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# A Motion Analysis of FPSO in Irregular Waves including Swells

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**Abstract**

Recently moored offshore vessels like as FPSO(Floating Production Storage Offloading) are frequently deployed in seas for a long time. For successful operation, the motion behavior of such a vessel in waves must be clarified in advance either theoretically or experimentally. It is of particular interest to examine the behavior, when swells are superposed to seas with different incident angle. Such a situation is actually reported in some offshore oilfield. In this paper, the motion of a FPSO in irregular waves including swells is studied in time domain.

Hydrodynamic coefficients and wave forces are calculated in frequency domain using three-dimensional singularity distribution method. Time memory function and added mass at infinite frequency are derived by Fourier transform utilizing hydrodynamic damping coefficients. In the process, the numerical accuracy of added mass at infinite frequency is carefully examined in association with free decay simulations.

It is found from numerical simulations that swells significantly affect the vertical motion of FPSO mainly because of their longer period compared to the ordinary sea waves. In particular, the roll motion is largely amplified because the dominant period of swell is closer to the roll natural period than that of seas.

**Keywords: FPSO, irregular waves, swells, hydrodynamic forces, Fourier inverse transform, time domain analysis**

## 1 Introduction

FPSO usually keeps its position by turret-mooring system, which enables FPSO weathervane to minimize environmental loads. However, large motions can be induced when FPSO is exposed to unpredicted swells coming from different angles. Workout due to swells is actually reported in some offshore oilfields like Brazilian Marlim field and West Africa.

In this study, the motion of FPSO in waves and swells is analyzed. The equation of motion in time domain is derived by Fourier transform. As well known, time domain analysis can easily accommodate different waves simultaneously as well as nonlinear parameters. Numerical simulations are made for realistic environmental conditions in order to examine the motion behavior of a FPSO. A particular attention is paid on the roll motion.

## 2 Mathematical description

The equations of 6 DOF motion of FPSO in frequency domain can be written as

$$-\omega^2\left(M_{ij}+a_{ij}(\omega)\right)X_j(\omega)+i\omega b_{ij}(\omega)X_j(\omega)+C_{ij}X_j(\omega)=F_i(\omega)$$

$$\begin{cases} i=1,2,\cdots,6 \\ j=1,2,\cdots,6 \end{cases} \quad (1)$$

where $M_{ij}$, $a_{ij}(\omega)$, $b_{ij}(\omega)$, $C_{ij}$ and $F_{ij}(\omega)$ are mass, added mass, wave damping, restoring coefficient and wave exciting force matrix, respectively[1]. Here $X_j(\omega)$ denotes the motion response of *j*-th mode. Hydrodynamic coefficients and wave forces are calculated in frequency domain using three-dimensional singularity distribution method. A constant panel method is adopted with sufficient number of panels, which shall ensure the convergence of resulting numerical solutions(Lee and Choi 2000).

The counterpart equation in time domain, (2), is obtained by inverse Fourier transformation of frequency domain equation, (1), and the causality condition.

$$\left(M_{ij}+a_{ij}(\infty)\right)\ddot{X}_j(t)+\int_{-\infty}^{t} k_{ij}(t-\tau)\dot{X}_j(\tau)d\tau+C_{ij}X_j=F_i(t)$$

$$\begin{cases} i=1,2,\cdots,6 \\ j=1,2,\cdots,6 \end{cases} \quad (2)$$

where time memory function, $k_{ij}(t)$, and added mass at infinite frequency, $a_{ij}(\infty)$, can be defined as

$$k_{ij}(t)=\frac{2}{\pi}\int_0^\infty b_{ij}(\omega)\cos\omega t d\omega \quad (3)$$

$$a_{ij}(\infty)=a_{ij}(\omega)+\frac{1}{\omega}\int_0^\infty k_{ij}(t)\sin\omega t dt \quad (4)$$

Wave exciting force, $F_i(t)$, is expressed as like (5) in the case of regular waves and (6) in the case of irregular waves.

$$F_t(t)=\mathrm{Re}\left[F_i(\omega)\cdot|\zeta|e^{i(\omega t+\varepsilon)}\right] \quad (5)$$

$$F_t(t)=\sum_{\omega_k=\omega_1}^{\omega_N}\mathrm{Re}\left[F_i(\omega_k)\cdot|\zeta_k|e^{i(\omega_k t+\varepsilon_k)}\right] \quad (6)$$

where
$F_i(\omega_k)$ : force or moment of $\omega_k$ wave
$|\zeta_k|$ : amplitude of $\omega_k$ wave
$\varepsilon_k$ : random phase of $\omega_k$ wave

