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# DYNAMICAL EVOLUTION OF ROTATING SINGLE-MASS STELLAR CLUSTER

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## ABSTRACT

We study the influence of rotation on the dynamical evolution of collisional single-mass stellar clusters up to core-collapse by using N-body simulations. Rotating King models which are characterized by dimensionless central potential parameter $W_0$ and the rotation parameter $\omega_0$ are used as initial models. Our results show that inner shells slowly contract until core-collapse phase is reached, followed by a slow expansion. Angular momentum is transported outward, while the core is rotating even faster than before, as predicted by gravogyro catastrophe theory. We confirm that rotation plays an important role in accelerating the dynamical evolution of stellar cluster, in particular in accelerating the core collapse.

*Key words :* stellar dynamics — methods: n-body simulations

## I. INTRODUCTION

The idea that rotation is an important property for stellar cluster is supported by some observational phenomena.

First, the flattening of globular clusters. White & Shawl (1987) found that mean of projected ellipticities of 99 clusters in Milky way is $e = 0.07 \pm 0.01$, while Staneva et al. (1996) reported ellipticities of $e = 0.086 \pm 0.038$ for 173 clusters in M31. Meylan & Mayor (1986) proposed that flattening of globular clusters may be explained in terms of rotation.

Second, the observational evidence that younger globular clusters (i.e. in Magellanic Clouds) are flatter than older clusters (i.e. in Milky Way) as reported by Frenk & Fall (1982). Davoust & Prugniel (1990) suggested that this phenomenon is caused by loss of angular momentum due to rotation of globular clusters.

Third, the increase of angular velocities toward the center of core of 47 Tuc. This is interpreted as a gradually faster rotating core due to contraction of an initially slowly rotating body (Gerhardt et al. 1995). Theoretically, this phenomenon appears also in the rotating cylinders, and is called as gravogyro catastrophe (Hachisu 1979). Compared to non-rotating models, rotating models are less studied.

Previous numerical studies are mainly based on the use of Fokker-Planck equation as an approximate model (Einsel & Spurzem 1999; Kim et al. 2002; Kim et al. (2004). Unfortunately, Fokker-Planck models rely on a number of physical and numerical approximations, and 2D rotating models neglect any of third integral. On the other hand, only one study has been conducted an $N$-body simulations (Akiyama & Sugimoto 1989). Therefore an intensive study on rotating stellar clusters using $N$-body simulations is urgently required. This study is aimed to examine dynamical processes of rotating stellar clusters during their evolution by N-body simulations, and by comparing our simulation results with previous results found by Fokker-Planck model.

## II. NUMERICAL MODELS AND INITIAL CONDITIONS

We present new, high accuracy collisional N-body simulations, using the fourth order Hermite integrator NBODY6++ which incorporates individual hierarchical time steps, Ahmad-Cohen neighbor scheme, regularization of close encounters, and hierarchical subsystems. We conduct simulations for single mass systems with N=1000, 5000, and 10000 stars. In order to get representative results, the statistical significance of the lower-$N$ simulations are improved by ensemble averages (see Gierz & Heggie 1994).

Tabel 1. shows the computational effort for these models, including the number of particles, the number of run for each case, the CPU time for each run, and the type of machines where simulations were conducted.

As initial models, we employ the rotating King models generalized for rotation, which are characterized by two parameters: dimensionless central potential $W_0$ and the rotation parameter $\omega_0$.

In order to compare with previous results found by Fokker-Planck model, we examine the evolution of cluster models with central potential $W_0 = 6$, each of

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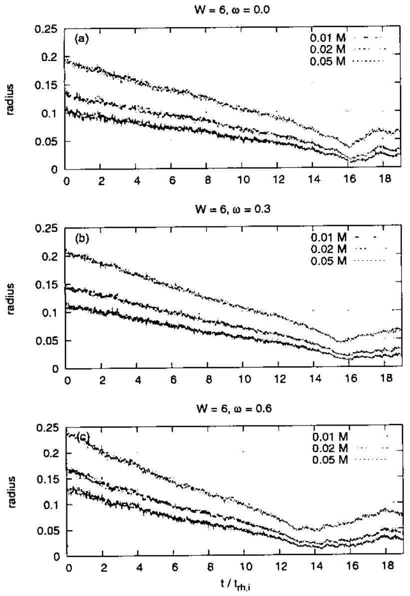

