# On Fuzzy Irresolute Functions

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#### **Abstract**

As a generalization of the notions of fuzzy  $\alpha$ -irresolute, fuzzy preirresolute, fuzzy irresolute and fuzzy  $\beta$ -irresolute functions, we introduce the notion of fuzzy  $\beta\alpha$ -continuous functions and investigate the relationships between fuzzy  $\beta\alpha$ -continuous functions and fuzzy separation axioms.

**Key words**: fuzzy  $\beta$ -open set, fuzzy  $\alpha$ -open set, fuzzy  $\beta\alpha$ -continuity

#### 1. Introduction and Preliminaries

The notions of fuzzy irresolute, fuzzy  $\alpha$ -irresolute and fuzzy preirresolute functions were introduced and studied by Mukherjee and Sinha [6], Thakur and Saraf [11] and Park and Park [8], respectively.

The aim of this paper is to introduce a new class of fuzzy functions which is called  $\beta\alpha$ -continuous functions including the classes of fuzzy irresolute, fuzzy  $\alpha$ -irresolute, fuzzy pre-irresolute and fuzzy  $\beta$ -irresolute functions. Furthermore, we obtain basic properties of  $\beta\alpha$ -continuous functions and investigate relationships between fuzzy  $\beta\alpha$ -continuity and fuzzy covering properties and fuzzy  $\beta\alpha$ -continuity and fuzzy separations axioms, respectively.

The class of fuzzy sets on a universe X will be denoted by  $I^X$  and fuzzy sets on X will be denoted by Greek letters as  $\mu$ ,  $\rho$ ,  $\eta$ , etc. A family  $\tau$  of fuzzy sets in X is called a fuzzy topology [3] for X iff  $(1) \oslash X \in \tau$  (2)  $\mu \land \rho \in \tau$  whenever  $\mu$ ,  $\rho \in \tau$  and (3)  $\bigvee \{\mu_a \colon \alpha \in I\} \in \tau$  whenever each  $\mu_a \in \tau(\alpha \in I)$ . In this case, the pair  $(X, \tau)$  (or simply X) is called a fuzzy topological space (for short, fuzzy space). Every member of  $\tau$  is called a fuzzy open set [7]. For a fuzzy set  $\mu$  in X, int  $\mu$  and  $\operatorname{cl} \mu$  will denote the interior and closure of  $\mu$ , respectively. A fuzzy set in X is called a fuzzy point iff it takes the value 0 for all  $y \in X$  except one, say,  $x \in X$ . If its value at x is  $\alpha(0 \land \alpha \leq 1)$ , we denote this fuzzy point by  $x_\alpha$ , where the point x is called its support [7]. For any fuzzy point  $x_\varepsilon$  and any fuzzy set  $\mu$ , we write  $x_\varepsilon \in \mu$  iff  $\varepsilon \leq \mu(x)$ .

**Definition 1.1.** A fuzzy set  $\mu$  in X is called:

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- (1) fuzzy  $\alpha$ -open [10] if  $\mu \leq \text{int cl int } (\mu)$ ;
- (2) fuzzy semiopen [1] if  $\mu \le cl$  int  $(\mu)$ ;
- (3) fuzzy preopen  $[10] \mu \leq \operatorname{int} \operatorname{cl}(\mu)$ ;
- (4) fuzzy  $\beta$ -open [5, 12] if  $\mu \le cl$  int  $cl(\mu)$ .

The complement of a fuzzy  $\alpha$ -open (resp. fuzzy semiopen, fuzzy preopen, fuzzy  $\beta$ -open) set is called fuzzy  $\alpha$ -closed (resp. fuzzy semiclosed, fuzzy preclosed, fuzzy  $\beta$ -closed).

