로 제안된 응력의존 및 변형 모형을 대상으로 유한요소법에 의한 도로기초 층변형 예측 알고리즘을 제안하였고 층변형 예측시 응력의존의 고려에 따른 영향을 분석하였다.

Abstract

There are several major practical consequences of stress-dependent properties of unbound pavement foundations. Among those are the stress-dependent modulus and Poisson's ratio's that may change, the compressive stresses that are generated in materials under load, the stiffening and strengthening effect of repeated loading to progressively increase the unbound pavement materials resistance to permanent deformation. In order to study these, the algorithm for predicting deformations on conventional flexible pavements are proposed and the stress-dependent effects on layer deformation are presented in this paper by the developed stress-dependent finite element analysis program with the selected models.

Keywords : Finite element analysis, Layer deformation, Pavement foundations, Stress-dependency, Unbound materials

1. Introduction

Unbound pavement materials are made up of a range of odd-shaped particles that tend to lie flat when they are compacted. The stiffness of this collection of particles and its volume change under load changes with stress state mainly because of the shape and gradation of the particles. This gives rise to stress-dependent stiffness and Poisson's ratios that may change depending on the applied stresses. These properties can be related to particle size, shape and orientation. It is based on this fact that the moduli of unbound pavement materials are stress-dependent, and the form of the non-linear stress-dependent is exponential (Uzan 1992; Lytton et al. 1993).

If the moduli are stress-dependent, then the Poisson's ratios must be stress-dependent as well. Earlier study by Allen (1973) shows measured Poisson's ratios rising above 1.0. The fact that the Poisson's ratio routinely rises above 0.5 is a major reason why unbound materials in pavement foundations work as well as they do. If unbound materials are prevented from expanding horizontally,

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it will build up a confining pressure that stiffens the surrounding materials under load. Therefore, these material properties of stress-dependent moduli and Poisson's ratio have a direct impact on the major long-term deformation damages of structural layers in pavement foundations under repeated traffic loading. In this paper, the algorithm for predicting deformations on conventional flexible pavements is presented and the stress-dependent effects are also illustrated.

2. Resilient and Deformation Models

It is well known that the behavior of unbound materials is non-linear and stress-dependent. The most commonly used non-linear elastic modulus model for characterizing stiffening behaviors of unbound materials is the K-θ and k-σ_d model has been used for characterizing softening behaviors of fine-grained soils (Hicks and Monismith 1971; Uzan 1992; Lytton et al. 1993). However, several studies have shown that the model does not accurately predict the response of unbound materials. Because the above models assume a constant Poisson's ratio of materials as an initial input data, and the effect of shear stresses on the resilient properties is not considered, nor are the dimensional problems of model itself. The shear stress component is especially responsible for dimensional change and consequently, permanent deformation.

Whereas, the Universal Soil Model (Uzan 1992) introduced the effect of octahedral she ar stress with the K-θ model and added atmospheric pressure as a normalizing factor to make the stress terms non-dimensional as shown in the Equation 1.

$$M_r = K_1\left(\frac{I_1}{P_a}\right)^{K_2}\left(\frac{\tau_{oct}}{P_a}\right)^{K_3} \qquad (1)$$

where:

M_r = resilient modulus for vertical direction,

P_a = atmospheric pressure,

I_1 = first stress invariant,

τ_{oct} = octahedral shear stress, and

K_i = material constants.

The octahedral shear stress term is believed to account for dilation effect that takes place when a pavement element is subjected to a large principal stress ratio. Depending on the level of stresses, the first stress invariant or bulk stress term considers the hardening effect associated with higher modulus, while the octahedral shear stress term considers the softening effect.

