International Journal of Fuzzy Logic and Intelligent Systems, vol. 5, no. 2, June 2005 pp. 136-139

Fuzzy (r, s)-preopen sets

Seung On Lee and Eun Pyo Lee

Department of Mathematics, Chungbuk National University, Cheongju 361-763, Korea
Department of Mathematics, Seonam University, Namwon 590-711, Korea

Abstract

In this paper, we introduce the concepts of fuzzy (r, s)-preopen sets and fuzzy (r, s)-precontinuous mappings on intuitionistic fuzzy topological spaces in Sostak's sense and then we investigate some of their characteristic properties.

Key words : fuzzy (r, s)-preopen sets, fuzzy (r, s)-precontinuous mappings

1. Introduction

The concept of fuzzy set was introduced by Zadeh [11]. These spaces and its generalizations are later studied by several authors, one of which, developed by Sostak [10], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay, Hazra, and Samanta [3], and by Ramadan [9].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Coker and his colleagues [4,5,7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets.

Using the idea of degree of openness and degree of nonopenness, Coker and Demirci [5] defined intuitionistic fuzzy topological spaces in Sostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

In this paper, we introduce the concepts of fuzzy (r, s)-preopen sets and fuzzy (r, s)-precontinuous mappings on intuitionistic fuzzy topological spaces in Sostak's sense and then we investigate some of their characteristic properties.

2. Preliminaries

Let I be the unit interval $[0, 1]$ of the real line. A member μ of I^X is called a fuzzy set of X. For any $\mu \in I^X$, μ^c denotes the complement $1-\mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant mappings on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An intuitionistic fuzzy set A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A: X \to I$ and $\gamma_A: X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

Definition 2.1. ([1]) Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets on X. Then

(1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.
(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
(3) $A^c = (\gamma_A, \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
(6) $0_\sim = (\tilde{0}, \tilde{1})$ and $1_\sim = (\tilde{1}, \tilde{0})$.

Let f be a mapping from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y. Then:

(1) The image of A under f, denoted by $f(A)$ is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

A smooth fuzzy topology on X is a mapping $T: I^X \to I$ which satisfies the following properties:

(1) $T(\tilde{0}) = T(\tilde{1}) = 1$
(2) $T(\mu_1 \wedge \mu_2) \geq T(\mu_1) \wedge T(\mu_2)$.
(3) $T(\vee \mu_i) \geq \wedge T(\mu_i)$.

The pair (X, T) is called a smoot fuzzy topological spaces.

Manuscript received Apr. 4, 2005; revised May. 9, 2005
This work was supported by the research grant of Chungbuk National University in 2005

An intuitionistic fuzzy topology X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $0_{\sim}, 1_{\sim} \in T$.

(2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.

(3) If $A_i \in T$ for all i, then $\cup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space.*

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r,s) such that $r, s \in I$ and $r+s \leq 1$.

Definition 2.2. ([5]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Sostak's sense* (SoIFT for short) $\tau = (\tau_1, \tau_2)$ on X is a mapping $\tau: I(X) \to I \otimes I$ which satisfies the following properties:

(1) $\tau_1(0_{\sim}) = \tau_1(1_{\sim}) = 1$ and $\tau_2(0_{\sim}) = \tau_2(1_{\sim}) = 0$.

(2) $\tau_1(A \cap B) \geq \tau_1(A) \wedge \tau_1(B)$ and
$\tau_2(A \cap B) \leq \tau_2(A) \vee \tau_2(B)$.

(3) $\tau_1(\cup A_i) \geq \wedge \tau_1(A_i)$ and
$\tau_2(\cup A_i) \leq \vee \tau_2(A_i)$.

The $(X, \tau) = (X, \tau_1, \tau_2)$ is said to be an intuitionistic fuzzy topological space in Sostak's sense (SoIFTS for short). Also, we call $\tau_1(A)$ a gradation of openness of A and $\tau_2(A)$ a gradation of nonopenness on A.

Definition 2.3. ([8]) Let A be an intuitionistic fuzzy set in a SoIFTS (X, τ_1, τ_2) and $(r,s) \in I \otimes I$. Then A is said to be

(1) fuzzy (r,s)*-open* if $\tau_1(A) \geq r$ and $\tau_2(A) \leq s$,

(2) fuzzy (r,s)*-closed* if $\tau_1(A^c) \geq r$ and $\tau_2(A^c) \leq s$.