#### **Definition 1.2.** A fuzzy function $f: X \rightarrow Y$ is said to be:

- (1) fuzzy open [3] (resp. always fuzzy  $\beta$ -open) if  $f(\rho)$  is fuzzy open (resp. fuzzy  $\beta$ -open) in Y for every fuzzy open (resp. fuzzy  $\beta$ -open) set  $\rho$  in X;
- (2) fuzzy irresolute [6] if  $f^{-1}(\rho)$  is fuzzy semiopen in X for each fuzzy semiopen set  $\rho$  in Y;
- (3) fuzzy  $\alpha$ -irresolute [11] if  $f^{-1}(\rho)$  is fuzzy  $\alpha$ -open in X for each fuzzy  $\alpha$ -open set  $\rho$  in Y;
- (4) fuzzy preirresolute [8] if  $f^{-1}(\rho)$  is fuzzy preopen in X for each fuzzy preopen set  $\rho$  in Y;
- (5) fuzzy  $\beta$ -irresolute if  $f^{-1}(\rho)$  is fuzzy  $\beta$ -open in X for each fuzzy  $\beta$ -open set  $\rho$  in Y.

# 2. Fuzzy $\beta \alpha$ -continuous functions

**Definition 2.1.** A fuzzy function  $f: X \to Y$  is said to be fuzzy  $\beta \alpha$ -continuous if for each fuzzy point  $x_{\varepsilon} \in X$  and each fuzzy  $\alpha$ -open set  $\rho$  in Y containing  $f(x_{\varepsilon})$ , there exists a fuzzy  $\beta$ -open set  $\mu$  in X containing  $x_{\varepsilon}$  such that  $f(\mu) \leq \rho$ .

**Theorem 2.2.** For a fuzzy function  $f: X \rightarrow Y$ , the following statements are equivalent:

- (1) f is fuzzy  $\beta \alpha$ -continuous;
- (2) for every fuzzy  $\alpha$ -open set  $\rho$  in Y,  $f^{-1}(\rho)$  is fuzzy

 $\beta$ -open;

(3) for every fuzzy  $\alpha$ -closed set  $\rho$  in Y,  $f^{-1}(\rho)$  is fuzzy  $\beta$ -closed.

**Proof.** (1)  $\Rightarrow$ (2): Let  $\rho$  be a fuzzy  $\alpha$ -open set in Y and let  $x_{\varepsilon} \in f^{-1}(\rho)$ . Since  $f(x_{\varepsilon}) \in \rho$ , by (1), there exists a fuzzy  $\beta$ -open set  $\mu_{x_{\varepsilon}}$  in X containing  $x_{\varepsilon}$  such that  $\mu_{x_{\varepsilon}} \leq f^{-1}(\rho)$ . We obtain that  $f^{-1}(\rho) = \bigvee_{x_{\varepsilon} \in f^{-1}(\rho)} \mu_{x_{\varepsilon}}$ . Thus,  $f^{-1}(\rho)$  is fuzzy  $\beta$ -open.

- (2)  $\Rightarrow$  (1): Let  $\rho$  be a fuzzy  $\alpha$ -open set in Y and let  $f(x_{\varepsilon}) \in \rho$ . We have  $x_{\varepsilon} \in f^{-1}(\rho)$ . By (2),  $f^{-1}(\rho)$  is a fuzzy  $\beta$ -open set. Take  $\eta = f^{-1}(\rho)$ . Then  $f(\eta) \leq \rho$ . Thus, f is fuzzy  $\beta \alpha$ -continuous.
- (2)  $\Rightarrow$  (3): Let  $\rho$  be a fuzzy  $\alpha$ -closed set in Y. Then  $Y \setminus \rho$  is fuzzy  $\alpha$ -open. By (2),  $f^{-1}(Y \setminus \rho) = X \setminus f^{-1}(\rho)$  is fuzzy  $\beta$ -closed.

 $(3) \Rightarrow (2)$ : Similar to  $(2) \Rightarrow (3)$ .

**Definition 2.3.** A fuzzy filter base  $\Lambda$  is said to be fuzzy  $\beta$ -convergent (resp. fuzzy  $\alpha$ -convergent) to a fuzzy point  $x_{\varepsilon} \in X$  if for any fuzzy  $\beta$ -open (resp. fuzzy  $\alpha$ -open) set  $\rho$  in X containing  $x_{\varepsilon}$ , there exists a fuzzy set  $\mu \in \Lambda$  such that  $\mu \leq \rho$ .