In addition, the Poisson's ratio of unbound pavement materials is also known to be the stress-dependent and should therefore be considered with the stress-dependent modulus simultaneously in a single framework. For this, a relationship between the Poisson's ratio and the resilient modulus is adopted based on a thermodynamic constraint suggested in previous studies (Lade and Nelson 1987; Liu 1993; Uzan 1992). This relationship was established between the resilient modulus as expressed in Equation 1 and the thermodynamic constraints to derive an expression that relates the stress state and the rate of change of the Poisson's ratio with a changing stress state as described in Equation 2.

$$\frac{2}{3}\frac{\partial\nu}{\partial J_2}+\frac{1}{I_1}\frac{\partial\nu}{\partial I_1}=\nu\left[\frac{1}{3}\frac{K_3'}{J_2}+\frac{K_2}{I_1^2}\right]+\left[-\frac{1}{6}\frac{K_3'}{J_2}+\frac{K_2}{I_1^2}\right] \qquad (2)$$

where:

ν = Poisson's ratio,

$K_3' = K_3/2$,

K_i = material parameters,

I_1 = normalized first stress invariant, and

J_2 = normalized second invariant of the deviatoric stress.

Typically, the permanent deformation caused by the number of loadings increases exponentially or asymptotes to a plateau value. Several prediction models have been proposed for characterizing the permanent deformation behavior of unbound materials under repeated loads. Among the various models, the VESYS is selected as a linear model (Kenis 1978; Kenis and Wang 1997). The VESYS model states that the ratio of vertical plastic strain per cycle, $d\varepsilon^p/dN$, to the resilient strain, ε_r, is an exponential function of the number of load cycles, N.

$$\frac{1}{\varepsilon_r}\frac{d\varepsilon^P}{dN} = \mu N^{-\alpha} \quad (3)$$

where:

ε_p = permanent deformation,

ε_r = elastic or/ resilient deformation,

N = the number of load applications,

μ = parameter representing the constant of proportionality of strains, and

α = parameter indicating the rate of decrease.

3. Modeling Layer Deformation

Permanent deformation in flexible pavements is the direct result of the passage of loads over the pavement surface and the strain induced by the load. This induced strain can be simplified as two components, the resilient strain and the permanent strain. It is suggested that the resilient strain remains fairly constant during the major part of the pavement's life, except for at a low number of load repetitions where the material undergoes conditioning and near failure (Uzan et al. 1988). It can also be assumed that the elastic strain is constant throughout the pavement life and the plastic strain per load application is assumed to decrease with the number of load applications as described in Figure 1.

For the deformation calculation, a 2-D nonlinear elastic finite element program is developed with the use of the layer strain approach. In this program, the nonlinear analysis is made using an incremental loading and an iterative solution technique for each load increment. Stress dependent modulus and Poisson's ratio are used. This algorithm handles both stress-dependency and equilibrium criteria in an incremental scheme. From the results of structural analysis, deformations at each layer is calculated by summing the products of the permanent strains and the corresponding difference in depths between the layers using the layer strain approach as presented in Figure 2.

The incremental strain for each load application is calculated by taking the derivative of the permanent strain with respect to load application and is shown in Equation 4. Two permanent deformation parameters that can define the characteristics of rut in relation to the materials are the alpha, (α), and the gnu, (μ). These parameters can then be used to substitute for I and S and the incremental strain is calculated by:

$$\Delta\varepsilon_a^{(N)} = \varepsilon_r \mu N^{-\alpha} \quad (4)$$

when N is equal to 1,

$$\mu = \frac{\varepsilon_a^{(1)}}{\varepsilon_r} S \quad (5)$$

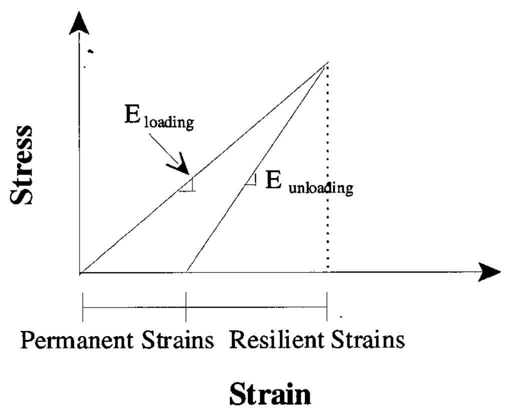

Fig. 1. Simplified Relationship of Incremental Permanent Strain under the Passage of a single Axle Load

