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Analysis of Three-Dimensional Cracks in Inhomogeneous Materials Using Fuzzy Theory

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Abstract

This paper describes a fuzzy-based system for analyzing the stress intensity factors (SIFs) of three-dimensional (3D) cracks. 3D finite element method(FEM) was used to obtain the SIF for subsurface cracks and surface cracks existing in inhomogeneous materials. A geometry model, i.e. a solid containing one or several 3D cracks is defined. Several distributions of local node density are chosen, and then automatically superposed on one another over the geometry model by using the fuzzy theory. Nodes are generated by the bucketing method, and ten-noded quadratic tetrahedral solid elements are generated by the Delaunay triangulation techniques. The singular elements such that the mid-point nodes near crack front are shifted at the quarter-points, and these are automatically placed along the 3D crack front. The complete FE model is generated, and a stress analysis is performed. The SIFs are calculated using the displacement extrapolation method. The results were compared with those surface cracks in homogeneous materials. Also, this system is applied to analyze cladding effect of surface cracks in inhomogeneous materials.

Key words : Stress Intensity Factor, Surface Crack, Inhomogeneous Materials, Subsurface Cracks, Fuzzy Theory, Finite Element Analysis, Singular Element

Nomenclature

a	depth of a semi-elliptical surface crack
c	half length of a semi-elliptical surface crack
K_I	the stress intensity factor(SIF) for a Mode I
Q	shape factor for a semi-elliptical crack
ϕ	parametric angle of the ellipse
σ	applied uniform tensile stress
E	Young's modulus

1. Introduction

Three-Dimensional Cracks such as surface or embedded cracks are more common flaws being found in practical structures. Analyses of the 3D cracks are desirable in structural integrity studies of practical structures. Various techniques have been developed for this purpose. For example, the stress intensity factors for an elliptical or a semi-elliptical crack have been obtained by the finite element method [1-3], the alternating method [4,5], the body force method [6] and so on.

Among them, the finite element method (FEM) seems the most promising method to deal with 3D crack problems because of its flexibility and extensibility. However, there are still some problems to be solved. The main concern for the FEM is a relatively higher computation cost, especially when dealing with 3D crack problems in inhomogeneous materials. To overcome this, several techniques such as the direct method[2], the virtual crack extension method [7], the superposition method [8] and the special singular element method [9] have been proposed in conjunction with the FEM. It should be also noted here that the data preparation for 3D crack analyses require special element arrangement near the crack front, and that much efforts are necessary to generate such special meshes. Dramatic progress in computer technology now shortens computation time. However in reality, labour intensive tasks to prepare a FE model of a structural component with 3D cracks are still a bottle neck. The author has proposed an automatic FE mesh generation method for 3D structures based on the fuzzy knowledge processing and computational geometry[10,11].

In the present study, by integrating this mesh generator, one of commercial FE analysis codes and some additional techniques to calculate the SIF, a new fuzzy-based system for analyzing the SIFs of 3D cracks in inhomogeneous materials was developed. In order to examine accuracy and efficiency of the present system, the SIF for a semi-elliptical surface crack in a inhomogeneous plate subjected to uniform tension is calculated, and compared with Raju-Newman's solution[1,12].

2. Outline of the System

The biggest advantage of the present analysis system is a very simple operation to analyze complex structures such as a plate with several semi-elliptical surface cracks. Fig. 1 shows a flowchart of SIF analysis system.

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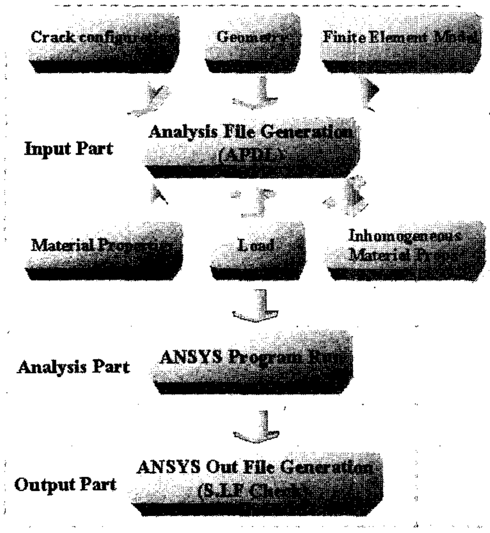