**Theorem 2.4.** If a fuzzy function  $f: X \to Y$  is fuzzy  $\beta \alpha$ -continuous, then for each fuzzy point  $x_{\varepsilon} \in X$  and each fuzzy filter base  $\Lambda$  in X which is  $\beta$ -convergent to  $x_{\varepsilon}$ , the fuzzy filter base  $f(\Lambda)$  is fuzzy  $\alpha$ -convergent to  $f(x_{\varepsilon})$ .

**Proof.** Let  $x_{\varepsilon} \in X$  and  $\Lambda$  be any fuzzy filter base in X which is  $\beta$ -convergent to  $x_{\varepsilon}$ . Since f is fuzzy  $\beta \alpha$ -continuous, then for any fuzzy  $\alpha$ -open set  $\lambda$  in Y containing  $f(x_{\varepsilon})$ , there exists a fuzzy  $\beta$ -open set  $\mu$  in X containing  $x_{\varepsilon}$  such that  $f(\mu) \leq \lambda$ . Since  $\Lambda$  is fuzzy  $\beta$ -convergent to  $x_{\varepsilon}$ , there exists a  $\rho \in \Lambda$  such that  $\rho \leq \mu$ . This means that  $f(\rho) \leq \lambda$  and therefore the fuzzy filter base  $f(\Lambda)$  is fuzzy  $\alpha$ -convergent to  $f(x_{\varepsilon})$ .

**Remark 2.5.** For  $f: X \rightarrow Y$ , the following diagram holds:

The following examples show that these implications are not reversible.

**Example 2.6.** Let  $X = \{a, b\}, Y = \{x, y\}$  and  $\lambda$ ,  $\mu$  are fuzzy sets defined as follows:

$$\lambda(a) = 0.3, \ \lambda(b) = 0.6,$$

$$\mu(x) = 0.7$$
,  $\mu(y) = 0.5$ .

Let  $\tau_1 = \{X, \emptyset, \lambda\}$  and  $\tau_2 = \{Y, \emptyset, \mu\}$ . Then the fuzzy function  $f: (X, \tau_1) \rightarrow (Y, \tau_2)$  defined by f(a) = x and f(b) = y is fuzzy  $\beta \alpha$ -continuous but neither fuzzy  $\alpha$ -irresolute nor irresolute.

## Example 2.7. In the above example, we take

$$\mu(x) = 0.7, \ \mu(y) = 0.6.$$

Then the fuzzy function  $f:(X, \tau_1) \rightarrow (Y, \tau_2)$  defined by f(a) = x and f(b) = y is fuzzy  $\beta \alpha$ - continuous but neither fuzzy preirresolute nor  $\beta$ - irresolute.

**Theorem 2.8.** Let  $f: X \to Y$  be a fuzzy function and let  $g: X \to X \times Y$  be the fuzzy graph function of f [1], defined by  $g(x_{\varepsilon}) = (x_{\varepsilon}, f(x_{\varepsilon}))$  for each  $x_{\varepsilon} \in X$ . If g is fuzzy  $\beta \alpha$ -continuous, then f is fuzzy  $\beta \alpha$ -continuous.

**Proof.** Let  $\rho$  be fuzzy  $\alpha$ -open set in Y. Then  $X \times \rho$  is fuzzy  $\alpha$ -open set in  $X \times Y$ . Since g is fuzzy  $\beta \alpha$ -continuous, then  $f^{-1}(\rho) = g^{-1}(X \times \rho)$  is fuzzy  $\beta$ -open in X. Thus, f is fuzzy  $\beta \alpha$ -continuous.

