Determination of stress state in chip formation zone by central slip-line field

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Stress state of chip formation zone is one of the main problems in metal cutting mechanics. In two-dimensional case this process is usually considered as consistent shears of work material along one of several shear surfaces, separating chip from workpiece. These shear planes are assumed to be trajectories of maximum shear stress forming corresponding slip-line field. This paper suggests a new approach to the constriction of slip-line field, which implies uniform compression in chip formation zone. Based on the given model it has been found that imaginary shear line in orthogonal cutting is close to the trajectory of maximum normal stress and the problem about its determination has been considered as well. It has been shown that there is a second central slip-line field inside chip, which corresponds well to experimental data about stress distribution on tool rake face and tool-chip contact length. The suggested model would be useful in understanding mechanistic problems in machining.

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NOMENCLATURE

k_s = value of plasticity corresponding to yield strength
k_f = value of plasticity corresponding to fracture strength
σ_y = yield strength of work material
ε = shear strain
σ_0 , n = parameters of hardening
a = undeformed chip thickness
a_1 = chip thickness
$\xi = a_1/a$ = chip thickness coefficient
α = tool rake angle
Φ = imaginary shear angle
N = normal force on tool rake face
F = friction force on tool rake face
L = tool-chip contact length
σ = average normal stress/hydrostatical pressure
$\sigma_n(x)$, $\tau_n(x)$ = normal and shear stress distribution on tool rake face
$\theta(x)$ = angle between tangent to α-slip line at current point M on tool-chip interface and X-axis
φ = angle between normal to the given contour and X-axis
S_f = true fracture strength of work material

1. Introduction

The importance of the understanding on stress distribution during machining has been emphasized, because the stress state of work material is interrelated with boundary conditions for stress. First of all, it concerns the stress distribution on tool faces, which influences temperature fields inside tool and workpiece, and defines cutting forces and tool strength, wear intensity and other engineering problems. It could be said that determination of stress state of work material in chip formation zone is the fundamental problem of metal cutting mechanics. Many studies have been devoted to solve this problem because of its importance[1-19]. In the case of two-dimensional cutting by a tool with unrestricted rake face, it is a general assumption that chip formation occurs as a result of shearing of work material along single shear surface, separating chip from workpiece. These shear surfaces (or shear lines in two-dimensional consideration) are assumed to be trajectories of maximum shear stress forming corresponding slip-line field. The main difficulty of slip-line method is that slip-line field is suggested in advance according to the engineering guess of author. Undoubtedly, the accuracy of every slip-line solution is determined by the initial suggestion of the model.

The first known model of two-dimensional (orthogonal) cutting suggests that chip formation occurs along single shear plane. This model was developed by Timae[1], Zvorykin[2], Merchant[3], Shaw et. al.[4], Oxley[5] and many other researchers[6-8]. The main problem of this model is that material particles accelerate infinitely in this plane while in reality the deformation occurs in a zone with definite size. To compensate the simplicity of the model, the models with several shear lines were developed[9-11]. However, these solutions concern only the deformation in a primary zone of chip formation. On the other hand, the phenomenon of chip curl suggests an idea that there is a second slip-line field inside chip. Probably Lee and Shaffer[12] were the first that proposed the continuation of plastic deformation after primary shear. Klushin[13] expressed the same opinion and stated that deformation of chip continues all the time when tool and chip contact each other. Kudo[14] proposed different slip-line fields inside chip depending on tool type (with unrestricted and restricted tool rake face), including chip curl and built-up-edge phenomena. However, all these solutions are based on the assumption of the uniform stress distribution on tool-chip interface, which is not really true. A new slip-line solution is suggested in this paper, which is consistent with experimental data of stress state in chip formation.

2. Suggested Slip-Line Field

The suggested slip-line field is presented in Fig.1. Primary deformation zone is composed of central slip-line field FABDEF and triangular field BCD. Point F lies on non-contact chip surface. Central slip-line field FABDEF includes mutually orthogonal family of straight β-slip lines and arcs of α-slip lines. Since chip surface DEF is free form stress, all rays of β-slip lines pass it at 45° this surface DEF to satisfy the condition that normal stress is zero. Line ABC is the initial boundary of primary deformation zone and AF is final one. Point H is imaginary intersection point between free surface and single shear plane. Angle Φ is imaginary shear angle. It is assumed that tangent to the chip surface at point F is parallel to the tool rake face. Since line AF is assumed to be straight and makes an angle 45° with free chip surface according to the boundary conditions of slip-lines, it follows that ∠FAG = 45°. The same condition is applied to the point G on the tool surface.