Fig. 1. The flowchart of S.I.F analysis system

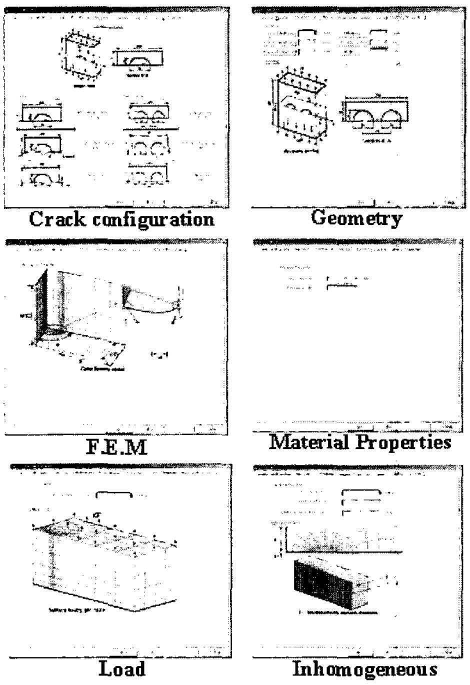


Fig. 2. Input screen of crack configuration and material properties

Also, Fig. 2 shows an input screen of crack configuration and material properties consisting geometry. Crack configuration, geometry, material properties and boundary conditions are directly attached using a mouse, and then by inputting values. The present system deals with displacement as well as force boundary conditions.

3. Principle and Algorithms

3.1 Control of optimum mesh patterns by fuzzy theory

In this section, the connecting process of locally- optimum mesh images is dealt with using the fuzzy knowledge processing technique. Performances of automatic mesh generation methods based on node generation algorithms depend on how to control node spacing functions or node density distributions and how to generate nodes.

In the present system, nodes are first generated, and then a finite element mesh is automatically built as shown in Figs. 3 and 4. In general, it is not so easy to well control element size for a complex geometry. The present system stores several local node patterns such as the pattern suitable to well capture stress concentration, the pattern to subdivide a finite domain uniformly, and the pattern to subdivide a whole domain uniformly. A user selects some of those local node patterns and designates where to locate them.

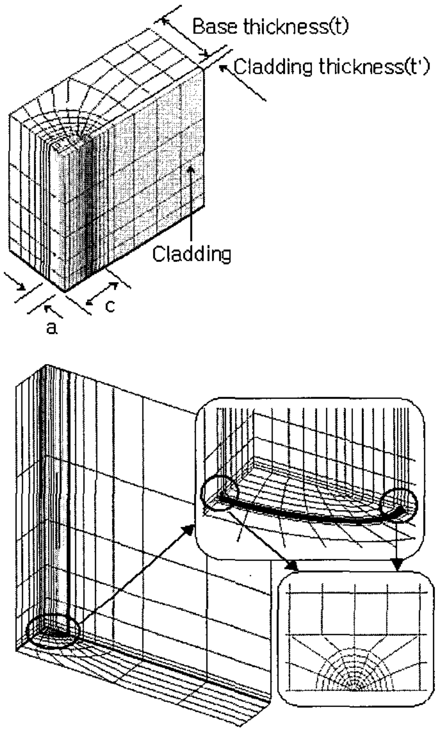


Fig. 3. Examples of locally-optimum mesh patterns for a crack

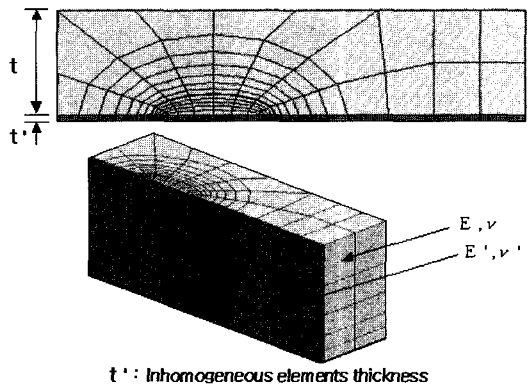


Fig. 4. Finite element mesh for twin cracks of inhomogeneous materials

3.2 Fuzzy control of node position

The fuzzy rules employed here can be generalized as :

RULE^i : IF p is A^i , THEN q is B^i

where RULE^i is the i-th fuzzy rule, A^i and B^i the fuzzy variables, p the value of node, and $\triangle$p the difference of the current and the next values of p, i.e. |p(n+1)-p(n)|(n : the iteration number of node), respectively. The labels of the fuzzy variables are defined as follows.