#### **Definition 2.9.** A fuzzy space X said to be:

- (1) fuzzy  $\beta$ -compact [4] (resp. fuzzy  $\alpha$ -compact [4, 11]) if every fuzzy  $\beta$ -open (resp. fuzzy  $\alpha$ -open) cover of X has a finite subcover;
- (2) fuzzy countably  $\beta$ -compact (resp. fuzzy countably  $\alpha$ -compact) if every fuzzy  $\beta$ -open (resp. fuzzy  $\alpha$ -open) countably cover of X has a finite subcover;
- (3) fuzzy  $\beta$ -Lindelof (resp. fuzzy  $\alpha$ -Lindelof) if every cover of X by fuzzy  $\beta$ -open (resp. fuzzy  $\alpha$ -open) sets has a countable subcover.

**Theorem 2.10.** Let  $f: X \rightarrow Y$  be a fuzzy  $\beta \alpha$ -continuous surjection. Then the following statements hold:

- (1) If X is fuzzy  $\beta$ -compact, then Y is fuzzy  $\alpha$ -compact.
- (2) If X is fuzzy  $\beta$ -Lindelof, then Y is fuzzy  $\alpha$ -Lindelof.
- (3) If X is fuzzy countably  $\beta$ -compact, then Y is fuzzy countably  $\alpha$ -compact.

**Proof.** We prove only (1). Let  $\{\mu_{\alpha} \colon \alpha \in I\}$  be any fuzzy  $\alpha$ -open cover of Y. Since f is fuzzy  $\beta\alpha$ - continuous, then  $\{f^{-1}(\mu_{\alpha}) \colon \alpha \in I\}$  is a fuzzy  $\beta$ - open cover of X. Since X is fuzzy  $\beta$ -compact, there exists a finite subset  $I_0$  of I such that  $X = \bigvee \{f^{-1}(\mu_{\alpha}) \colon \alpha \in I_0\}$ . Then we have  $Y = \bigvee \{\mu_{\alpha} \colon \alpha \in I_0\}$  and thus Y is fuzzy  $\alpha$ -compact.

**Definition 2.11.** A fuzzy space X is said to be fuzzy  $\beta$ -

connected (resp. fuzzy connected [9]) if it cannot be expressed as the union of two nonempty, disjoint fuzzy  $\beta$ -open (resp. fuzzy open) sets.

**Theorem 2.12.** If  $f: X \rightarrow Y$  is fuzzy  $\beta \alpha$ -continuous surjective function and X is fuzzy  $\beta$ - connected space, then Y is fuzzy connected space.

**Proof.** Suppose that Y is not fuzzy connected space. Then there exists nonempty disjoint fuzzy open sets  $\beta$  and  $\mu$  such that  $Y = \beta \vee \mu$ . Hence,  $\beta$  and  $\mu$  are fuzzy  $\alpha$ -open sets in Y. Since f is fuzzy  $\beta \alpha$ -continuous,  $f^{-1}(\beta)$  and  $f^{-1}(\mu)$  are fuzzy  $\beta$ -closed and  $\beta$ -open. Moreover,  $f^{-1}(\beta)$  and  $f^{-1}(\mu)$  are nonempty disjoint and  $X = f^{-1}(\beta) \vee f^{-1}(\mu)$ . This shows that X is not fuzzy  $\beta$ -connected. This is a contradiction. Therefore, Y is fuzzy connected.

**Definition 2.13.** A fuzzy space X is called hyperconnected [2] if every fuzzy open set is dense.

**Remark 2.14.** The following example shows that fuzzy  $\beta\alpha$ -continuous surjection do not necessarily preserve fuzzy hyperconnectedness.

**Example 2.15.** Let  $X = \{a, b\}$ ,  $Y = \{x, y\}$  and  $\lambda$ ,  $\mu$  are fuzzy sets defined as follows:

$$\lambda(a) = 0.3, \ \lambda(b) = 0.6,$$
  
 $\mu(x) = 0.5, \ \mu(y) = 0.5.$ 

Let  $\tau_1 = \{X, \emptyset, \lambda\}$  and  $\tau_2 = \{Y, \emptyset, \mu\}$ . Then the fuzzy function  $f: (X, \tau_1) \rightarrow (Y, \tau_2)$  defined by f(a) = x and f(b) = y is fuzzy  $\beta \alpha$ -continuous surjective and  $(X, \tau_1)$  is hyperconnected. But  $(Y, \tau_2)$  is not hyperconnected.