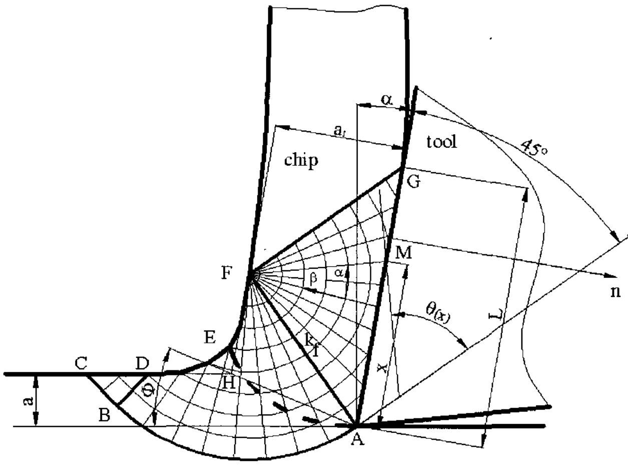

Fig.1 Slip-line solution for orthogonal cutting by tool with unrestricted rake face

There is a second central slip-line field AFG inside chip body, which is continuation of primary deformation zone. Line GF is a final boundary of plastic zone in the chip. It is obvious that at point G, which is the end of tool-chip contact, shear stress on tool-chip contact is zero. Thus according to the boundary conditions for slip lines, this line GF passes tool rake face at angle 45°. Since chip surface is free from stress, line GF passes this surface at the same angle 45° according to the boundary conditions for stress.

In the previous experimental studies[9-11] the primary deformation zone is simplified by central "fan" slip-line field, which is composed of rays of β-slip lines and arcs of α-slip lines, where the "fan" center is located on the tool edge. The same suggestion has been done in our previous work[15]. However this assumption implies that curves of plastic flow in primary deformation zone change their curvature on final boundary of the area (line AF in Fig.1). This situation does not correspond to the real behavior of material in cutting. For this reason, this paper suggests modified slip-line solution for primary deformation zone, as shown in Fig.1. This slip-line field is probably more realistic. In particular, many researchers proved the similar curvilinear view of initial boundary of primary deformation zone as presented in a given slip-line solution. It was shown by metallographic analysis and microhardness tests[16, 20] and by cutting photoelastic materials[17] that initial boundary of primary zone has the same curvilinear character and part of this curve passes under the line of tool path as in the suggested model. In these works it was also shown that final boundary of deformation zone is close to straight line AF presented as shown in Fig.1.

3. Stress State of Chip Formation Zone

It is known that material particles are hardened intensively within primary deformation zone ABCDEF. Stress state of work material on the line ABC can be presented by shear yield strength for given temperature-stress-strain rate conditions of deformation. In the theory of plasticity this shear strength is usually called as the value of plasticity k_s. For its determination, Von Mises criterion is usually applied:

$$k_s = \frac{\sigma_y}{\sqrt{3}}, \tag{1}$$

where σ_y is the yield strength of work material for given temperature-stress-strain rate conditions of deformation.

As material particle moves towards the chip body, the value of plasticity is changed due to the hardening effect. In this case Eq. (1) can be presented in the form:

$$k_s = \frac{\sigma_1 \varepsilon^n}{\sqrt{3}}. \tag{2}$$

The values of parameters σ_l and n depend on temperature-velocity factors of deformation and can be found by special tests[18].

On the final boundary AF of primary deformation zone the material hardening is saturated and chip can be considered as ideal plastic body. In this case the new value of plasticity k_f includes ultimate strength. The saturation of hardening in cutting was proved by microhardness tests of quick-stop chip microsections[16]. It was shown that material hardness does not change after deformation in primary zone (area ABDEF, Fig.1), as a result of ultimate hardening of chip material. The presence of extreme hardness itself was also proved by Rozenberg and Rozenberg[19] using the method of multiple indentations.

It is obvious that work material is compressed in front of the tool. Thus the uniform compression can be expected in area BCD. According to the theory of plasticity, the average normal stress is $\sigma_{BCD} = -k_s$ in this field. Since all β-slip lines in area ABDEF are straight and passing free chip surface DEF at an angle 45°, then according to the properties of slip-lines, the entire zone ABCDEF is under uniform compression.

Since the chip body is also compressed by the tool, the average normal stress on line GF is $\sigma_{GF} = -k_f$. The same normal stress acts on final boundary AF of primary deformation zone: $\sigma_{AF} = -k_f$ because the hardening is assumed to be saturated here. Hence area AFG is under the uniform compression.