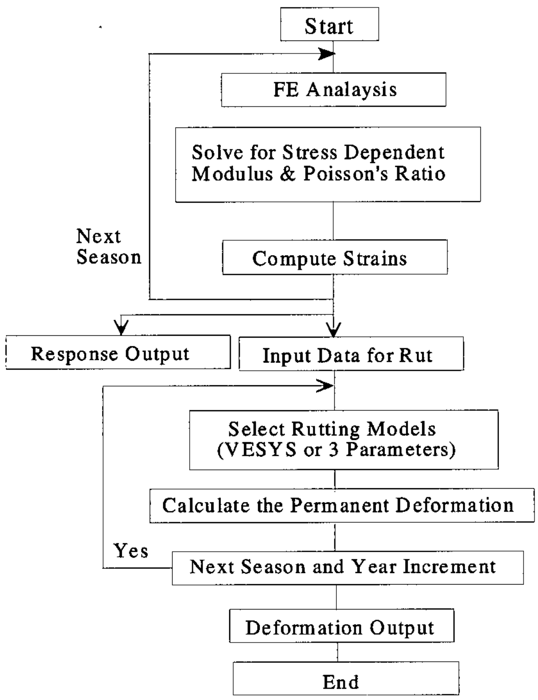

Fig. 2. Flow Chart for Predicting Deformations Based on Finite Element Analysis Program

From Figure 1,

$$E_{lo(N)} = \frac{\sigma}{\varepsilon_r + \Delta\varepsilon_a} = \text{modulus during loading} \quad (6)$$

$$E_{un} = \frac{\sigma}{\epsilon_r} = \text{modulus during unloading} \quad (7)$$

By rewriting Equations 4 through 7, modulus during loading can be written as:

$$E_{lo(N)} = \frac{\sigma}{\Delta\varepsilon_a^{(N)} + \varepsilon_r} = \frac{\sigma}{\varepsilon_r(\mu N^{-\alpha}+1)} = \frac{E_{un}}{1+\mu N^{-\alpha}} \quad (8)$$

The Equation 8 gives a relation between loading modulus and unloading modulus as a function of permanent deformation characteristics of pavement materials, α and μ, and the number of load applications.

To estimate the incremental permanent strain from a single axle load application, it is assumed that the stress-strain relationship is of the form shown in Figure 1. Using geometry, it can be shown that:

$$\frac{\Delta\varepsilon_a}{\Delta N} = \sigma\left[\frac{1}{E_l} - \frac{1}{E_u}\right] \quad (9)$$

The fractional increase of the total strain, F(N), which is the permanent strain with load repetition, is given by:

$$F(N) = \frac{\Delta\varepsilon_a}{\varepsilon_r + \Delta\varepsilon_a} \quad (10)$$

It is assumed that the resilient strain is large in comparison to the increase of the permanent strain with each load repetition. With this assumption, the following approximation can be used:

$$F(N) \cong \frac{\Delta\varepsilon_a}{\varepsilon_r} \quad (11)$$

Since

$$\frac{\Delta\varepsilon_a}{\Delta N} \approx \frac{\partial\varepsilon_a}{\partial N} \quad (12)$$

the fractional increase of the total strain, F(N), can be expressed as:

$$F(N) \cong \frac{\Delta\varepsilon_a}{\varepsilon_r \partial N} = \frac{E_u}{E_I} - 1 = \mu N^{-\alpha} \quad (13)$$

The rut depth may be then estimated by:

$$\delta_a(N) = \int_0^N \int_0^{Z_{max}} \varepsilon_c(z) F(N) dz\, dN \quad (14)$$

where

Z_{max} = depth of the pavement layer, and

$\varepsilon_c(Z)$ = compressive strain at depth z.

Equation 14 may be extended to include all layers as follows:

$$\delta_a(N) = \int_0^N \int_0^{d_1} \varepsilon_c(z) F_1(N) dz\, dN + \int_0^N \int_{d_1}^{d_2} \varepsilon_c(z) F_2(N) dz\, dN + \cdots + \int_0^N \int_{d_{n-1}}^{d_n} \varepsilon_c(z) F_1(N) dz\, dN \quad (15)$$

where d_1, d_2, ..., d_n are the depths of each layer in the pavement, and F_1, F_2, ..., Fn are the fractional increases of the total strain for each layer. So, the rut depth becomes:

$$\delta_a(N) = \sum_{i=1}^{n}\left[\int_0^N \mu_i N^{-\alpha_i} dN \int_{d_{i-1}}^{d_i} \varepsilon_c(z) dz\right] \quad (16)$$