## 3. Several properties

In this section, we investigate the relationships between fuzzy  $\beta\alpha$ -continuous functions and separation axioms and those graphs.

**Definition 3.1.** A fuzzy space X is said to be fuzzy  $\beta$ - $T_1$  (resp. fuzzy  $\alpha$ - $T_1$ ) if for each pair of distinct fuzzy points  $x_{\varepsilon}$  and  $y_{\nu}$  of X, there exist fuzzy  $\beta$ -open (resp.  $\alpha$ -open) sets  $\beta$  and  $\mu$  containing  $x_{\varepsilon}$  and  $y_{\nu}$ , respectively, such that  $y_{\nu} \notin \beta$  and  $x_{\varepsilon} \notin \mu$ .

**Theorem 3.2.** If  $f: X \to Y$  is a fuzzy  $\beta \alpha$ -continuous injection and Y is fuzzy  $\alpha - T_1$ , then X is fuzzy  $\beta - T_1$ .

**Proof.** Suppose that Y is fuzzy  $\alpha$ - $T_1$ . For any distinct fuzzy points  $x_{\varepsilon}$  and  $y_{\nu}$  in X, there exist fuzzy  $\alpha$ -open sets  $\mu$  and  $\rho$  in Y such that  $f(x_{\varepsilon}) \in \mu$ ,  $f(y_{\nu}) \notin \mu$ ,  $f(x_{\varepsilon}) \notin \rho$ 

and  $f(y_{\nu}) \in \rho$ . Since f is fuzzy  $\beta \alpha$ -continuous,  $f^{-1}(\mu)$  and  $f^{-1}(\rho)$  are  $\beta$ -open sets in X such that  $x_{\varepsilon} \in f^{-1}(\mu)$ ,  $y_{\nu} \notin f^{-1}(\mu)$ ,  $x_{\varepsilon} \notin f^{-1}(\rho)$  and  $y_{\nu} \in f^{-1}(\rho)$ . This shows that X is fuzzy  $\beta$ - $T_1$ .

**Definition 3.3.** A fuzzy space X is said to be fuzzy  $\beta$ - $T_2$  (resp. fuzzy  $\alpha$ - $T_2$ ) if for each pair of distinct fuzzy points  $x_{\varepsilon}$  and  $y_{\nu}$  of X, there exist disjoint fuzzy  $\beta$ -open (resp. fuzzy  $\alpha$ -open) sets  $\beta$  and  $\mu$  in X such that  $x_{\varepsilon} \in \beta$  and  $y_{\nu} \in \mu$ .

**Theorem 3.4.** If  $f: X \rightarrow Y$  is a fuzzy  $\beta \alpha$ -continuous injection and Y is fuzzy  $\alpha - T_2$ , then X is fuzzy  $\beta - T_2$ .

**Proof.** For any pair of distinct fuzzy points  $x_{\varepsilon}$  and  $y_{\nu}$  in X, there exist disjoint fuzzy  $\alpha$ -open sets  $\beta$  and  $\mu$  in Y such that  $f(x_{\varepsilon}) \in \beta$  and  $f(y_{\nu}) \in \mu$ . Since f is fuzzy  $\beta \alpha$ -continuous,  $f^{-1}(\beta)$  and  $f^{-1}(\mu)$  is fuzzy  $\beta$ -open in X containing  $x_{\varepsilon}$  and  $y_{\nu}$ , respectively. Then we obtain  $f^{-1}(\beta) \wedge f^{-1}(\mu) = \emptyset$ . This shows that X is fuzzy  $\beta \cdot T_2$ .

**Definition 3.5.** A fuzzy space X is said to be fuzzy strongly  $\alpha$ -regular (resp. fuzzy strongly  $\beta$ - regular) if for each fuzzy  $\alpha$ -closed (resp. fuzzy  $\beta$ -closed) set  $\eta$  and each fuzzy point  $x_{\varepsilon} \in \eta$ , there exist disjoint fuzzy open sets  $\beta$  and  $\mu$  such that  $\eta \leq \beta$  and  $x_{\varepsilon} \in \mu$ .