When swells approch in addition to waves, the total wave force becomes

$$F_i(t) = F_i(t)_{\text{Head Sea}} + F_i(t)_{\text{Swell}} \tag{7}$$

## 3 Validation of simulation code

Based on (2), a simulation code has been developed, where the 4th-order Runge-Kutta scheme is implemented for time integration. In order to validate the code, free decay and regular motion simulations are carried out. The purpose of the free decay simulation is to confirm the natural period of motion modes and at the same time to estimate the damping coefficient. From the motion simulation in regular waves, the resulting motion response is compared with those obtained from frequency-domain analysis. As will be shown later, the results of time-domain simulation in regular waves correspond quite well with those of frequency-domain analysis. For this purpose, a typical offshore barge was taken, of which specifications are shown in Table 1 and its lateral configuration in Figure 1 [3].

**Table 1:** Particulars of barge

| | |
|---|---|
| L (m) | 150 |
| B (m) | 50 |
| D (m) | 10 |
| V (m3) | 73,750 |
| KG (m) | 10 |
| k44 (m) | 19 |
| k55 (m) | 39 |
| k66 (m) | 39 |

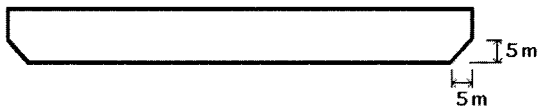


**Figure 1:** Lateral configuration of barge

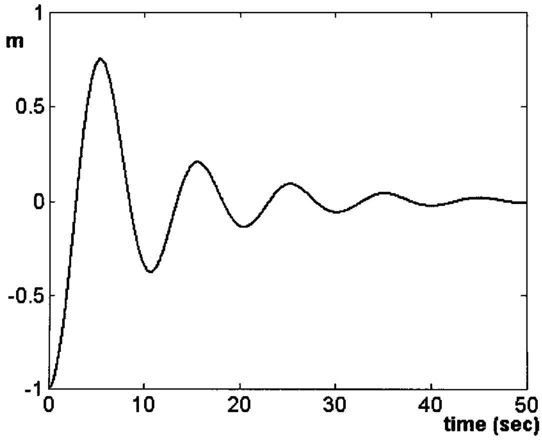


**Figure 2:** Heave free decay simulation

Figure 2 shows the result of heave free decay simulation of the barge. Based on the concept of logarithmic decrement, the damping coefficient is estimated as $b_{33}(\omega_M)/\omega_M \rho V = 0.725$, where $\omega_M$ is the motion frequency and $\rho V$ is the displacement. The natural period measured from the simulation turns out to be 9.9 s, while the calculated value is 10.2s ($T_3 = 2\pi\sqrt{[M + a_{33}(\omega_M)]/C_{33}}$). It is to note that there is an apparent discrepancy between them. The reason for this small difference is supposed to be caused by uncertainty in the value of added mass at infinite frequency. It is well known in ship hydrodynamics that the added mass at infinite frequency is hard to evaluate correctly. Since free decay simulation is indirectly affected by the value of added mass at infinity, it can be assumed that the natural period calculated by the above formula is closer to the "true" value, i.e $T_3 = 10.2$ s. The added mass at infinity frequency can be estimated by using (4), and its accuracy is examined by comparing the natural period.

Simulations in regular waves are carried out in order to validate the code by comparing the result with those obtained from well-established frequency-domain analysis. In the process, numerical filters are introduced in order to avoid numerical over-shootings at the beginning of computation. Motions in regular waves of 0.1~1.2 rad/s are simulated and it is found that the results agree quite well with their counterparts of frequency domain. For example, the motion amplitude for waves of 0.5 rad/s and $180^o$ incident angle is as follows; $0.39m$ for surge, $0.50m$ for heave, $1.07^o$ for pitch, which are almost the same as those resulting from frequency-domain analysis.

