A New Excitation Control for Multimachine Power Systems II: Robustness and Disturbance Attenuation Analysis

Haris E. Psillakis and Antonio T. Alexandridis

Abstract: In this paper a new adaptive, decentralized excitation control scheme proposed to enhance the transient stability of multimachine power systems is extensively analyzed with respect to its robustness and disturbance attenuation. As shown in the paper, both robustness and disturbance attenuation can be effectively improved by suitably selecting the design parameters of the proposed controller. Particularly, some simple rules for the selection of the control gains and the adaptation parameters are extracted which, as it is proven, may be essential for the system performance. Simulation tests on a two generator infinite bus power system absolutely confirm the theoretical results.

Keywords: Multimachine power system control, disturbance attenuation, robust control.

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1. INTRODUCTION

The design of the excitation control of power system generators has been given great attention by many researchers. Designs based on conventional and advanced nonlinear control theory have been effectively used. For details, refer to [1] and the references therein. However, in many cases the design approach is generally developed on mathematical models with fixed structure and parameters without considering system uncertainties or external disturbances. This may be crucial for the system performance and the controller effectiveness [2,3].

In this paper, we examine a new decentralized, adaptive excitation control as proposed in the companion paper [1], assuming that the system operates under parameter uncertainties as well as external disturbances. First, we analyze in detail the proposed controller performance under these circumstances. In both cases we result in some simple rules (Theorems 2 and 3) that indicate that by regulating some design parameters of the proposed controller we can significantly improve the robustness and the disturbance attenuation capability of the system. These rules are taken into account in the design of such controllers for an illustrative example of a multimachine system, and their effectiveness is tested by extensive simulations. As it is expected from the theoretical analysis, the proposed control scheme appears to be stable in the face of parameter uncertainties and external disturbances while it verifies robustness and significant disturbance attenuation capability by providing highly satisfactory responses.

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2. PRELIMINARIES

For an n-generator power system, the dynamic model of the i-th generator is [1]

$$\dot{\delta}_i(t) = \omega_i(t) - \omega_0, \tag{1}$$

$$\dot{\omega}_i(t) = -\frac{D_i}{M_i}\left(\omega_i(t) - \omega_0\right) + \frac{\omega_0}{M_i}\left(P_{mi} - P_{ei}(t)\right), \tag{2}$$

$$\dot{E}'_{qi}(t) = \frac{1}{T'_{d0i}}\left(E_{fi}(t) - E_{qi}(t)\right), \tag{3}$$

where

$$E_{qi}(t) = E'_{qi}(t) + \left(x_{di} - x'_{di}\right)I_{di}(t), \tag{4}$$

$$E_{fi}(t) = k_{ci}u_{fi}(t), \tag{5}$$

$$I_{qi}(t) = \sum_{j=1}^{n} E'_{qj}\left(B_{ij}\sin\delta_{ij}(t) + G_{ij}\cos\delta_{ij}(t)\right), \tag{6}$$

$$I_{di}(t) = \sum_{j=1}^{n} E'_{qj}\left(G_{ij}\sin\delta_{ij}(t) - B_{ij}\cos\delta_{ij}(t)\right), \tag{7}$$

$$P_{ei}(t) = E'_{qi}(t)I_{qi}(t), \tag{8}$$

$$Q_{ei} = E'_{qi}I_{di}(t), \tag{9}$$

$$E_{qi}(t) = x_{adi}I_{fi}(t), \tag{10}$$

$$V_{tqi}(t) = E'_{qi}(t) - x'_{di}I_{di}(t), \tag{11}$$

$$V_{tdi}(t) = x'_{di}I_{qi}(t), \tag{12}$$

$$V_{ti}(t) = \sqrt{V_{tqi}{}^2(t) + V_{tdi}{}^2(t)}. \tag{13}$$

The symbols used in the above equations are explained in the Appendix of [1].

Using the backstepping design and the feedback linearization technique, as it is explained in the companion paper [1], we transform the original system into a partially linear system with state variables z_{i1}, z_{i2}, z_{i3}:

$$\left.\begin{aligned} z_{i1} &= \Delta\delta_i \\ z_{i2} &= \Delta\omega_i + c_{i1}\Delta\delta_i \\ z_{i3} &= \Delta P_{ei} - \frac{M_i}{\omega_0}(1+c_{i1}c_{i2})\Delta\delta_i \\ &\quad - \frac{M_i}{\omega_0}\left(c_{i1}+c_{i2}-\frac{D_i}{M_i}\right)\Delta\omega_i \end{aligned}\right\} \tag{14}$$

and we result in the following excitation control law for each machine

$$E_{fi}(t) = \frac{T'_{d0i}}{I_{qi}}\left(k_{i1}\Delta\delta_i + k_{i2}\Delta\omega_i - k_{i3}\Delta P_{ei} + v_i\right). \tag{15}$$

Controller (15) is fully decentralized since it uses only local measurements (a method for measuring the power angle δ_i can be found in [4]).