As for Ai,
- LARGE → p is much larger than 1.0.
- MEDIUM → p is larger than 1.0.
- SMALL → p is little larger than 1.0.

As for Bi,
- LARGE → q is positive and large.
- MEDIUM → q is positive and medium.
- SMALL → q is positive and small.

As shown in Fig. 5, trapezoid type membership functions are utilized as those of labels of A^i and B^i from the viewpoint of simplicity.

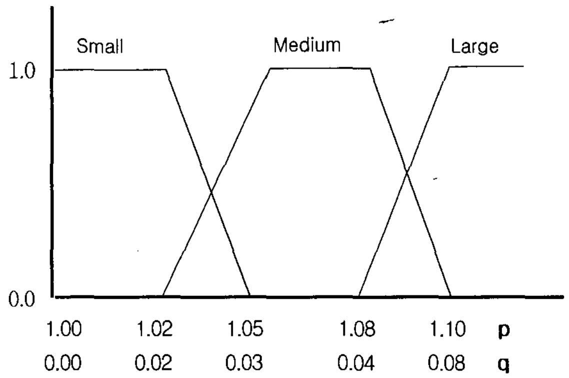


Fig. 5. Membership functions of labels of A^i(p) and B^i(q)

3.3 Singular element

When ordinary quadratic tetrahedral elements are employed to calculated the stress intensity factor, a very fine mesh is required near crack front to capture $\sqrt{r}$ variation in displacements $1/\sqrt{r}$ and variation in stresses where r denotes the distance from crack front. To relax this situation, singular elements as shown in Fig. 6 are adopted [9]. In the singular element, the mid-point nodes near a crack front are shifted to the quarter-points. This conversion of ordinary tetrahedral elements along a front of 3D crack to the singular elements is automatically performed in the last stage of the creation of a FE model.

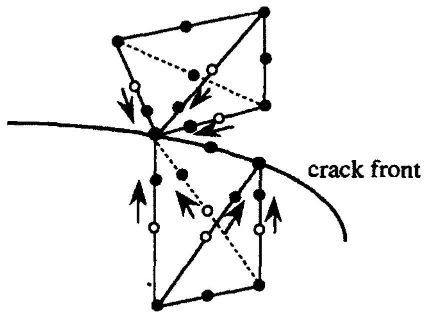


Fig. 6. Conversion of ordinary quadratic tetrahedral elements along crack front into singular elements

3.4 Calculation of Stress Intensity Factor

The FE model generated is automatically analyzed using ANSYS, and then displacements, strains and stresses are calculated. To obtain the stress intensity factors accurately as well as automatically, some techniques are employed.

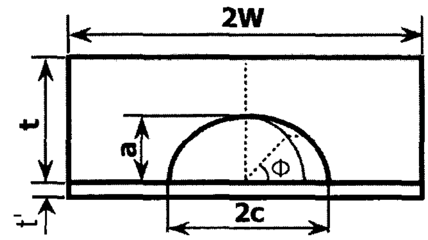
Stress intensity factors are computed using the displacement extrapolation method [2]. Nodal displace- ments calculated along the crack face are substituted in the following crack tip displacement equation :

$$K = \frac{E'}{4}\sqrt{2\pi}\lim_{r\to 0}\frac{w}{\sqrt{r}}$$

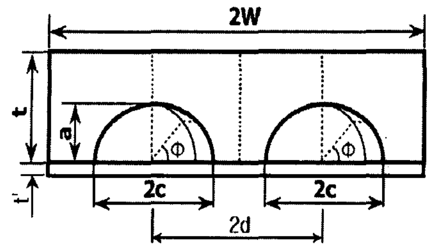
$$E' = \begin{cases} \dfrac{E}{1-\nu^2} & (\text{for } plane\ strain) \\ E & (\text{for } plane\ stress) \end{cases} \quad (1)$$

where w is a nodal displacement, and E' is equal to E in the plane stress condition or $E/(1-\nu^2)$ in the plane strain condition. Only positions of free surface intersection are regarded as in the plane stress condition, while other positions are in the plane strain condition.

In this least square operation, the K value evaluated at the shifted quarter point is neglected. This displacement extrapolation method is popularly used to calculate the stress intensity factor. In the present study, this process is fully automated. When a crack is designated by a user in the definition process of a geometry model, radial lines for the crack front as shown in Fig. 7 are automatically determined. After the stress analysis using ANSYS, displacement distributions are interpolated along the radial lines, on each of which the stress intensity factor is calculated by the least square method.