Definition 2.4. Let (X, τ_1, τ_2) be a SoIFTS. For each $(r,s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy* (r,s)*-interior* is defined by

$$\text{int}(A, r, s) = \cup \{B \in I(X) | A \supseteq B, B \text{ is fuzzy}(r,s)-\text{open}\}$$

and the *fuzzy* (r,s)*-closure* is defined by

$$\text{cl}(A, r, s) = \cap \{B \in I(X) | A \subseteq B, B \text{ is fuzzy}(r,s)-\text{closed}\}.$$

The operators $\text{int}: I(X) \times I \otimes I \to I(X)$ and $\text{cl}: I(X) \times I \otimes I \to I(X)$ are called the *fuzzy interior operator* and *fuzzy closure operator* in (X, τ_1, τ_2), respectively.

Lemma 2.5 [8] For an intuitionistic fuzzy set A in a SoIFTS (X, τ_1, τ_2) and $(r,s) \in I \otimes I$,

(1) $\text{int}(A, r, s)^c = \text{cl}(A^c, r, s)$.

(2) $\text{cl}(A, r, s)^c = \text{int}(A^c, r, s)$.

Let (X, τ) be an intuitionistic fuzzy topological space in Sostak's sense. Then it is easy to see that for each $(r,s) \in I \otimes I$, the family $\tau_{(r,s)}$ defined by

$$\tau_{(r,s)} = A \in I(X) | \tau_1(A) \geq r \quad \text{and} \quad \tau_2(A) \leq s$$

is an intuitionistic fuzzy topology on X.

Let (X, T) be an intuitionistic fuzzy topological space and $(r,s) \in I \otimes I$. Then the map $T^{(r,s)}: I(X) \to I \otimes I$ defined by

$$T^{(r,s)}(A) = \begin{cases} (1,0) & \text{if } A = 0_{\sim}, 1_{\sim}, \\ (r,s) & \text{if } A \in T - \{0_{\sim}, 1_{\sim}\}, \\ (0,1) & \text{otherwise} \end{cases}$$

becomes an intuitionistic fuzzy topology in Sostak's sense on X.

Definition 2.6. Let A be an intuitionistic fuzzy set in a SoIFTS (X, τ_1, τ_2) and $(r,s) \in I \otimes I$. Then A is said to be

(1) *fuzzy* (r,s)*-semiopen* if there is a fuzzy (r,s)-open set B in X such that $B \subseteq A \subseteq \text{cl}(B, r, s)$,

(2) *fuzzy* (r,s)*-semiclosed* if there is a fuzzy (r,s)-closed set B in X such that $\text{int}(B, r, s) \subseteq A \subseteq B$.

3. Fuzzy (r,s)-semiopen sets

Definition 3.1. Let A be an intuitionistic fuzzy set in a SoIFTS (X, τ_1, τ_2) and $(r,s) \in I \otimes I$. Then A is said to be

(1) *fuzzy* (r,s)*-preopen* if $A \subseteq \text{int}(cl(A, r, s), r, s)$,

(2) *fuzzy* (r,s)*-semiclosed* if $\text{cl}(int(A, r, s), r, s) \subseteq A$.

Theorem 3.2. Let A be an intuitionistic fuzzy set in a SoIFTS (X, τ_1, τ_2) and $(r,s) \in I \otimes I$. Then the following statements are equivalent:

(1) A is a fuzzy (r,s)-preopen set.

(2) A^c is a fuzzy (r,s)-preclosed set.

Proof. It follows from Lemma 2.5.

It is obvious that every fuzzy (r,s)-open ((r,s)-closed) set is a fuzzy (r,s)-preopen ((r,s)-preclosed) set but the converse need not be true which is shown by the following example.

Example 3.3. Let $X = \{x, y\}$ and let A_1 and A_2 be intuitionistic fuzzy set of X defined as

$$A_1(x) = (0.5, 0.2), \quad A_1(y) = (0.1, 0.7);$$

and

$$A_2(x) = (0.6, 0.2), \quad A_2(y) = (0.5, 0.3).$$

Define $\tau: I(X) \to I \otimes I$ by

$$\tau(A) = (\tau_1(A), \tau_2(A)) = \begin{cases} (1,0) & \text{if } A = 0_{\sim}, 1_{\sim}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly (τ_1, τ_2) is a SoIFT on X. The intuitionistic fuzzy set A_2 is a fuzzy $(\frac{1}{2},\frac{1}{3})$-preopen set which is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-open set. Also, A_2^c is a fuzzy $(\frac{1}{2},\frac{1}{3})$-preclosed set which is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-closed set.

Theorem 3.4. Let (X,τ_1,τ_2) be a SoIFTS and $(r,s)\in I\otimes I$.