# 4 Motion analysis of FPSO

## 4.1 Simulation conditions

Time-domain simulation is made for a FPSO in an oil field. Principal dimensions of full loaded FPSO are listed in Table 2. It is assumed that swells are superposed to seas with different incident angles. In computation, the significant wave height, $H_s$, is assumed to be $1.2m$ with peak period, $T_P$, of 8s. In addition to waves, swells are considered of which the significant wave height ranges from 2.0m to 3.6m with corresponding period of from 12.1s to 15.1s. Ochi-Hubble spectra as given in (8) are used for waves, where the peak enhancement factor, $\lambda$, is taken as 6 for waves and 8 for swells[4]. To examine the effects of swell direction, three incident angles are chosen. These are 0°, 15° and 35°.

**Table 2:** Specifications of FPSO

| | |
|---|---|
| L (m) | 285.0 |
| B (m) | 64.2 |
| D (m) | 23.4 |
| V (m3) | 427,300 |
| KB (m) | 11.8 |
| KG (m) | 15.9 |
| k44 | 0.35 B |
| K55 | 0.25 L |
| k66 | 0.25 L |

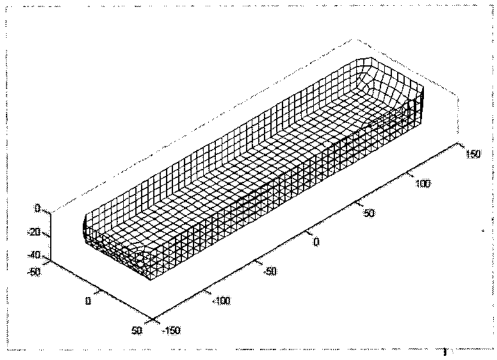


**Figure 3:** Elements of FPSO

$$S(\omega)=\frac{1}{4}\frac{[(4\lambda+1)]\omega_P^4)^{\lambda}}{\Gamma(\lambda)}\frac{H_S^2}{\omega_P^{(4\lambda+1)}}\exp\left[-\left(\frac{4\lambda+1}{4}\right)\left(\frac{\omega_P}{\omega}\right)^4\right] \tag{8}$$

where $\omega_p$ : peak frequency ( $rad/s$ )
$H_s$ : significant wave height ($m$)
$\lambda$ : peak enhancement factor

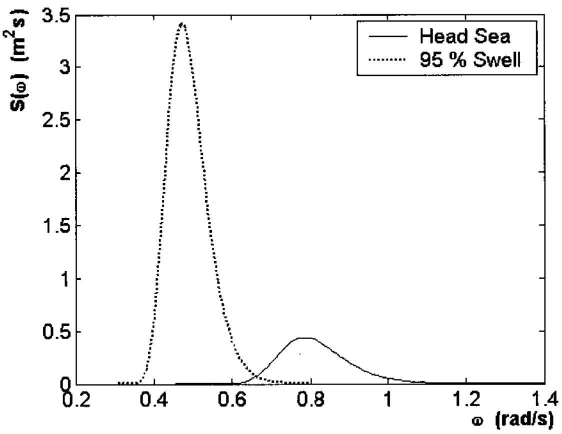


**Figure 4:** Wave spectrum for head waves and swells

## 4.2 Results and discussion

When the FPSO is exposed to head sea waves only, the maximum surge and heave amplitudes are $0.04m$ and $0.02m$, respectively. It implies that the operability of FPSO is not seriously deteriorated by its motion. But the motion increases drastically when swells are superposed as shown in Figure 5-8. It happens mainly because the FPSO responds more sensitively to longer waves than shorter waves.

It is found that the motion amplitude generally increases as the period of swells becomes longer. It is particularly the case for the heave motion. From the figures, the maximum heave amplitude is estimated as $0.73m$ for 10 year swells as shown in Figure 5 and $0.92m$ for 100 year swells as shown in Figure 6. Likely wise the maximum pitch is $0.94^{\circ}$ for 10 year swells and $1.03^{\circ}$ for 100 year swells (see Figure 7 and 8).