The constant gains are given by

$$\begin{aligned} k_{i1} &= \frac{M_i}{\omega_0}c_{i3}\left(1+c_{i1}c_{i2}\right), \\ k_{i2} &= \frac{M_i}{\omega_0}\left[\left(c_{i3}-\frac{D_i}{M_i}\right)\left(c_{i1}+c_{i2}-\frac{D_i}{M_i}\right)+c_{i1}c_{i2}+1\right], \\ k_{i3} &= c_{i1}+c_{i2}+c_{i3}-\frac{D_i}{M_i} \end{aligned} \tag{16}$$

with c_{i1}, c_{i2}, c_{i3} arbitrary positive scalars.

Input $v_i = v_i(t)$ is a controlled external input obtained by an adaptation mechanism as described in Theorem 1 of [1], i.e.

$$v_i = -\frac{\rho_i^2 z_{i3}}{\rho_i |z_{i3}| + l_i}, \tag{17}$$

where l_i a small positive scalar and

$$\rho_i = |c_{i3}-\lambda_i||z_{i3}| + \hat{\sigma}_i(t) + \sum_{j=1}^{3}\hat{\xi}_{ij}(t)|z_{ij}| \tag{18}$$

with λ_i a constant related to the adaptive control design through the following estimates' update laws

$$\dot{\hat{\sigma}}_i(t) = \begin{cases} \alpha_i |z_{i3}|, & \text{if} \quad |z_{i3}| > \sqrt{\dfrac{2l_i}{\lambda_i}}, \\ 0, & \text{otherwise} \end{cases} \tag{19}$$

$\alpha_i > 0 \quad i = 1,2,\ldots,n \quad \text{with} \quad \hat{\sigma}_i(t_0) \geq 0,$

$$\dot{\hat{\xi}}_{ij}(t) = \begin{cases} \gamma_{ij} |z_{i3}z_{ij}|, & \text{if} \quad |z_{i3}| > \sqrt{\dfrac{2l_i}{\lambda_i}}, \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

$\gamma_{ij} > 0 \quad j = 1,2,3 \quad i = 1,2,\ldots,n, \hat{\xi}_{ij}(t_0) \geq 0.$

3. ROBUSTNESS ANALYSIS

Now, consider for the multimachine system the presence of the following uncertain parameters

$$\eta_{i1} = \frac{\omega_0}{M_i}, \quad \eta_{i2} = \frac{D_i}{M_i} \tag{21}$$

with nominal values η_{i1}^*, η_{i2}^* and parameter errors

$$\tilde{\eta}_{i1} = \eta_{i1} - \eta_{i1}^*, \tilde{\eta}_{i2} = \eta_{i2} - \eta_{i2}^*. \tag{22}$$

Then, it can be shown with an analysis similar to that given in [1] that for the control law (15) the zi3-dynamics take the form

$$\dot{z}_{i3} = -c_{i3}z_{i3} + \bar{f}_{i1} + v_i, \tag{23}$$

where

$$\bar{f}_{i1} = \tilde{f}_i + \frac{\left(c_{i1}+c_{i2}-\eta_{i2}^*\right)}{\eta_{i1}^*}\left(\tilde{\eta}_{i2}\Delta\omega_i + \tilde{\eta}_{i1}\Delta P_{ei}\right). \tag{24}$$

The linear terms with respect to $\Delta\omega_i, \Delta P_{ei}$ in $\bar{f}_{i1}$ do not affect our analysis for z_{i3} (Theorem 1 of [1]). This is due to the fact that they only change the values of the unknown constants $\xi_{i1}, \xi_{i2}, \xi_{i3}$ as both $\Delta\omega_i$ and ΔP_{ei} can be written as a linear combination of z_{i1}, z_{i2}, z_{i3}. Defining now

$$\begin{aligned} \bar{f}_{i2} &= \bar{f}_{i1} - \eta_{i1}z_{i2} = \\ &\tilde{f}_i + \frac{\left(c_{i1}+c_{i2}-\eta_{i2}^*\right)}{\eta_{i1}^*}\left(\tilde{\eta}_{i2}\Delta\omega_i + \tilde{\eta}_{i1}\Delta P_{ei}\right) - \eta_{i1}z_{i2}, \end{aligned} \tag{25}$$

it can be easily seen from (25) that an analogous bound for $|\tilde{f}_i(t)|$ as that given in [1], also exists for $|\bar{f}_{i2}(t)|$, i.e. there exist unknown positive constants $\bar{\sigma}_i, \bar{\xi}_{i1}, \bar{\xi}_{i2}, \bar{\xi}_{i3}$ such that

$$|\bar{f}_{i2}(t)| \leq \bar{\sigma}_i + \sum_{j=1}^{3}\bar{\xi}_{ij}|z_{ij}|, \quad i = 1,2,\ldots,n. \tag{26}$$