**Fig. 1.**— Lagrangian radii of inner part for N=5000 clusters with $W_0$=6, $\omega_0$= 0.0, 0.3 and 0.6.

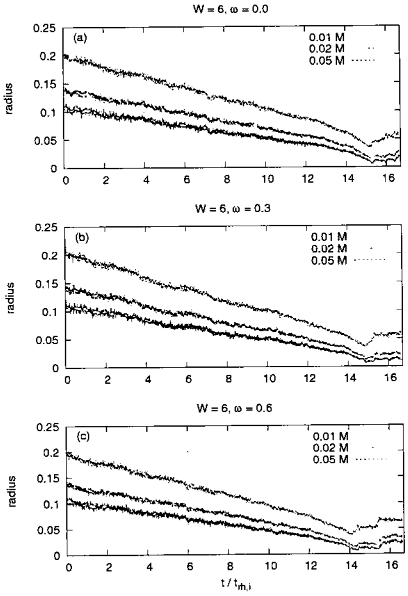

**Fig. 2.**— Lagrangian radii of inner part for N=10000 clusters with $W_0$=6, $\omega_0$= 0.0, 0.3 and 0.6.

which having rotation parameter $\omega_0$ = 0.0, 0.3, and 0.6. As computational units, we take gravitational constant $G$=1 and cluster mass $M$=1. The unit of time is expressed in terms of the initial half-mass relaxation time, which is taken from Eq. 2-63 in Spitzer (1987):

$$t_{\rm rh,i} = 0.138\sqrt{\frac{N r_{\rm hi}{}^3}{Gm}}\frac{1}{\ln(\gamma N)}, \qquad (1)$$

Here $N$ is number of particles, $m$ is the mean mass of particles, $r_{\rm hi}$ is the initial half-mass radius, and $\ln(\gamma N)$ is the Coulomb logarithm. The value of the coefficient in the Coulomb logarithm is chosen to be $\gamma$ = 0.11 (Gierz & Heggie 1994).

## III. RESULTS

Evolutions of inner part of non-rotating, intermediate rotating and rapidly rotating models for N=5000, and 10000 are shown in Figs 1 and 2. Contractions which produce core structures are observed. Shells which contain 5%, 2% and 1% of total mass of the clusters slowly contract until core-collapse phase is reached, followed by a slow expansion. Comparing these three models, as long as we fix potential, we find that core-collapse happens earlier in rapidly rotating model. The stronger the initial rotating parameter is, the shorter the time required for core-collapse.

Table 1.
Outline of N-body runs

| Particle number | Number of run/case | CPU time/run (hrs) | Machines |
|---|---|---|---|
| 1000 | 20 | 1 | Pentium III and Alpha |
| 5000 | 10 | 82 | Pentium III and Alpha |
| 10000 | 5 | 77 | Hydra Parallel Comp |

As the clusters evolve into core collapse phase, the core density is getting higher, while the velocity dispersion inside the core also increases. Figure 3 shows this evolution for clusters with $W_0$=6 and $\omega_0$= 0.0, 0.3 and 0.6.

In the early phase of their evolutions, which are shown on the left side of the diagram, the the growth of velocity dispersion within core shows slight differences between the three models, while these differences becomes smaller in the advance phase of evolution.

This diagrams inform about the collapse characteristic of the clusters, which is described by the quantity

$$\gamma = \frac{d\ln{\sigma_c}^2}{d\ln n_c}, \qquad (2)$$

where $\sigma_c$ is the velocity dispersion inside the core and $n_c$ is the core density. We find that $\gamma$ = 0.021, 0.028 and 0.033 for models with $W_0$=6 and $\omega_0$= 0.0, 0.3 and 0.6, respectively. This indicates that, although clusters with high amount of initial rotation collapse faster, the collapse characteristics themselves are similar. Initial rotation contributes only minor effect on the collapse characteristic. Clusters with different initial rotations evolve similarly toward the collapse phase.