(1) If $\{A_i\}$ is a family of fuzzy (r,s)-preopen sets of X, then $\cup A_i$ is fuzzy (r,s)-preopen.

(2) If $\{A_i\}$ is a family of fuzzy (r,s)-preclosed sets of X, then $\cap A_i$ is fuzzy (r,s)-preclosed.

Proof. (1) Let $\{A_i\}$ be a collection of fuzzy (r,s)-preopen sets. Then for each i,

$$A_i\subseteq \operatorname{int}(\operatorname{cl}(A_i,r,s),r,s).$$

So

$$\cup A_i\subseteq\cup\operatorname{int}(\operatorname{cl}(A,r,s),r,s)\subseteq\operatorname{int}(\operatorname{cl}(\cup A_i,r,s),r,s).$$

Thus $\cup A_i$ is a fuzzy (r,s)-preopen set.

(2) It follows from (1) using Theorem 3.2.

That fuzzy (r,s)-semiopen sets and fuzzy (r,s)-preopen sets are independent notions is shown by the following two examples.

Example 3.5. Let $X=\{x,y\}$ and let A_1 and A_2 be intuitionistic fuzzy set of X defined as

$$A_1(x)=(0.2,0.6),\ A_1(y)=(0.4,0.3);$$

and

$$A_2(x)=(0.7,0.2),\ A_2(y)=(0.2,0.5).$$

Define $\tau: I(X)\to I\otimes I$ by

$$\tau(A)=(\tau_1(A),\tau_2(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{3}) & \text{if } A=A_1,\\ (0,1) & \text{otherwise}.\end{cases}$$

Then clearly (τ_1,τ_2) is a SoIFT on X. The intuitionistic fuzzy set A_2 is a $(\frac{1}{2},\frac{1}{3})$-preopen set which is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-semiopen.

Example 3.6. Let $X=\{x,y\}$ and let A_1 and A_2 be intuitionistic fuzzy set of X defined as

$$A_1(x)=(0.2,0.7),\ A_1(y)=(0.1,0.8);$$

and

$$A_2(x)=(0.5,0.3),\ A_2(y)=(0.7,0.2).$$

Define $\tau: I(X)\to I\otimes I$ by

$$\tau(A)=(\tau_1(A),\tau_2(A))=\begin{cases}(1,0) & \text{if } A=0_\sim,1_\sim,\\ (\frac{1}{2},\frac{1}{3}) & \text{if } A=A_1,\\ (0,1) & \text{otherwise}.\end{cases}$$

Then clearly (τ_1,τ_2) is a SoIFT on X. The intuitionistic fuzzy set A_2 is a fuzzy $(\frac{1}{2},\frac{1}{3})$-semiopen set which is not a fuzzy $(\frac{1}{2},\frac{1}{3})$-preopen set.

Definition 3.7. Let (X,τ_1,τ_2) be a SoIFTS. For each $(r,s)\in I\otimes I$ and for each $A\in I(X)$, the *fuzzy* (r,s)*-preinterior* is defined by

$$\operatorname{pint}(A,r,s)=\cup\{B\in I(X)|A\supseteq B,\ B\text{ is fuzzy}(r,s)-\text{preopen}\}$$

and the fuzzy (r,s)-preclosure is defined by

$$\operatorname{pcl}(A,r,s)=\cap\{B\in I(X)|A\subseteq B,\ B\text{ is fuzzy (r,s) }-<\text{losed}\}.$$

Obviously $\operatorname{pcl}(A,r,s)$ is the smallest fuzzy (r,s)-preclosed set which contains A and $\operatorname{pint}(A,r,s)$ is the greatest fuzzy (r,s)-preopen set which is contained in A. Also, $\operatorname{pcl}(A,r,s)=A$ for any fuzzy (r,s)-preclosed set A and $\operatorname{pint}(A,r,s)=A$ for any fuzzy (r,s)-preopen set A. Moreover, we have

$$\begin{aligned}\operatorname{int}(A,r,s)&\subseteq\operatorname{int}(A,r,s)\subseteq A\\ &\subseteq\operatorname{pcl}(A,r,s)\subseteq\operatorname{cl}(A,r,s).\end{aligned}$$

Also, we have the following results:

(1) $\operatorname{pcl}(0_\sim,r,s)=0_\sim$, $\operatorname{pcl}(1_\sim,r,s)=1_\sim$.

(2) $\operatorname{pcl}(A,r,s)\supseteq A$.

(3) $\operatorname{pcl}(A\cup B,r,s)\supseteq\operatorname{pcl}(A,r,s)\cup\operatorname{pcl}(B,r,s)$.