Now, consider the nonnegative function $\bar{V}_i$

$$\begin{aligned} \bar{V}_i &= \frac{z_{i1}^2}{2} + \frac{z_{i2}^2}{2} + \frac{z_{i3}^2}{2} \\ &+ \frac{\left[\hat{\sigma}_i(t)-\bar{\sigma}_i\right]^2}{2\alpha_i} + \sum_{j=1}^{3}\frac{\left[\hat{\xi}_{ij}(t)-\bar{\xi}_{ij}\right]^2}{2\gamma_{ij}}, \end{aligned} \tag{27}$$

its time derivative for $|z_{i3}| > \sqrt{2l_i/\lambda_i}$ is

$$\begin{aligned}\dot{\bar{V}}_i &= -c_{i1}z_{i1}^2 - (c_{i2}+l_{i22})z_{i2}^2 - c_{i3}z_{i3}^2 \\ &\quad - l_{i12}z_{i1}z_{i2} + z_{i3}\left[\bar{f}_{i2}(t) + v_i(t)\right] \\ &\quad + \left[\hat{\sigma}_i(t) - \bar{\sigma}_i\right]|z_{i3}| + \sum_{j=1}^{3}\left[\hat{\xi}_{ij}(t) - \bar{\xi}_{ij}\right]|z_{i3}z_{ij}|,\end{aligned} \tag{28}$$

where

$$l_{i22} = \tilde{\eta}_{i2} + \frac{c_{i1}+c_{i2}-\overset{*}{\eta}_{i2}}{\overset{*}{\eta}_{i1}}\tilde{\eta}_{i1}, \tag{29}$$

$$l_{i12} = -c_{i1}\tilde{\eta}_{i2} + \frac{1+c_{i1}\overset{*}{\eta}_{i2}-c_{i1}^2}{\overset{*}{\eta}_{i1}}\tilde{\eta}_{i1}. \tag{30}$$

Therefore from (28), the following inequality results

$$\begin{aligned}\dot{\bar{V}}_i &\le -\begin{bmatrix} z_{i1} & z_{i2}\end{bmatrix}\begin{bmatrix} c_{i1} & l_{i12}/2 \\ l_{i12}/2 & c_{i2}+l_{i22}\end{bmatrix}\begin{bmatrix} z_{i1} \\ z_{i2}\end{bmatrix} \\ &\quad - \lambda_i z_{i3}^2 + |z_{i3}|\left[\left|\bar{f}_{i2}(t)\right| - \bar{\sigma}_i - \sum_{j=1}^{3}\bar{\xi}_{ij}|z_{ij}|\right] + z_{i3}v_i(t) \\ &\quad + |z_{i3}|\underbrace{\left[\left|\lambda_i - c_{i3}\right||z_{i3}| + \hat{\sigma}_i(t) + \sum_{j=1}^{3}\hat{\xi}_{ij}(t)|z_{ij}|\right]}_{\rho_i}.\end{aligned} \tag{31}$$

However, since the third term in the right-hand side of the above inequality is nonpositive we can write

$$\begin{aligned}\dot{\bar{V}}_i &\le -\begin{bmatrix} z_{i1} & z_{i2}\end{bmatrix}\begin{bmatrix} c_{i1} & l_{i12}/2 \\ l_{i12}/2 & c_{i2}+l_{i22}\end{bmatrix}\begin{bmatrix} z_{i1} \\ z_{i2}\end{bmatrix} \\ &\quad - \lambda_i z_{i3}^2 + z_{i3}v_i(t) + |z_{i3}|\rho_i\end{aligned} \tag{32}$$

and substituting $v_i(t)$ from (16) and choosing c_{i1}, c_{i2} such that

$$\begin{bmatrix} c_{i1} & l_{i12}/2 \\ l_{i12}/2 & c_{i2}+l_{i22}\end{bmatrix} > 0, \tag{33}$$

we result in

$$\begin{aligned}\dot{\bar{V}}_i &\le -\lambda_i z_{i3}^2 - \frac{\rho_i^2 z_{i3}^2}{\rho_i|z_{i3}| + l_i} + |z_{i3}|\rho_i \\ &= -\lambda_i z_{i3}^2 + \frac{l_i\rho_i|z_{i3}|}{\rho_i|z_{i3}| + l_i},\end{aligned} \tag{34}$$

i.e. for $|z_{i3}| > \sqrt{2l_i/\lambda_i}$ it is

$$\dot{\bar{V}}_i \le -2l_i + \frac{l_i\rho_i|z_{i3}| + l_i^2}{\rho_i|z_{i3}| + l_i} = -2l_i + l_i = -l_i. \tag{35}$$