The first integral on the right-hand side of Equation 16 becomes:

$$\int_0^N \mu_i N^{-\alpha_i} dN = \frac{\mu_i N^{1-\alpha_i}}{1-\alpha_i} \quad (17)$$

Therefore, the total deformation becomes:

$$\delta_a(N) = \sum_{i=1}^{n}\left[\frac{\mu_i N^{1-\alpha_i}}{1-\alpha_i} \int_{d_{i-1}}^{d} \varepsilon_c(z) dz\right] \quad (18)$$

Equation 18 can be integrated numerically in each layer beneath the center of loads. Then, the predicted deformation at the surface can be the sum of the vertical permanent strains of all the layers.

A finite element analysis program has been adopted to take the K_1 through K_3 values for vertical direction as input and convert them into stiffness that vary with stress state. Both the moduli and the Poisson's ratios vary within a pavement structure from element to element beneath a load. The stresses are different from what are predicted with linear finite element, linear layered elastic, and non-linear layered elastic methods. The radial strains calculated in the granular materials using the stress-sensitive approaches are commonly smaller than those calculated by other methods. These results are a consequence of the larger Poisson's ratios and higher continuing pressures that are generated in the stress-dependent finite element analysis program (Park and Lytton 2004).

4. Layer Deformation with Respect to Stress-dependent Response

Deformation of a flexible pavement occurs mainly due to the AC layer and/or unbound foundation layers. Currently, the limiting strain method is widely used for the evaluation of rutting performance. This approach assumes that layer deformation is only a function of vertical compressive strain at the top of the subgrade, and if these values are less than a critical value, then rutting will not occur at a given traffic level. Therefore, only rutting in the subgrade is considered.

However, unbound base layers are the main structural components in most flexible pavements, and the total rutting performance is closely tied to base layer rutting. Heavy loads on a thin or structurally weaken pavements, such as low-volume roads, may induce excessive rutting in unbound foundation layers. It is also a well-known fact that unbound foundation materials are very dependent on the stress state within their layers. When a pavement is modeled using elastic models based on a constant moduli and Poisson's ratio within layers, this may indicate tension in the bottom of unbound base layer and thus underestimate the rutting at a given layer.

Therefore, to understand the permanent deformations within unbound foundation layers, realistic models that can simulate expected field conditions on those layers are developed as described earlier. A developed finite element model, which accounts for the stress-dependent modulus and Poisson's ratio models, can significantly reduce tension in the bottom half of the base layer (Park and Lytton 2004). It is evident that this approach may improve the layer deformation prediction and can identify contributions of each layer to the deformation more closely.

An analysis was conducted to demonstrate the fact that the stress-dependent behavior has a significant influence on the predicted layer deformation in flexible pavements. Layer deformation was estimated using the VESYS model. Because of the unavailability of some of the material properties, typical values for the VESYS rutting parameters, α and μ, are assumed for each layer in the analyses and were kept the same. A single load of 40.0 kN was applied over a circular area with a radius of 135 mm, which corresponds to a surface pressure of 689 kPa. The average values used for an example analysis are noted in Table 1.

The stress dependency of the predicted rutting was compared to the rutting predicted without stress dependency. Figure 3 illustrates the overall rutting prediction obtained. As can be seen in the figure, the increase in overall rutting with time is predicted with the stress dependency at a given condition. It would be adequate to assess not only overall rutting, but also rutting in each layer. This may indicate where

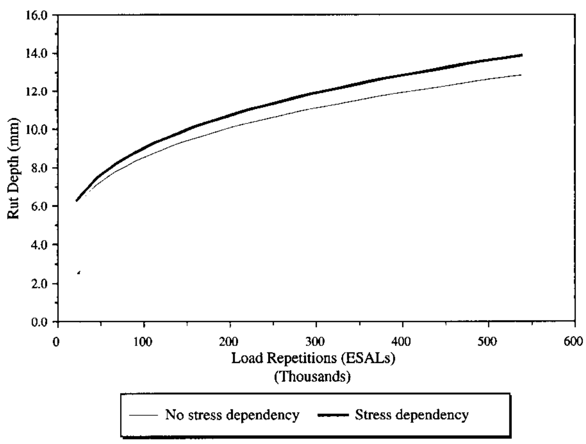

Fig. 3. Predicted Overall Rutting with Respect to Stress Dependency

Table 1. Resilient and Deformation Parameters for an Example Analysis

Layer	Thickness (mm)	K_1	K_2	K_3	α	μ
AC Surface	100	50,000	0.1	0.0	0.60	0.75
Granular Base	200	700	0.6	−0.3	0.84	0.53
Subgrade	2300	400	0.0	−0.3	0.81	0.08

Note: A single load of 40 kN was applied over a circular area with a radius of 135 mm.

the major rutting occurs and identify how they progress.