Finally, whole deformation area ABCDEFGA of chip formation is the zone of uniform compression. In the primary zone ABCDEFA the value of plasticity changes as a result of deformation hardening. In the zone AFGA the behavior of work material can be considered as ideal plastic deformation under the compression $\sigma = -k_f$ with the value of plasticity k_f.

4. Consideration of Conventional Shear Line Zone

The principal stresses act at 45° to every slip line in the field at every point, which form trajectories of maximum normal stress[20]. Dotted line AE in Fig.1 presents this trajectory from tool cutting edge. It is easy to see that the path of this line is close to the point H that defines the location of conventional "shear" line. Thus from the suggested slip-line model, it follows that conventional shear line is not a shear or slip line but trajectory of maximum normal stress. Since FABDEF is central slip-line field, it is easy to show that line AE is logarithmic curve. From the general relations of theory of plasticity, it can be found that the principal stress

$$\sigma_2 = -2k_2 \tag{3}$$

acts on this line. k_2 implies value of plasticity which explains current strain-strain rate-temperature state of work material along this line.

Since material is ruptured in chip formation, the maximum normal stress σ_2 on trajectory AE may correspond to the ultimate stress state of material, which is its ultimate strength. The last statement must be verified experimentally. However some researchers have already shown that stress in imaginary shear zone close to the true tensile strength or fracture tensile strength of work material[10, 17, 22]. This situation becomes possible when maximum normal stress acts in imaginary shear line and thus this line becomes the actual trajectory of principal stress and not a slip-line.

5. Stress Distribution on Tool Rake Face

The most practically important factor in stress state of chip formation zone is the contact loads on tool-chip interface. The suggested slip-line model gives possibility to determine these loads analytically.

As it has been found above, AFG area (see Fig.1) is the region of uniform compression where average normal stress/hydrostatical pressure $\sigma_{AGF} = -k_f$ acts. From the geometry, the angle $\theta(x)$ (see Fig.1) can be derived according to the distance x from tool edge as:

$$\theta(x) = \arctan\frac{x}{L-x}\cdot \tag{4}$$

X-axis for convenience makes 45° with the rake face.

From the theory of plasticity[20], the normal and shear stresses on the rake face are related with the hydrostatical pressure σ and angle θ in the general form as follows

$$\begin{aligned}\sigma_n &= \sigma - k\cdot\sin 2(\theta-\varphi)\\ \tau_n &= k\cdot\cos 2(\theta-\varphi)\end{aligned}. \tag{5}$$

In our case the contour is concaved along chip surface AG (see Fig.1. It is obvious that in the given case angle φ is equal to $-45°$, thus the law of normal and shear stress distribution on tool rake face are obtained as

$$\begin{aligned}\sigma_n(x) &= k_f\left(1+\cos(2\theta(x))\right)\\ \tau_n(x) &= k_f\,\sin(2\theta(x))\end{aligned}. \tag{6}$$

The general theoretical view of stress distribution is presented in Fig.2 (a) according Eqs. (6). The form of shear stress distribution, that shear stress increases from the tool tip to some maximum value and then decreases to zero at the end of chip-tool contact, was experimentally proved by many researchers[16, 23-25]. Bagchi and Wright[23] carried out the experiments about stress distribution on tool rake face using photoelastic sapphire. They found that in cutting 1020 steel and 12L14 steel the characteristic of stress distribution is independent of the depth of cut and cutting velocity and has the form presented on Fig.2 (b). In this research[23] it is well verified that the shear stress is zero at the tool tip and at the end of chip-tool contact, and the maximum shear stress is located approximately at the center of chip-tool contact. The experimental result can be explained by the suggested slip line field model in Fig.1

According to the Eqs. (6), theoretical normal stress on chip-tool interface is maximum close to the tool tip and then it decreases up to zero at the end of chip-tool contact. The form of normal stress distribution corresponds well to the experimental data especially at the end of tool-chip contact (see Fig.2). Close to the tool tip, experimental curves of normal stress usually have some pick[16, 23-25]. This difference of experimental and theoretical data can be explained by errors of stress measurement by known methods. These methods, especially cutting by split tool, don't consider the action of forces on clearance face, which can be very significant and lead to the serious distortion of real graph of stress distribution, especially close to the tool tip. Neglecting these forces on the clearance face, the "additional" error stresses are added to the real stresses on tool rake face and make wrong picture of stresses. The method of cutting by photoelastic sapphire using by Bagchi and Wright[23] seems to be most reliable because it can reduce the influence of forces acted on tool clearance face in comparison with the split tool method.