From (35) it is clear that there exists $\bar{T}_i \le t_0 + \bar{V}_i(t_0)/l_i$, such that for every $t \ge \bar{T}_i$, it holds true that $|z_{i3}(t)| \le \sqrt{2l_i/\lambda_i}$. The uniform boundedness of $z_{ij}(t), \hat{\sigma}_i(t), \hat{\xi}_{ij}(t), j = 1,2,3$ in the time interval $[t_0, \bar{T}_i]$ becomes obvious from the definition of $\bar{V}_i$ in (27). From (19)-(20), it can be seen that for $t > \bar{T}_i$ we have $\hat{\sigma}_i(t) = \hat{\sigma}_i(\bar{T}_i) (i = 1,2,\ldots,n)$ and $\hat{\xi}_{ij}(t) = \hat{\xi}_{ij}(\bar{T}_i)$ $(j = 1,2,3,\ i = 1,2,\ldots,n)$. Hence, the estimates $\hat{\sigma}_i(t), \hat{\xi}_{ij}(t)$ are uniformly bounded for all $t \ge t_0$. Therefore, we have proven the following theorem.

Theorem 1: If the excitation input (15) is applied on the n-machine system described by equations (1)-(13), under the parameter uncertainties of (21) and (22), then, there exists $\bar{T}_i > t_0$ such that

$$|z_{i3}(t)| \le \sqrt{\frac{2l_i}{\lambda_i}} \quad \forall t \ge \bar{T}_i, \quad i = 1,2,\ldots,n$$

and the other two error variables $z_{i1}(t), z_{i2}(t)$ are uniformly bounded for $t \in [t_0, \bar{T}_i]$ and the signals $\hat{\sigma}_i(t), \hat{\xi}_{i1}(t), \hat{\xi}_{i2}(t), \hat{\xi}_{i3}(t)$ are all uniformly bounded for $t \ge t_0$.

The dynamics for the z_{i1}, z_{i2} variables are now given by

$$\begin{cases} \dot{z}_{i1} = z_{i2} - c_{i1}z_{i1} \\ \dot{z}_{i2} = -(1+l_{i12})z_{i1} - (c_{i2}+l_{i22})z_{i2} - \eta_{i1}z_{i3}. \end{cases} \tag{36}$$

So the time derivative of $V_{i1} = z_{i1}^2/2 + z_{i2}^2/2$ for $t \ge \bar{T}_i$ sequentially is

$$\begin{aligned}\dot{V}_{i1} &= -c_{i1}z_{i1}^2 - (c_{i2}+l_{i22})z_{i2}^2 - l_{i12}z_{i1}z_{i2} - \eta_{i1}z_{i2}z_{i3} \\ &\le -c_{i1}z_{i1}^2 - (c_{i2}+l_{i22})z_{i2}^2 - l_{i12}z_{i1}z_{i2} + \eta_{i1}\sqrt{\frac{2l_i}{\lambda_i}}|z_{i2}| \\ &\le -c_{i1}z_{i1}^2 - \left[c_{i2}(1-\varepsilon_i) + l_{i22}\right]z_{i2}^2 - l_{i12}z_{i1}z_{i2} \\ &\quad - \varepsilon_i c_{i2}\left(|z_{i2}| - \frac{\eta_{i1}}{2\varepsilon_i c_{i2}}\sqrt{\frac{2l_i}{\lambda_i}}\right)^2 + \frac{l_i\eta_{i1}^2}{2\varepsilon_i c_{i2}\lambda_i} \\ &\le -\begin{bmatrix} z_{i1} & z_{i2}\end{bmatrix}\begin{bmatrix} c_{i1} & l_{i12}/2 \\ l_{i12}/2 & c_{i2}(1-\varepsilon_i)+l_{i22}\end{bmatrix}\begin{bmatrix} z_{i1} \\ z_{i2}\end{bmatrix} + \frac{l_i\eta_{i1}^2}{2\varepsilon_i c_{i2}\lambda_i}\end{aligned} \tag{37}$$

Choosing large enough c_{i1}, c_{i2}, the matrix

$$P_i = \begin{bmatrix} c_{i1} & l_{i12}/2 \\ l_{i12}/2 & c_{i2}(1-\varepsilon_i)+l_{i22}\end{bmatrix} \tag{38}$$

is positive definite so that for some $\bar{m}_i > 0$, it is

$$P_i \ge \bar{m}_i I \tag{39}$$

and (37) yields

$$\dot{V}_{i1} \le -\bar{m}_i\left(z_{i1}^2 + z_{i2}^2\right) + \frac{l_i\eta_{i1}^2}{2\varepsilon_i c_{i2}\lambda_i} = -2\bar{m}_i V_{i1} + \frac{l_i\eta_{i1}^2}{2\varepsilon_i c_{i2}\lambda_i},$$

$$\dot{V}_{i1} \le -2\bar{m}_i\left(V_{i1} - \frac{l_i\eta_{i1}^2}{4\bar{m}_i\varepsilon_i c_{i2}\lambda_i}\right). \tag{40}$$