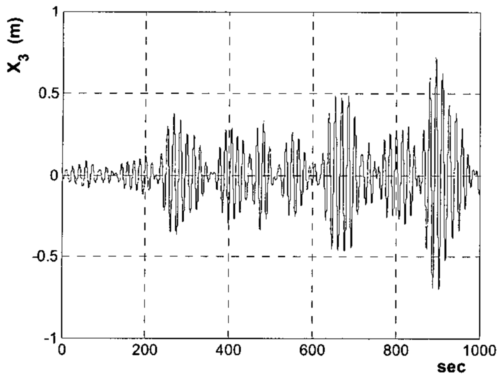


**Figure 5:** Heave motion for head seas with incident angle=0° and 10 year swell

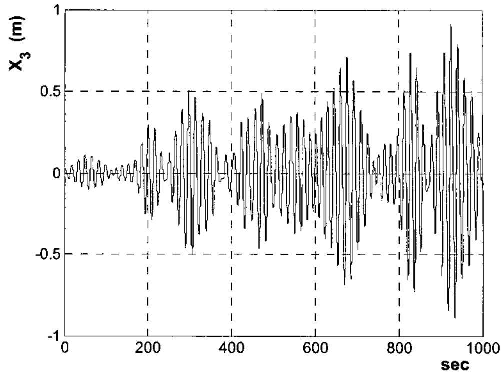


**Figure 6:** Heave motion for head seas with incident angle=0° and 100 year swell

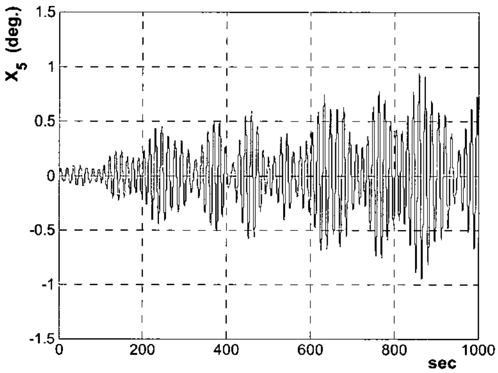


**Figure 7:** Pitch motion for head seas with incident angle=0° and 10 year swell

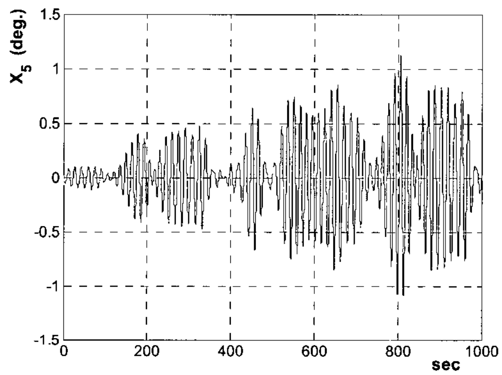


**Figure 8:** Pitch motion for head seas with incident angle=0° and 100 year swell

Sway, roll and yaw motions occur when the direction of swells deviates from the centerline of FPSO, and their magnitudes become larger as the deviation angle increases. In 100 year swells, the maximum sway is $0.20m$ for incident angle 15° and $0.52m$ for incident angle 35° (Figure 9 and 10), while the maximum yaw angle is 0.34° for incident angle 15° and 0.75° for heading angle 35°, respectively as indicated in Figure 11 and 12.

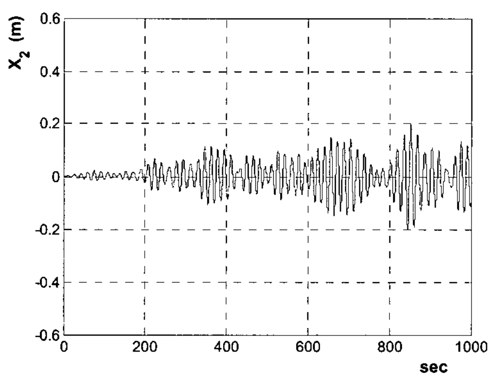


**Figure 9:** Sway motion for head seas with incident angle=15°and 100 year swell

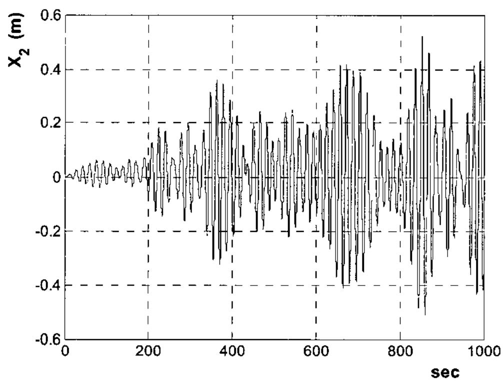


**Figure 10:** Sway motion for head seas with incident angle=35°and 100 year swell

Our special interest lies in the roll motion because roll can easily be amplified due to small damping. It is particularly the case when swells attack FPSO from the beam direction. It is to note that the natural roll period of FPSO is about $T_4 = 16s$ and the mean period of swells is in the range between 12.1s and 15.1s. The maximum roll angle turns out to be as much as 7.5° in waves with significant height of 3.6m and peak period of 15.6 s overlapped by 100 year swells. This result implies that the operation of FPSO must be suspended in this condition. Otherwise anti-roll devices must be provided.