**Definition 3.6.** A fuzzy space X is said to be fuzzy strongly  $\alpha$ -normal (resp. fuzzy strongly  $\beta$ -normal) if for every pair of disjoint fuzzy  $\alpha$ -closed (resp. fuzzy  $\beta$ -closed) sets  $\eta_1$  and  $\eta_2$  in X, there exist disjoint fuzzy open sets  $\beta$  and  $\mu$  such that  $\eta_1 \leq \beta$  and  $\eta_2 \leq \mu$ .

**Theorem 3.7.** If  $f: X \rightarrow Y$  is fuzzy  $\beta \alpha$ -continuous, fuzzy open bijection and X is a fuzzy strongly  $\beta$ -regular space, then Y is fuzzy strongly  $\alpha$ -regular.

**Proof.** Let  $\eta$  be fuzzy  $\alpha$ -closed set in Y and  $y_{\nu} \notin \eta$ . Take  $y_{\varepsilon} = f(x_{\varepsilon})$ . Since f is fuzzy  $\beta \alpha$ -continuous,  $f^{-1}(\eta)$  is a fuzzy  $\beta$ -closed set. Take  $\lambda = f^{-1}(\eta)$ . We have  $x_{\varepsilon} \notin \lambda$ . Since X is fuzzy strongly  $\beta$ -regular, there exist disjoint fuzzy open sets  $\beta$  and  $\mu$  such that  $\lambda \leq \beta$  and  $x_{\varepsilon} \in \mu$ . Thus, we obtain that  $\eta = f(\lambda) \leq f(\beta)$  and  $y_{\varepsilon} = f(x_{\varepsilon}) \in f(\mu)$  such that  $f(\beta)$  and  $f(\mu)$  are disjoint fuzzy open sets. This shows that Y is fuzzy strongly  $\alpha$ -regular.

**Theorem 3.8.** If  $f: X \rightarrow Y$  is fuzzy  $\beta \alpha$ -continuous, fuzzy open bijection and X is a fuzzy strongly  $\beta$ -normal space, then Y is fuzzy strongly  $\alpha$ -normal.

**Proof.** Let  $\eta_1$  and  $\eta_2$  be disjoint fuzzy  $\alpha$ -closed sets in

Y. Since f is fuzzy  $\beta\alpha$ -continuous,  $f^{-1}(\eta_1)$  and  $f^{-1}(\eta_2)$  are fuzzy  $\beta$ -closed sets. Take  $\beta=f^{-1}(\eta_1)$  and  $\mu=f^{-1}(\eta_2)$ . We have  $\beta\wedge\mu=\emptyset$ . Since X is fuzzy strongly  $\beta$ -normal, there exist disjoint fuzzy open sets  $\lambda$  and  $\rho$  such that  $\beta\leq\lambda$  and  $\mu\leq\rho$ . We obtain that  $\eta_1=f(\beta)\leq f(\lambda)$  and  $\eta_2=f(\mu)\leq f(\rho)$  such that  $f(\lambda)$  and  $f(\rho)$  are disjoint fuzzy open sets. Thus, Y is fuzzy strongly  $\alpha$ -normal.

Recall that for a fuzzy function  $f: X \to Y$ , the subset  $\{(x_{\varepsilon}, f(x_{\varepsilon})) : x_{\varepsilon} \in X\} \leq X \times Y$  is called the graph of f and is denoted by G(f).

**Definition 3.9.** A graph G(f) of a fuzzy function  $f: X \to Y$  is said to be fuzzy  $\beta$ - $\alpha$ -closed if for each  $(x_{\varepsilon}, y_{\nu}) \in (X \times Y) \setminus G(f)$ , there exist a fuzzy  $\beta$ -open set  $\beta$  in X containing  $x_{\varepsilon}$  and a fuzzy  $\alpha$ -open set  $\mu$  in Y containing  $y_{\nu}$  such that  $(\beta \times \mu) \wedge G(f) = \emptyset$ .