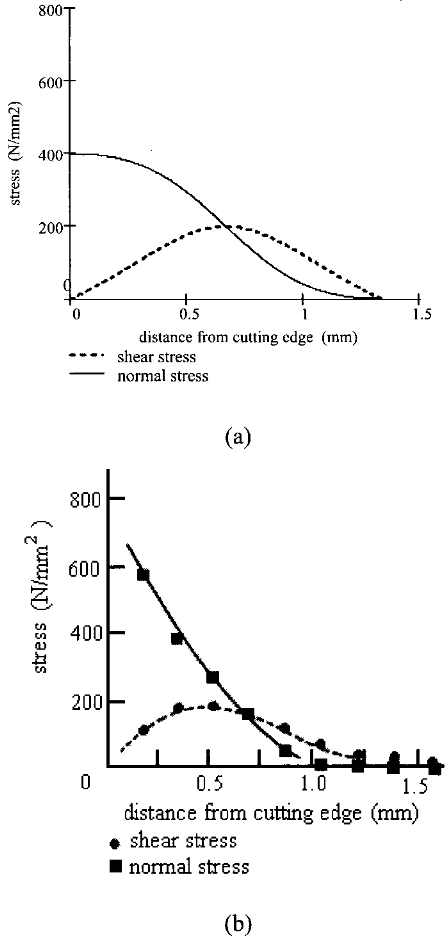

Fig. 2 Theoretical (a) [according to Eqs. (6)] and experimental[22] (b) stress distributions on the tool rake face surface during the machining of 1020 steel at 10 m/min and an uncut chip thickn ess of 0.132 mm (rake angle −5°)

However it also creates some parasitical stresses, which probably are the reason of difference between theoretical and experimental graph of stresses, especially concerning normal stress close to the tool tip.

6. Tool-Chip Contact Length

This is one of the important parameters in machining. From geometry of suggested slip-line solution (see Fig.1) it can be obtained that tool-chip contact length is defined by the formula:

$$L = \frac{2\cdot a\cdot\cos(\alpha-\Phi)}{\sin\Phi}. \tag{7}$$

It was proved[15] that Eq. (7) corresponds well to general experimental formula found by Rozenberg and Rozenberg[19]. It was also shown that Eq. (7) has perfect coincidence with experimental formula by Kato[26]. Our experiments with 4 different materials such as aluminum alloy A6061, copper, carbon steel SM45C and stainless steel STS304 have also shown that Eq. (7) predicts tool-chip contact length very well as shown in Fig.3.

Table 1 Conditions of the experiment

Work material	Cutting velocity (m/min)	Undeformed chip thickness (mm)	Tool rake angle (°)	Tool clearance angle (°)
A6061	200 - 1000	0, 0.1, 0.15, 0.2, 0.25	-5, 0, 5, 10, 20	5
Copper	200 - 800	0, 0.1, 0.15, 0.2, 0.25	-5, 0, 5, 10, 20	5
SM45C	80 - 300	0, 0.1, 0.15, 0.2, 0.25	-5, 0, 5, 10, 20	5
STS304	80 - 140	0, 0.1, 0.15, 0.2	-5, 0, 5, 10, 20	5

The imaginary shear angle Φ can be expressed by chip thickness coefficient ξ as:

$$\Phi = \arctan\left(\frac{\cos\alpha}{\xi - \sin\alpha}\right). \tag{8}$$

Chip thickness coefficient defines a ratio $\xi = a_1/a$, where a_1 is chip thickness (see Fig.1). Eq. (7) can be reduced to the form using Eq. (8)

$$\frac{L}{a} = 2\cdot\xi. \tag{9}$$

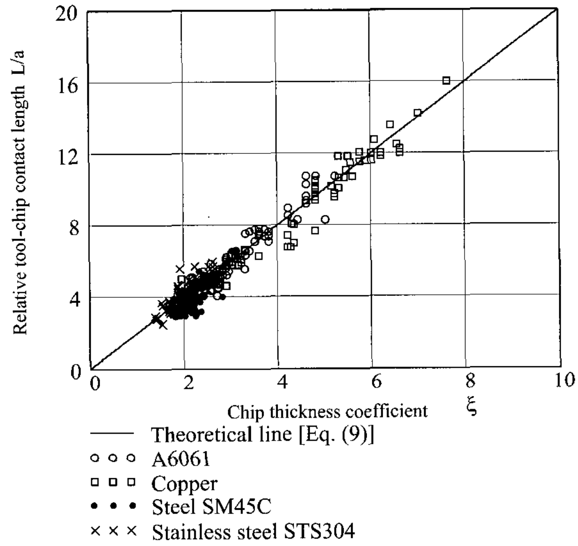

Fig. 3 Comparison of Eq. (9) with experimental data for different materials

Fig. 3 presents theoretical prediction by Eq. (9) and experimental data as dependence of the relative tool-chip contact length L/a on the chip thickness coefficient ξ. The conditions of experiment are listed in Table 1.