Figure 4 shows progressive rut development in pavement layers with depth and the variation of predicted layer rutting within the layers. This figure shows that the rutting increases with the stress dependency, and clearly indicates that the predicted rutting is relatively critical at the base layer. It should also be noted that the layer rutting decreases in the subgrade layer in the case of the stress dependency. This is shown in Figures 5 and 6, which show the layer rutting contribution at each layer. As can be seen in Figure 6, the trends of the percent layer rutting within the base layer follows a pattern similar to that for the case of no stress dependency, as shown in Figure 5. However, an increase in layer rutting in AC becomes higher than those of other layers in the case of stress dependency. This trend indicates that the rutting on the subgrade decreases with traffic, and the rutting in AC

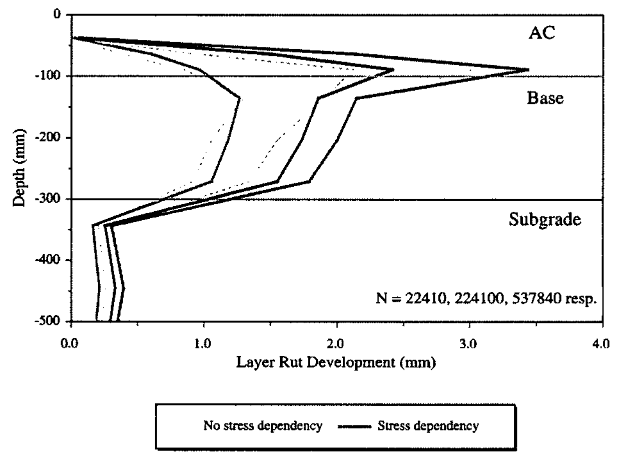

Fig. 4. Predicted Progressive Rut Development with Respect to Stress Dependency

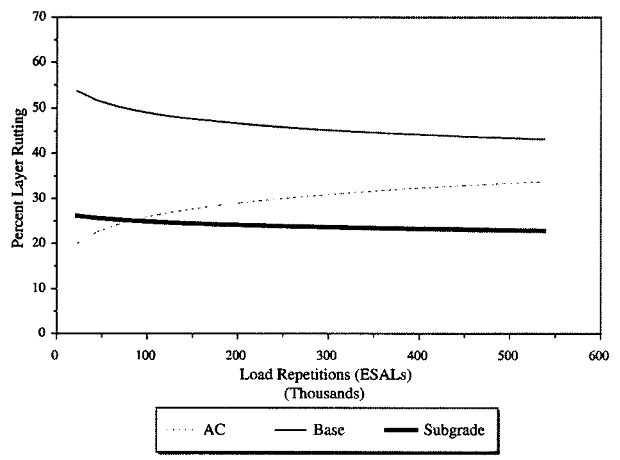

Fig. 5. Predicted Percent Layer Rutting without Stress Dependency

layer increases due to stress dependent moduli and Poisson's ratios within layers.

Figures 7 and 8 show the variation of predicted stress dependent moduli in the AC and foundation layers. As