Figure 4 shows that the rotation velocity of the core increases so much as contraction of cluster proceeds.

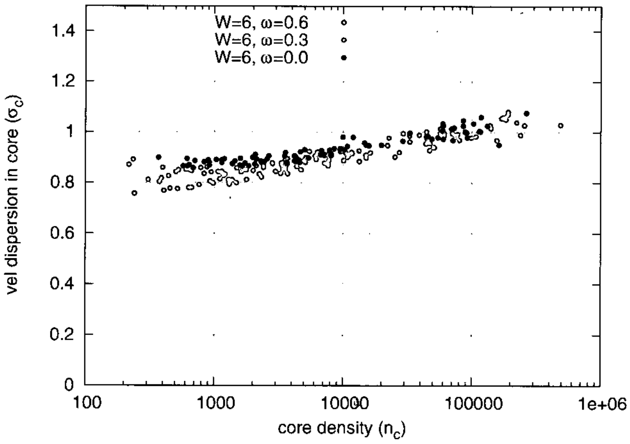

**Fig. 3.**— Evolution of velocity dispersion inside the core versus core density for clusters with $W_0$=6, $\omega_0$= 0.0, 0.3 and 0.6.

The core rotates even much faster than in its early evolution. Shells which contain 5% and 20% of total mass of the clusters even rotate two and a half times faster than their initial rotation velocities.

The phenomena that the core rotates faster than its initial rotation could be linked with the gravogyro catastrophe found by Hachisu (1979). In his theoretical study of rotating cylinders, he states that, if angular momentum is removed from a shell, gravitational contraction results an increase of angular velocity which leads to angular momentum transport and central contraction. This effect is called as gravogyro catastrophe caused by negative specific moment of inertia.

The gravogyro catastrophe does not proceed indefinitely, because the angular momentum contained in the core is limited. It prevents the system from contracting indefinitely and leads to a gradual transport of angular momentum outward. Figure 4 shows that rotation velocity of inner shells decrease slowly after the core collapse phase.

We present the evolution of initial angular momentum and transport of angular momentum in figures 5 and 6. Each line in figure 5 represents a certain amount of angular momentum of the initial model. Stars with high angular momentum move outward, so the angular momentum contained within a certain radius decreases with time as seen in figure 5. Angular momentum within outer shells decreases faster than in the inner shells. Within the inner shells, although angular momentum decreases slowly, after core-collapse the core loses almost all of its angular momentum.

## IV. DISCUSSION

Our simulation results in figures 1–2 show that rotation indeed accelerates the core collapse. It can be understood since, the intermediate and rapid rotating clusters use part of their total energy to support the rotation. For example, models with $W_0 = 6$ and $\omega = 0.6$ spend about 36 % of their total energy as rotational energy, while rotational energy takes about 10 % of total energy in cluster models with $W_0 = 6$ and $\omega_0 = 0.3$. Consequently, clusters with higher rotations produce lower velocity dispersions in their core, as shown in figure 3. This condition promotes the decrease of core-collapse time.