**Lemma 3.10.** A graph G(f) of  $f: X \to Y$  is fuzzy  $\beta - \alpha$  -closed in  $X \times Y$  if and only if for each  $(x_{\varepsilon}, y_{\nu}) \in (X \times Y) \setminus G(f)$ , there exist a fuzzy  $\beta$ -open set  $\beta$  in X containing  $x_{\varepsilon}$  and a fuzzy  $\alpha$ -open set  $\mu$  in Y containing  $y_{\nu}$  such that  $f(\beta) \wedge \mu = \emptyset$ .

**Theorem 3.11.** If  $f: X \to Y$  is fuzzy  $\beta \alpha$ -continuous and Y is fuzzy  $\alpha$ -Hausdorff, then G(f) is fuzzy  $\beta$ - $\alpha$ -closed in  $X \times Y$ .

**Proof.** Let  $(x_{\varepsilon},y_{\nu}) \in (X \times Y) \setminus G(f)$ . Then  $f(x_{\varepsilon}) \neq y_{\nu}$ . Since Y is fuzzy  $\alpha$ -Hausdorff, there exist disjoint fuzzy  $\alpha$ -open sets  $\beta$  and  $\mu$  in Y such that  $f(x_{\varepsilon}) \in \beta$  and  $y_{\nu} \in \mu$ . Since f is fuzzy  $\beta\alpha$ -continuous, there exists a  $\beta$ -open set  $\rho$  in X containing  $x_{\varepsilon}$  such that  $f(\rho) \leq \beta$ . Therefore, we obtain  $y_{\nu} \in \mu$  and  $f(\rho) \wedge \mu = \emptyset$ . This shows that G(f) is fuzzy  $\beta$ - $\alpha$ -closed.

**Theorem 3.12.** Let  $f: X \to Y$  has a fuzzy  $\beta$ - $\alpha$ -closed graph G(f). If f is injective, then X is fuzzy  $\beta$ - $T_1$ .

**Proof.** Let  $x_{\varepsilon}$  and  $y_{\nu}$  be any two distinct fuzzy points of X. Then,  $(x_{\varepsilon}, f(y_{\nu})) \in (X \times Y) \setminus G(f)$ . By definition of fuzzy  $\beta$ - $\alpha$ -closed graph, there exist a fuzzy  $\beta$ -open set  $\beta$  in X and a fuzzy  $\alpha$ -open set  $\mu$  in Y such that  $f(x_{\varepsilon}) \in \beta$ ,  $y_{\nu} \in \mu$  and  $f(\rho) \wedge \mu = \emptyset$  and hence  $\beta \wedge f^{-1}(\mu) = \emptyset$ . Therefore, we have  $y_{\nu} \notin \beta$ . This implies that X is fuzzy  $\beta$ - $T_1$ .

**Theorem 3.13.** Let  $f: X \to Y$  has a fuzzy  $\beta$ - $\alpha$ -closed graph G(f). If f is surjective always fuzzy  $\beta$ -open function, then Y is fuzzy  $\beta$ - $T_2$ .

**Proof.** Let  $y_{\nu}$  and  $y_{\xi}$  be any distinct points of Y. Since f is surjective,  $f(x_{\nu}) = y_{\nu}$  for some  $x_{\nu} \in X$  and  $(x_{nu}, y_{\xi}) \in (X \times Y) \setminus G(f)$ . By the fuzzy  $\beta$ -  $\alpha$ -closedness of graph G(f), there exists a fuzzy  $\beta$ -open set  $\beta$  in X and a fuzzy  $\alpha$ -open set  $\mu$  in Y such that  $x_{\nu} \in \beta$ ,  $y_{\xi} \in \mu$  and  $(\beta \times \mu) \wedge G(f) = \emptyset$ . Then, we have  $f(\beta) \wedge \mu = \emptyset$ . Since f is always fuzzy  $\beta$ -open, then  $f(\beta)$  is fuzzy  $\beta$ -open such that  $f(x_{\nu}) = y_{\nu} \in f(\beta)$ . This implies that Y is fuzzy  $\beta$ -  $T_2$ .

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