Using the comparison principle [5] we have that

$$V_{i1}(t) - \frac{l_i\eta_{i1}^2}{4\bar{m}_i\varepsilon_i c_{i2}\lambda_i} \le \left[V_{i1}(\bar{T}_i) - \frac{l_i\eta_{i1}^2}{4\bar{m}_i\varepsilon_i c_{i2}\lambda_i}\right] e^{-2\bar{m}_i(t-\bar{T}_i)}, \tag{41}$$

$$V_{i1}(t) \le V_{i1}(\bar{T}_i) e^{-2\bar{m}_i(t-\bar{T}_i)} + \frac{l_i\eta_{i1}^2}{4\bar{m}_i\varepsilon_i c_{i2}\lambda_i}, \tag{42}$$

so that for $t \ge \bar{T}_{i1}$ where

$$\bar{T}_{i1} = \max\left\{\bar{T}_i, \bar{T}_i + \frac{1}{\bar{m}_i}\ln\left[\frac{V_{i1}^{1/2}(\bar{T}_i)}{\frac{\eta_{i1}}{2}\sqrt{\frac{l_i}{\bar{m}_i\varepsilon_i c_{i2}\lambda_i}}}\right]\right\}, \tag{43}$$

it holds true that

$$V_{i1}(t) \le \frac{l_i\eta_{i1}^2}{2\bar{m}_i\varepsilon_i c_{i2}\lambda_i}. \tag{44}$$

Thus, we have established the following theorem.

Theorem 2: For a multimachine power system with parameter uncertainties as given by (21) and (22), the control law (15) ensures that the error variables z_{i1}, z_{i2} enter in finite-time inside the sphere

$$S = \left\{(z_{i1}, z_{i2}) \mid z_{i1}^2 + z_{i2}^2 \le \left(\eta_{i1}\sqrt{\frac{l_i}{\bar{m}_i\varepsilon_i c_{i2}\lambda_i}}\right)^2\right\} \tag{45}$$

with the center as the origin and radius

$$\bar{r}_i := \eta_{i1}\sqrt{\frac{l_i}{\bar{m}_i\varepsilon_i c_{i2}\lambda_i}}. \tag{46}$$

Remark 1: For the power angle deviations we have the simple bound form

$$|\Delta\delta_i(t)| \le \eta_{i1}\sqrt{\frac{l_i}{\bar{m}_i\varepsilon_i c_{i2}\lambda_i}} \quad \forall t \ge \bar{T}_i,$$

which clearly indicates that the bigger c_{i1}, c_{i2} are, the smaller the radius $\bar{r}_i$ is.

4. DISTURBANCE ATTENUATION

Power systems are usually under disturbances such as sudden load changes or variations in the input mechanical power, which can be modeled in the ωi-dynamics as an unknown input $d_i(t)$:

$$\dot{\omega}_i(t) = -\frac{D_i}{M_i}(\omega_i(t) - \omega_0) + \frac{\omega_0}{M_i}(P_{mi} - P_{ei}(t)) + \frac{\omega_0}{M_i}d_i(t). \tag{47}$$

Now, for the control law (15) with gains given by (16) the zi3-dynamics take on the form

$$\dot{z}_{i3} = -c_{i3}z_{i3} + \tilde{f}_{i1} + v_i, \tag{48}$$

where

$$\tilde{f}_{i1} = \tilde{f}_i - \left(c_{i1} + c_{i2} - \frac{D_i}{M_i}\right)d_i(t). \tag{49}$$

Defining now

$$\tilde{f}_{i2} = \tilde{f}_{i1} - \frac{\omega_0}{M_i}z_{i2} = \tilde{f}_i - \left(c_{i1} + c_{i2} - \frac{D_i}{M_i}\right)d_i(t) - \frac{\omega_0}{M_i}z_{i2}, \tag{50}$$

it can be easily seen from (50) that an analogous bound for $|\tilde{f}_i(t)|$ as that given in [1] also exists for $|\tilde{f}_{i2}(t)|$, i.e. there exist unknown positive constants $\tilde{\sigma}_i, \tilde{\xi}_{i1}, \tilde{\xi}_{i2}, \tilde{\xi}_{i3}$ such that

$$|\tilde{f}_{i2}(t)| \le \tilde{\sigma}_i + \sum_{j=1}^{3}\tilde{\xi}_{ij}|z_{ij}|, \quad i = 1, 2, \ldots, n. \tag{51}$$