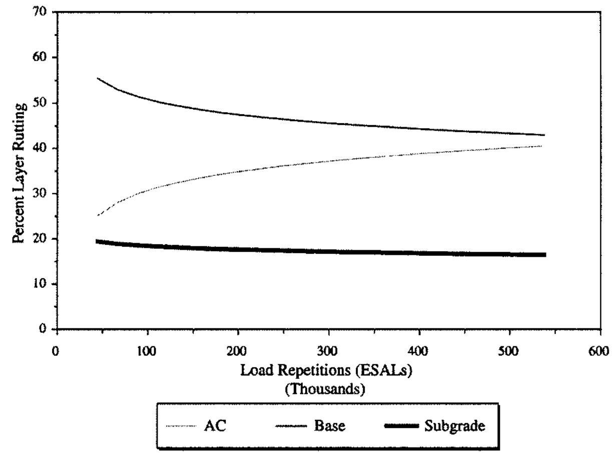

Fig. 6. Predicted Percent Layer Rutting with Stress Dependency

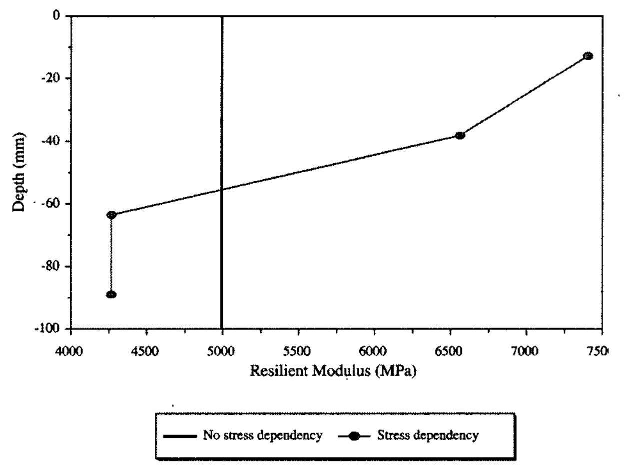

Fig. 7. Variation in Resilient Modulus with Respect to Stress Dependency (AC)

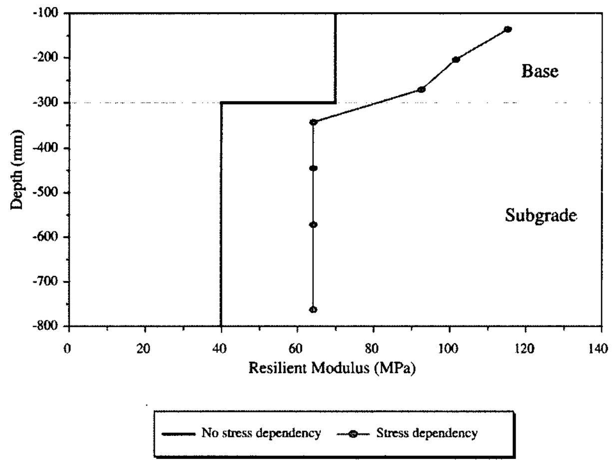

Fig. 8. Variation in Resilient Modulus with Respect to Stress Dependency (Foundation)

these results illustrate, the predicted resilient moduli vary considerably with depth. The figures indicate that the changes are very clear due to stress state with depth. This is expected since the first stress invariant is higher than the shear stress component which reduces the resilient modulus, while the resilient moduli remain unchanged in the case of no stress dependency.

Since the moduli values are varied rather than a constant value, this may significantly affect the predicted stresses and strains with layers, as shown in Figures 9 and 10. The vertical stresses increase with the stress dependency, while the strains increase at the AC and base layer and decrease in the subgrade layer with the stress dependency because of higher moduli due to cutoff values at very low stresses. At the lower part of the base layer, significant increases in strains are predicted as shown in Figure 10.