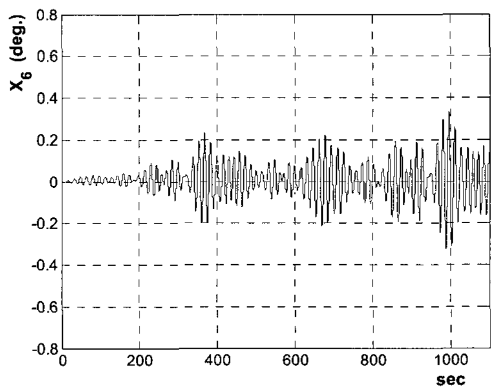


**Figure 11:** Yaw motion for head seas with incident angle=15°and 100 year swell

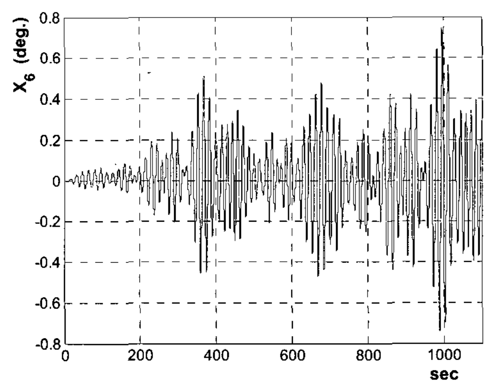


**Figure 12:** Yaw motion for head seas with incident angle=35°and 100 year swell

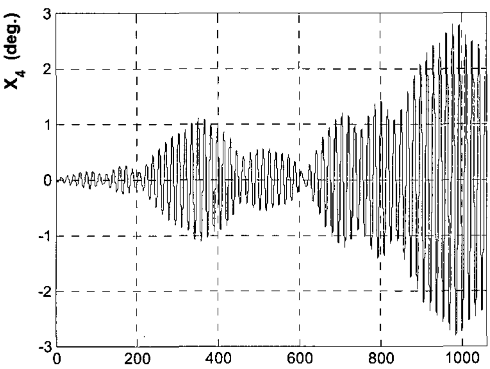


**Figure 13:** Roll motion for head seas with incident angle=35°and 10 year swell

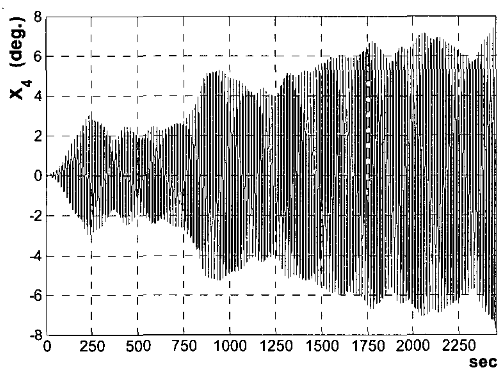


**Figure 14:** Roll motion for head seas with incident angle=35°and 100 year swell

# 5 Concluding remarks

Through motion simulations of a FPSO in waves and swells, it is found that swells affect the vertical motions of FPSO significantly. Sway, roll and yaw motions also occur when the direction of waves or swells deviates from the centerline of FPSO as expected. Among different motion modes, the roll motion is of particular importance from the viewpoint of operability and safety. Based on numerical simulations, it is found that the roll angle of FPSO exceeds the usual criteria in the design weather condition. Therefore the operation must be limited or some anti-roll measures must be made.

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## References

Bhattacharyya, J. 1978. Dynamics of Marine Vehicles, Wiley International.
Faltinsen, O.M. 1990. Sea Loads on Ship and Offshore Structures. Cambridge University Press. Cambridge.
Lee, D.H. and Choi, H.S. 2000. A dynamic analysis of FPSO-shuttle tanker system. 10th International Offshore & Polar Engineering Conference, Seattle.
Pinkster, J.A. 1978. Computation of the First and Second Order Wave Forces on Oscillating Bodies in Regular Waves.