(4) $\operatorname{pcl}(\operatorname{pcl}(A,r,s),r,s)=\operatorname{pcl}(A,r,s)$.

(5) $\operatorname{pint}(0_\sim,r,s)=0_\sim$, $\operatorname{pint}(1_\sim,r,s)=1_\sim$.

(6) $\operatorname{pint}(A,r,s)\subseteq A$.

(7) $\operatorname{pint}(A\cap B,r,s)\subseteq\operatorname{pint}(A,r,s)\cap\operatorname{pint}(B,r,s)$.

(8) $\operatorname{pint}(\operatorname{pint}(A,r,s),r,s)=\operatorname{pint}(A,r,s)$.

Theorem 3.8. For an intuitionistic fuzzy set A of a SoIFTS (X,τ_1,τ_2) and $(r,s)\in I\otimes I$, we have:

(1) $\operatorname{pint}(A,r,s)^c=\operatorname{pcl}(A^c,r,s)$.

(2) $\operatorname{pcl}(A,r,s)^c=\operatorname{pint}(A^c,r,s)$.

Proof. (1) Since $\operatorname{pint}(A,r,s)\subseteq A$ and $\operatorname{pint}(A,r,s)$ is fuzzy (r,s)-preopen in X, $A^c\subseteq\operatorname{pint}(A,r,s)^c$ and $\operatorname{pint}(A,r,s)^c$ is fuzzy (r,s)-preclosed in X. Thus

$$\begin{aligned}\operatorname{pcl}(A^c,r,s)&\subseteq\operatorname{pcl}(\operatorname{pint}(A,r,s)^c,r,s)\\ &=\operatorname{pint}(A,r,s)^c.\end{aligned}$$

Conversely, since $A^c\subseteq\operatorname{pcl}(A^c,r,s)$ and $\operatorname{pcl}(A^c,r,s)$ is fuzzy (r,s)-preclosed in X, $\operatorname{pcl}(A^c,r,s)^c\subseteq A$ and $\operatorname{pcl}(A^c,r,s)^c$ is fuzzy (r,s)-preopen in X. Thus

$$\begin{aligned}\operatorname{pcl}(A^c,r,s)^c&=\operatorname{pint}(\operatorname{scl}(A^c,r,s)^c,r,s)\\ &\subseteq\operatorname{pint}(A,r,s)\end{aligned}$$

and hence $\operatorname{pint}(A,r,s)^c\subseteq\operatorname{pcl}(A^c,r,s)$.

(2) Similar to (1).

Definition 3.9. Let $f: (X, \tau_1, \tau_2) \to (Y, \omega_1, \omega_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is said to be

(1) fuzzy (r, s)-precontinuous if $f^{-1}(B)$ is a fuzzy (r, s)-preopen set of X for each fuzzy (r, s)-open set B of Y,

(2) fuzzy (r, s)-preopen if $f(A)$ is a fuzzy (r, s)-preopen set of Y for each fuzzy (r, s)-open set A of X,

(3) fuzzy (r, s)-preclosed if $f(A)$ is a fuzzy (r, s)-preclosed set of Y for each fuzzy (r, s)-closed set A of X.

In general, it need not be true that f and g are fuzzy (r, s)-precontinuous ((r, s)-preopen and (r, s)-preclosed, respectively) mappings then so is $g \circ f$. But we have the following theorem.

Theorem 3.10. Let (X, τ_1, τ_2), (Y, ω_1, ω_2) and (Z, σ_1, σ_2) be SoIFTSs and let $f: X \to Y$ and $g: Y \to Z$ be mappings and $(r, s) \in I \otimes I$. Then the following statements are true:

(1) If f is a fuzzy (r, s)-precontinuous mapping and g is a fuzzy (r, s)-continuous mapping, then $g \circ f$ is a fuzzy (r, s)-precontinuous mapping.

(2) If f is a fuzzy (r, s)-open mapping and g is a fuzzy (r, s)-preopen mapping, then $g \circ f$ is a fuzzy (r, s)-preopen mapping.

(3) If f is a fuzzy (r, s)-closed mapping and g is a fuzzy (r, s)-preclosed mapping, then $g \circ f$ is a fuzzy (r, s)-preclosed mapping.

Proof. Straightforward.

Theorem 3.11. Let $f: (X, \tau_1, \tau_2) \to (Y, \omega_1, \omega_2)$ be a mapping and $(r, s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is a fuzzy (r, s)-precontinuous mapping.

(2) $f^{-1}(B)$ is a fuzzy (r, s)-preclosed set of X for each fuzzy (r, s)-closed set B of Y.