Now, consider the nonnegative function $\tilde{V}_i$

$$\tilde{V}_i = \frac{z_{i1}^2}{2} + \frac{z_{i2}^2}{2} + \frac{z_{i3}^2}{2} + \frac{[\hat{\sigma}_i(t) - \tilde{\sigma}_i]^2}{2\alpha_i} + \sum_{j=1}^{3}\frac{[\hat{\xi}_{ij}(t) - \tilde{\xi}_{ij}]^2}{2\gamma_{ij}}, \tag{52}$$

then, for the case wherein $|z_{i3}| > \sqrt{2l_i/\lambda_i}$, the excitation input is given by eqs. (15)-(18) and the update laws are given by (19)-(20), the time derivative of $\tilde{V}_i$ is as follows

$$\dot{\tilde{V}}_i = -c_{i1}z_{i1}^2 - c_{i2}z_{i2}^2 - c_{i3}z_{i3}^2 + \frac{\omega_0}{M_i}d_i(t)z_{i2} + z_{i3}\left[\tilde{f}_{i2}(t) + v_i(t)\right] + [\hat{\sigma}_i(t) - \tilde{\sigma}_i]|z_{i3}| + \sum_{j=1}^{3}\left[\hat{\xi}_{ij}(t) - \tilde{\xi}_{ij}\right]|z_{i3}z_{ij}|, \tag{53}$$

which results in the inequality

$$\dot{\tilde{V}}_i \le -c_{i1}z_{i1}^2 - \left(c_{i2} - \frac{\omega_0^2 d_i^2(t)}{2M_i^2 l_i}\right) z_{i2}^2 + \frac{l_i}{2} - \lambda_i z_{i3}^2 + |z_{i3}|\left[\left|\tilde{f}_{i2}(t)\right| - \tilde{\sigma}_i - \sum_{j=1}^{3} \tilde{\xi}_{ij}\left|z_{ij}\right|\right] + z_{i3}v_i(t) + |z_{i3}|\underbrace{\left[|\lambda_i - c_{i3}||z_{i3}| + \hat{\sigma}_i(t) + \sum_{j=1}^{3}\hat{\xi}_{ij}(t)|z_{ij}|\right]}_{\rho_i}. \quad (54)$$

The fifth term on the right-hand side of inequality (54) is nonpositive and therefore

$$\dot{\tilde{V}}_i \le -c_{i1}z_{i1}^2 - \left(c_{i2} - \frac{\omega_0^2 d_i^2(t)}{2M_i^2 l_i}\right) z_{i2}^2 + \frac{l_i}{2} - \lambda_i z_{i3}^2 + z_{i3}v_i(t) + |z_{i3}|\rho_i. \quad (55)$$

Substituting $v_i(t)$ from (17) and considering

$$c_{i1} > 0, \quad c_{i2} > \omega_0^2 d_i^2(t)/2M_i^2 l_i, \quad (56)$$

we have

$$\begin{aligned}\dot{\tilde{V}}_i &\le \frac{l_i}{2} - \lambda_i z_{i3}^2 - \frac{\rho_i^2 z_{i3}^2}{\rho_i|z_{i3}| + l_i} + |z_{i3}|\rho_i \\ &= \frac{l_i}{2} - \lambda_i z_{i3}^2 + \frac{l_i\rho_i|z_{i3}|}{\rho_i|z_{i3}| + l_i},\end{aligned} \quad (57)$$

i.e. for $|z_{i3}| > \sqrt{2l_i/\lambda_i}$

$$\dot{\tilde{V}}_i \le \frac{l_i}{2} - 2l_i + \frac{l_i\rho_i|z_{i3}| + l_i^2}{\rho_i|z_{i3}| + l_i} = \frac{l_i}{2} - 2l_i + l_i = -\frac{l_i}{2}. \quad (58)$$

From (58) it is clear that there exists $\tilde{T}_i \le t_0 + 2\tilde{V}_i(t_0)/l_i$ such that for every $t \ge \tilde{T}_i$, it holds true that $|z_{i3}(t)| \le \sqrt{2l_i/\lambda_i}$. The uniform boundedness of $z_{ij}(t), \hat{\sigma}_i(t), \hat{\xi}_{ij}(t), j = 1,2,3$ in the time interval $\left[t_0, \tilde{T}_i\right]$ is obvious from the definition of $\tilde{V}_i$ in (52). From (18)-(19), it can be seen that for $t > \tilde{T}_i$ we have that $\hat{\sigma}_i(t)\ \hat{\sigma}_i(\tilde{T}_i)(i = 1,2,\ldots,n)$ and $\hat{\xi}_{ij}(t) = \hat{\xi}_{ij}(\tilde{T}_i)$ $(j = 1,2,3,\ i = 1,2,\ldots,n)$. Hence, the estimates $\hat{\sigma}_i(t), \hat{\xi}_{ij}(t)$ are uniformly bounded for all $t \ge t_0$.