Good correspondence of predicted and experimental data proves that contours of suggested slip-line solution can be applied generally for all the work materials, cutting conditions, and rake angles of tools with unrestricted rake face.

7. Determination of Forces on Tool-Chip Interface and the Value of Plasticity

Integration of normal stress $\sigma_n(x)$ and shear stress $\tau_n(x)$ stresses (see Eqs.(6)) on contact length L gives expressions for the normal and friction forces on tool rake face:

$$N = \int_0^L \sigma_n(x)dx = L\cdot k_f, \tag{10}$$

$$F = \int_0^L \tau_n(x)dx = L\cdot k_f\left(\frac{\pi}{2}-1\right). \tag{11}$$

From Eqs. (10) and (11) it follows that average coefficient of friction f on tool-chip interface is independent of cutting conditions and has a constant value

$$f = const = \frac{\pi}{2} - 1. \tag{12}$$

From (10) and (11) it follows that the cutting forces depend on mechanical property of work material (value of plasticity k_f), undeformed chip thickness a, tool rake angle α and conventional shear angle Φ.

It is assumed that the value of plasticity k_f probably corresponds to the ultimate stress state of material when its hardening is almost saturated and its behavior can be considered as ideal plastic. The suggested theory makes it possible to find the value of plasticity k_f and connect it with general theory of plasticity.

From Eq. (7) for tool-chip contact length and Eq. (11) for total friction force, it can be obtained that average shear stress on tool-chip interface is defined as:

$$\tau_{naver} = k_f\cdot\left(\frac{\pi}{2}-1\right). \tag{13}$$

Poletika[16] found experimentally that for 22 different materials as carbon, alloyed and superalloyed steels, copper, aluminum, cadmium and bronze with different hardness, the value of average shear stress on tool-chip interface is independent of cutting conditions and tool geometry and defined by tensile strength S_f of work material at the moment of fracture as

$$\tau_{naver} = const = 0.28\cdot S_f. \tag{14}$$

From the analysis of Eqs. (13) and (14) it can be concluded that the value of plasticity k_f of chip material is constant and defined as:

$$k_f = const = \frac{S_f}{2}. \tag{15}$$

This is very important result, which verifies our suggestion. Eq. (15) represents Tresca criterion of plasticity considering hardening effect during deformation. In other words, Eq. (15) expresses the plastic flow state of material at the extreme moment when material is "ruptured". Probably these conditions occur at the final boundary AF of primary deformation zone (see Fig.1). After passing through primary deformation area ABCDEFA, material reaches to the extreme hardening state at AF and it becomes like an ideal plastic material but it is not ruptured as in tensile test because chip formation zone is compressed and the material particles "weld" or "stick" each other under the compression stress.

Thus, according to Poletika's experimental data, the stress state of chip formation zone on tool-chip interface corresponds to extreme compression stress, and the value of plasticity k_f in cutting is defined by Tresca criterion of plasticity, where tensile strength S_f must be used according to Eq. (15).

8. Conclusion and Future Work

New slip-line solution is suggested in this paper, which gives answer to the various problems of metal cutting mechanics. Using this model, the stress state of chip formation zone is found, especially on tool-chip interface in which the tool and manufacturing industry have great interests. It was suggested that imaginary shear line in cutting possibly corresponds to the trajectory of principle stress. The experimental data regarding tool-chip contact length, and stress distribution on that area show good correspondence to those of the suggested model.

Further experimental research must be done to verify the accuracy of cutting force prediction according to Eqs. (10) and (11), and exact value of plasticity k_f must be checked.

One of the open questions in the model is the analytical determination of imaginary shear angle Φ, which is one of the main parameter in the proposed solution. The analytical way to find this factor must be found to solve all the mechanical problems in cutting without special preliminary experiment, relying only on standard material properties.

Suggested slip-line solution in primary deformation zone will be useful for finding chip form, in particular for determination of chip radius.

Found stress distribution on tool rake face can be applied as a basis for the analysis of tool crater wear, temperature in that area, and tool strength determination.

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