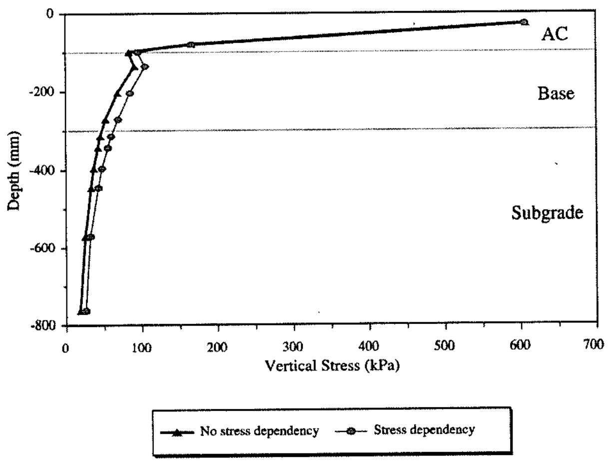

Fig. 9. Predicted Vertical Stresses with Respect to Stress Dependency

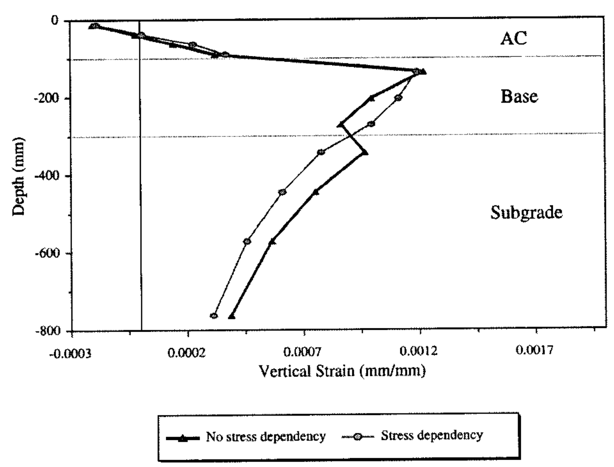

Fig. 10. Predicted Vertical Strains with Respect to Stress Dependency

It is logical that the higher moduli in a layer will lead to reducing vertical strains within layers. The trends of the layer rutting within the base do not follow the same pattern and results in more rutting. However, this tendency is counteracted by the Poisson's ratio which also varies within layers. This is illustrated in Figure 11. It is observed that the Poisson's ratios vary greatly, especially with the base layer. However, it was decided to use the values less than 0.48 as a maximum instead of using values higher than 0.5. Poisson's ratio greater than 0.5 is not compatible with a finite element model currently used for the calculations in this study. This means that an increase in Poisson's ratio within the 0.5 range leads to an increase in vertical strains. In addition, the predicted Poisson's ratio is not only dependent on the stress state, but also on the resilient material properties, k1, k2, k3. Therefore, it is conceivable that the opposite effect may be observed in some cases in this analysis depending on the relationships between modulus and induced stress levels for a given layer.

However, it is also conceivable that the stresses considered change significantly due to the variation of resilient modulus and Poisson's ratio within layers. Therefore, higher vertical compressive stresses are expected which may cause more rutting. Consequently, while the stress dependency of each layer is to affect the predicted resilient modulus and Poisson's ratio, the stresses and strains depend on the interactions between the stress dependent moduli, Poisson's ratios, the load levels, and the pavement geometry.

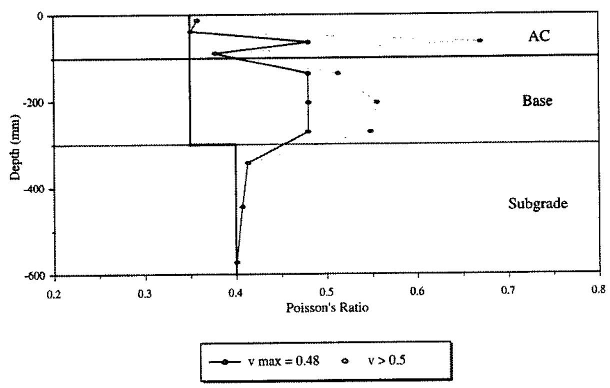

Fig. 11. Variation in Poisson's Ratio with Respect to Stress Dependency

5. Conclusions

The stress-dependent modulus and Poisson's ratios of the unbound pavement materials, when represented properly in a finite element program, are able to duplicate the deformational response of these materials under repeated loading. Both the stress dependent modulus and the Poisson's ratio vary considerably within the layers and this may significantly change the predicted rutting of unbound pavement foundations. The variations of stress dependent moduli and Poisson's ratio may affect significantly rutting and load spreading capability in the base at the outside of traffic loadings as well. This observation suggests that a methodology for analyzing the lateral deformation on unbound foundation layers is needed.

However, the question still arises whether the stress -dependent approach for analyzing the deformational response of conventional flexible pavement is more realistic. As an answer, the stress-dependent approach is more realistic because it represents actual observed material properties. Therefore, many of the important variables and material properties are still needed to make an accurate evaluation, and a more detailed study have to be made.

Acknowledgements

The present research was conducted by the research fund of Dankook University in 2004. Their support was gratefully recognized.

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(received on Jul. 28, 2005, accepted on Sep. 26, 2005)