The dynamics for the z_{i1}, z_{i2} variables are now given by

$$\begin{cases}\dot{z}_{i1} = z_{i2} - c_{i1}z_{i1} \\ \dot{z}_{i2} = -z_{i1} - c_{i2}z_{i2} - \dfrac{\omega_0}{M_i}z_{i3} + \dfrac{\omega_0}{M_i}d_i(t).\end{cases} \quad (59)$$

So the time derivative of V_{i1} for $t \ge \tilde{T}_i$ is

$$\begin{aligned}\dot{V}_{i1} &= -c_{i1}z_{i1}^2 - c_{i2}z_{i2}^2 - \frac{\omega_0}{M_i}z_{i2}z_{i3} + \frac{\omega_0}{M_i}z_{i2}d_i(t) \\ &\le -c_{i1}z_{i1}^2 - c_{i2}z_{i2}^2 + \frac{\omega_0}{M_i}|z_{i2}|\left(|d_i(t)| + \sqrt{\frac{2l_i}{\lambda_i}}\right) \\ &\le -c_{i2}(1-\varepsilon_i)z_{i2}^2 + \frac{\omega_0^2}{4\varepsilon_i c_{i2}M_i^2}\left(|d_i(t)| + \sqrt{\frac{2l_i}{\lambda_i}}\right)^2 \\ &\quad - c_{i1}z_{i1}^2 - \varepsilon_i c_{i2}\left[|z_{i2}| - \frac{\omega_0}{2\varepsilon_i c_{i2}M_i}\left(|d_i(t)| + \sqrt{\frac{2l_i}{\lambda_i}}\right)\right]^2.\end{aligned}$$

Defining $m_i = \min\{c_{i1}, c_{i2}(1-\varepsilon_i)\}$ we sequentially have

$$\begin{aligned}\dot{V}_{i1} &\le -m_i\left(z_{i1}^2 + z_{i2}^2\right) + \frac{\omega_0^2}{4\varepsilon_i c_{i2}M_i^2}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right)^2 \\ &= -2m_iV_{i1} + \frac{\omega_0^2}{4\varepsilon_i c_{i2}M_i^2}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right)^2,\end{aligned}$$

$$\dot{V}_{i1} \le -2m_i\left[V_{i1} - \frac{\omega_0^2}{8m_i\varepsilon_i c_{i2}M_i^2}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right)^2\right], \quad (60)$$

and using the comparison principle [5] we have that

$$\begin{aligned}&V_{i1}(t) - \frac{\omega_0^2}{8m_i\varepsilon_i c_{i2}M_i^2}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right)^2 \\ &\le \left[V_{i1}(\tilde{T}_i) - \frac{\omega_0^2}{8m_i\varepsilon_i c_{i2}M_i^2}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right)^2\right]e^{-2m_i(t-\tilde{T}_i)} \\ &V_{i1}(t) \le V_{i1}(\tilde{T}_i)e^{-2m_i(t-\tilde{T}_i)} + \frac{\omega_0^2}{8m_i\varepsilon_i c_{i2}M_i^2}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right)^2,\end{aligned} \quad (61)$$

so that for $t \ge \tilde{T}_{i1}$ where

$$\tilde{T}_{i1} = \max\left\{\tilde{T}_i, \tilde{T}_i + \frac{1}{2m_i}\ln\left[\frac{8m_i\varepsilon_i c_{i2}M_i^2 V_{i1}(\tilde{T}_i)}{\omega_0^2\left(\|d_i\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right)^2}\right]\right\}, \quad (62)$$

it holds true that

$$V_{i1}(t) \le \frac{\omega_0^2}{4m_i\varepsilon_i c_{i2} M_i^2}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right)^2, \quad (63)$$

i.e. the error variables z_{i1}, z_{i2} enter in finite-time inside the sphere

$$S = \left\{(z_{i1}, z_{i2}) \mid z_{i1}^2 + z_{i2}^2 \le \frac{\omega_0^2}{2m_i\varepsilon_i c_{i2} M_i^2}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right)^2\right\} \quad (64)$$

with the center as the origin and radius

$$\bar{R}_{i1} := \frac{\omega_0}{M_i\sqrt{2m_i\varepsilon_i c_{i2}}}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right). \quad (65)$$

Thus, we have proven the following theorem.