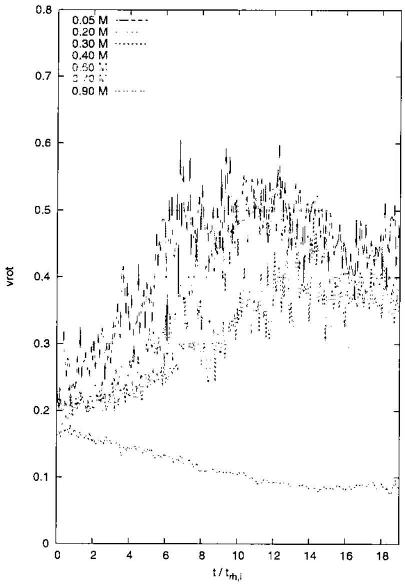

**Fig. 4.**— Evolution of rotation velocity inside the core of a rotating $W_0$=6 $\omega_0$= 0.6 cluster.

Our result shows that the rotation accelerates the core collapse, and this is in agreement with the one obtained by Fokker-Planck models (Einsel & Spurzem, 1999). As a quantitative comparison, we can compare one model with $W_0 = 6$ and $\omega_0 = 0.6$. By N-body simulations with N=1000 ,5000 and 10000, these reach the core collapse 7.5 % – 25 % faster than the non-rotating model, while by Foker-Planck approximation, the corresponding model reached the core-collapse 36 % earlier than the non-rotating model (Fig.2 in Einsel & Spurzem, 1999). That Fokker-Planck model drives the rotating clusters to collapse slightly earlier than the non-rotating clusters, this can be understood because these models suffer very high mass-loss (Fig.6 in Einsel & Spurzem 1999) which is too high compared to our N-body simulations. Our N-body simulations also show the evolution of the core after collapse or the reverse collapse phase. The core expands while its density decreases. This phenomena does not appear in the Fokker-Planck model. Instead of decreasing, the core density increase infinitely. One reason is that the Fokker-Planck model assumes that particles are only coupled via the smooth global gravitational potential which avoids the formation of hard binaries. In fact, binaries might support the reverse core collapse by pro-

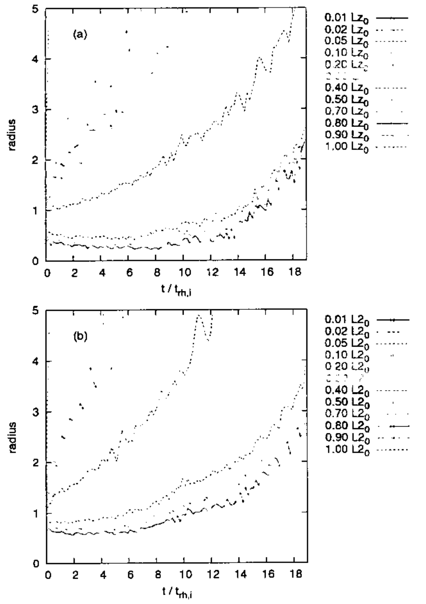

**Fig. 5.**— Evolution of (a) $z$-component of initial angular momentum (Lz) (b)initial total angular momentum in a rotating $W_0$=6 $\omega_0$= 0.6 cluster.

viding the single stars with enough kinetic energy to balance the energy lost by heat convection from core to halo (Hut 1993).

Transport of angular momentum, as presented in figures 5 and 6, plays an important role on the rotating stellar cluster. Previously, the outward movement of angular momentum brings the gradient of density which motivates contraction of inner shells. While contraction proceeds, the core rotates even faster than before, as shown in figure 4. This phenomena which could be linked with the gravogyro catastrophe do not proceed indefinitely. Core rotation produces a gradient of angular velocity $\omega$. In order to reduce the gradient of angular velocity, the remaining angular momentum of the core is transported further outward. We support the suggestion given by Akiyama & Sugimoto (1989) about the role of transport of angular momentum, that previously transport of angular momentum motivates contraction of the core, but in the later phase transport of angular momentum is the result of contraction. Angular momentum is squished out from the core until it loses all of its angular momentum after the core-collapse phase. The loss of angular momentum may change the shape of clusters, so that old-clusters become rounder than younger ones.

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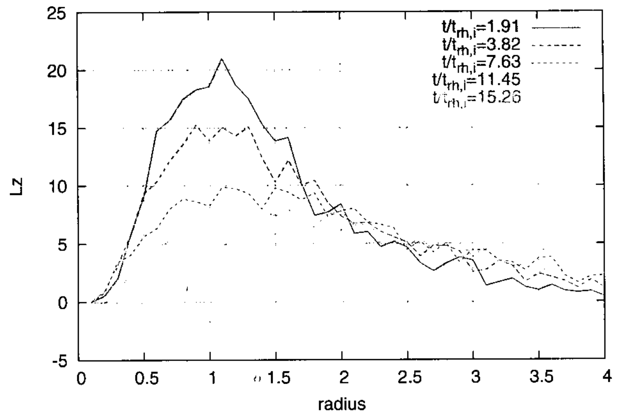

**Fig. 6.**— Transport of angular momentum in a rotating $W_0$=6 $\omega_0$= 0.6 cluster.

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