(3) $f^{-1}(\mathrm{cl}(B, r, s)) \supseteq \mathrm{cl}(\mathrm{int}(f^{-1}(B), r, s), r, s)$ for each intuitionistic fuzzy set B of Y.

(4) $\mathrm{cl}(f(A), r, s) \supseteq f(\mathrm{cl}(\mathrm{int}(A, r, s), r, s))$ for each intuitionistic fuzzy set A of X.

Proof.(1) $\leftrightarrow$ (2) It is obvious.

(2) $\to$ (3) Let B be any intuitionistic fuzzy set of Y. Then $\mathrm{cl}(B, r, s)$ is a fuzzy (r, s)-closed set of Y. By (2), $f^{-1}(\mathrm{cl}(B, r, s))$ is a fuzzy (r, s)-preclosed set of X. Thus

$$\begin{aligned} f^{-1}(\mathrm{cl}(B, r, s)) &\supseteq \mathrm{cl}(\mathrm{int}(f^{-1}(\mathrm{cl}(B, r, s)), r, s), r, s) \\ &\supseteq \mathrm{cl}(\mathrm{int}(f^{-1}(B), r, s), r, s). \end{aligned}$$

(3) $\to$ (4) Let A be any intuitionistic fuzzy set of X. Then $f(A)$ is an intuitionistic fuzzy set of Y. By (3),

$$\begin{aligned} f^{-1}(\mathrm{cl}(f(A), r, s)) &\supseteq \mathrm{cl}(\mathrm{int}(f^{-1}f(A), r, s), r, s) \\ &\supseteq \mathrm{cl}(\mathrm{int}(A, r, s), r, s). \end{aligned}$$

Hence

$$\begin{aligned} \mathrm{cl}(f(A), r, s) &\supseteq ff^{-1}(\mathrm{cl}(f(A), r, s)) \\ &\supseteq f(\mathrm{cl}(\mathrm{int}(A, r, s), r, s)). \end{aligned}$$

(4) $\to$ (2) Let B be any fuzzy (r, s)-closed set of Y. Then $f^{-1}(B)$ is an intuitionistic fuzzy set of X. By (4),

$$\begin{aligned} f(\mathrm{cl}(\mathrm{int}(f^{-1}(B), r, s), r, s)) &\subseteq \mathrm{cl}(ff^{-1}(B), r, s) \\ &\subseteq \mathrm{cl}(B, r, s) = B \end{aligned}$$

and hence

$$\begin{aligned} \mathrm{cl}(\mathrm{int}(f^{-1}(B), r, s), r, s) &\subseteq f^{-1}f(\mathrm{cl}(\mathrm{int}(f^{-1}(B), r, s), r, s)) \\ &\subseteq f^{-1}(B). \end{aligned}$$

Thus $f^{-1}(B)$ is a fuzzy (r, s)-preclosed set of X.

References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems 20 (1986), 87-90.

[2] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. 24 (1968), 182-190.

[3] K. C. Chattopadhyay, R. N. Hazra and S. K. Samanta, *Gradation of openness : fuzzy topology*, Fuzzy Sets and Systems 49 (1992), 237-242.

[4] D. Coker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems 88 (1997), 81-89.

[5] D. Coker and M. Demirci, *An introduction to intuitionistic fuzzy topological spaces in Sostak's sense*, BUSEFAL 67 (1996), 67-76.

[6] D. Coker and A. Haydar Es, On fuzzy compactness in intuitionistic fuzzy topological spaces, J. Fuzzy Math. 3 (1995), 899-909.

[7] H. Gurcay, D. Coker and A. Haydar Es, *On fuzzy continuity in intuitionistic fuzzy topological spaces*, J. Fuzzy Math. 5 (1997), 365-378.

[8] E. P. Lee, *Semiopen sets on intuitionistic fuzzy topological spaces in Sostack's sense*, J. of Fuzzy Logic and Intelligent Systems 14 (2004), 234-238.

[9] A. A. Ramadan, *Smooth topological spaces*, Fuzzy Sets and Systems 48 (1992), 371-375.

[10] A. Sostak, *On a fuzzy topological structure*, Supp. Rend. Circ. Mat. Palermo (Ser. II) 11 (1985), 89-103.

[11] L. A. Zadeh, *Fuzzy sets*, Inform. and Control 8 (1965), 338-353.

Seung On Lee
Department of Mathematics, Chungbuk National University, Cheongju 361-763, Korea

E-mail : solee@chungbuk.ac.kr

Eun Pyo Lee
Department of Mathematics, Seonam University, Namwon 590-711, Korea

E-mail : eplee@seonam.ac.kr