(a)



(b)

Fig. 7. Single and twin surface crack of inhomogeneous plate

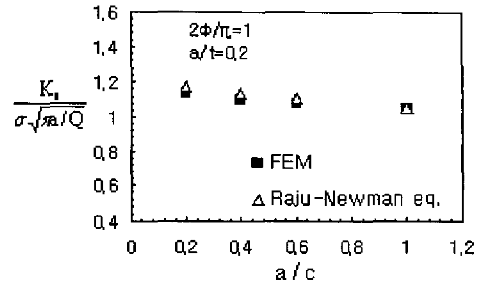


Fig. 8. Comparison of SIFs from FEM and Raju-Newman solutions

4. Results and Discussion

In order to examine efficiency and accuracy of the present system, a semi-elliptical surface crack in a inhomogeneous plate of width 2b, thickness t and height 2h subjected to uniform tension.

The analysis was performed for aspect ratio(a/c) of 0.2 to 1.0 and crack depth of a/t=0.2. Young's modulus E and Poisson's ratio ν were assumed to be 210 GPa and 0.3, respectively. Table 1 and Fig. 8 show the comparison at the deepest point(2ϕ/π=1.0) between the present solutions and Newman-Raju's solution [1,12]. The SIF K for this crack configura- tion can be often expressed as :

$$K = \sigma\sqrt{\pi a/Q}\,F(a/c,\ \phi,\ a/t) \qquad (2)$$

where Q is the squared complete elliptical integral of the second kind, and its approximate form is given as :

$$Q = 1 + 1.464\left(\frac{a}{c}\right)^{1.65} \qquad (3)$$

It can be seen from the figure that the present results using the singular elements agree well with Raju-Newman's solutions within 3 to 4% difference.

In practice, small surface cracks are formed and coalesced before being developed as a through-wall crack. Because of complexities in experimental procedures and difficulties in numerical analyses, there is little research dealing with interaction effects of two surface cracks. The present system is applied to solve the SIFs of twin surface cracks in inhomogeneous plate as shown in Fig. 7(b).

Table 1. Difference present S.I.F from Raju-Newman solutions at the deepest point (a/t=0.2)

a/c	$F=\frac{K_I}{\sigma\sqrt{\pi a/Q}}$		
	Present Sol.	Raju-Newman Sol.	Difference (%)
0.2	1.133	1.173	3.41
0.4	1.100	1.138	3.33
0.6	1.077	1.110	2.97
0.8	1.050	1.049	0.95

Table 2 shows a material properties of base and cladding materials. Fig. 9 presents analysis results for thickness ratio, t'/t and spacing, c/d at the surface point. This figure shows that the K value is gradually increased as the thickness ratio is increased. The interaction effect gradually increases when approaching the crack tip. Also, for contact case, the increase of K value was more noticeable.

Table 2. Material properties

Base materials [Steel]		Cladding materials [Stainless steel]	
Young's modulus E [GPa]	Poisson's ratio	Young's modulus E [GPa]	Poisson's ratio
170	0.3	190	0.3

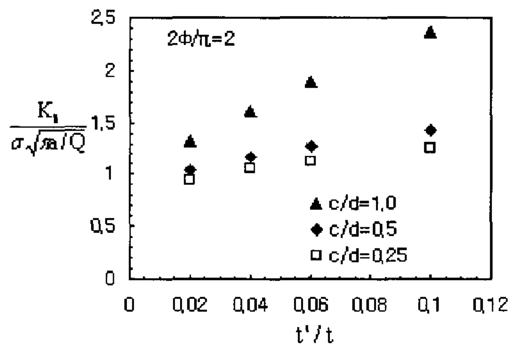


Fig. 9. S.I.F at the surface point(2Φ/π=2) for twin surface cracks in inhomogeneous plate

5. Conclusions

A new automated SIF analysis system for 3D cracks in inhomogeneous materials was developed. The automatic finite element mesh generation technique based on the fuzzy theory was integrated in the system, together with one of commercial finite element programs. Here interactive operations to be done by a user can be performed in about few minutes even for complicated problems of surface crack in a inhomogeneous materials. To demonstrate practical performances of the present system, the system was used to the analyses of surface crack in inhomogeneous plate subjected to uniform tension.

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