Theorem 3: For the n-machine system with excitation control given by (15) and unknown bounded external disturbances $d_i(t)$, all the error variables and the signals $\hat{\sigma}_i(t)$, $\hat{\xi}_{i1}(t)$, $\hat{\xi}_{i2}(t)$, $\hat{\xi}_{i3}(t)$ are uniformly bounded, and there exists $\tilde{T}_{i1} \ge t_0$ so that for every $t \ge \tilde{T}_{i1}$ it holds true

$$|z_{i3}(t)| \le \sqrt{2l_i/\lambda_i}$$

$$z_{i1}^2(t) + z_{i2}^2(t) \le \frac{\omega_0^2}{2m_i\varepsilon_i c_{i2} M_i^2}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right)^2$$

i.e.

$$|\Delta\delta_i(t)| \le \bar{R}_{i1} = \frac{\omega_0}{M_i\sqrt{2m_i\varepsilon_i c_{i2}}}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right)$$

wherein

$$c_{i1} > 0, \quad c_{i2} > \frac{\omega_0^2 d_i^2(t)}{2M_i^2 l_i}, \quad c_{i3} > 0.$$

Remark 2: From Theorem 3 it is easily seen that we can attenuate the effect of the disturbances by choosing large enough gains c_{i1}, c_{i2} correspondingly.

However, a tighter bound on the angle deviations can be established in a similar way to that used in the proof of Corollary 1 of [1].

Corollary 1: For the n-machine system with excitation control voltages given by (15) and bounded disturbances $d_i(t)$, the following tighter (for $c_{i1} \ge 1$) bounds on the angle deviation limits hold

$$\lim_{t\to\infty}|\Delta\delta_i(t)| \le \bar{R}_{i2} = \frac{\omega_0}{M_i c_{i1}\sqrt{2m_i\varepsilon_i c_{i2}}}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right) \quad (66)$$

wherein the gains are chosen so that

$$c_{i1} > 0, \quad c_{i2} > \frac{\omega_0^2 d_i^2(t)}{2M_i^2 l_i}, \quad c_{i3} > 0.$$

Remark 3: For $\varepsilon_i = 1/2$ and $c_{i1} \ge c_{i2}/2$ the above inequality takes the simple form

$$\lim_{t\to\infty}|\Delta\delta_i(t)| \le \frac{\sqrt{2}\omega_0}{M_i c_{i1} c_{i2}}\left(\|d_i(t)\|_\infty + \sqrt{\frac{2l_i}{\lambda_i}}\right). \quad (67)$$

Inequality (67) implies that the disturbance attenuation capability of the proposed controller is directly related to the product of the design constants c_{i1} and c_{i2}.

Note that for a disturbance with constant steady-state such as a step change occurring, for example by the addition or removal of a load, it is easily proven that

$$\lim_{t\to\infty}\Delta\omega_i(t) = 0,$$
$$\lim_{t\to\infty}\Delta P_{ei}(t) = d_i.$$

Additionally, since from the previous analysis it has been shown that

$$\lim_{t\to\infty}|z_{i3}(t)| \le \sqrt{\frac{2l_i}{\lambda_i}}$$

and since

$$\lim_{t\to\infty} z_{i3}(t) = -\frac{M_i}{\omega_0}(1 + c_{i1}c_{i2})\lim_{t\to\infty}\Delta\delta_i(t) - \frac{M_i}{\omega_0}\lim_{t\to\infty}\left(c_{i1} + c_{i2} - \frac{D_i}{M_i}\right)\Delta\omega_i(t) + \lim_{t\to\infty}\Delta P_{ei}(t),$$

we have that

$$\lim_{t\to\infty}|\Delta\delta_i(t)| \le \bar{R}_i = \frac{\omega_0}{M_i(1 + c_{i1}c_{i2})}\left(|d_i| + \sqrt{\frac{2l_i}{\lambda_i}}\right). \quad (68)$$

In accordance to (67) and (68), the bigger the selection of the values of the product $c_{i1}c_{i2}$ is, the smaller the deviation from the nominal power angle is, avoiding the risk of losing synchronism. For example, for a generator with M = 8sec, the choice of c_1, c_2 so that $c_1 c_2 > 149$ (e.g. $c_1 = 10$ and $c_2 = 15$) and $\delta_i = 0.01$, $\lambda_i = 100$ will lead to less than 80 variation for a 50% change of the input power from its nominal value (1p.u.). The last bound is obviously tighter than (67) (it holds $\bar{R}_{i1} < \bar{R}_{i2} < \bar{R}_i$ since $1/(1+x) < 1/x < \sqrt{2}/x \quad \forall x > 0$) as it is expected due to the additional assumption for the constant steady-state value of the disturbance.

5. CASE STUDIES

The two generator infinite bus power system used in [1] is considered to demonstrate the robustness and disturbance attenuation capability of the system.

The following two cases are simulated.

Case 1: Permanent serious fault with large parameter uncertainties: A symmetrical three phase short circuit fault occurs on one of the transmission lines between Generator #1 and Generator #2. The fault characteristics are exactly the same as that referred to in the case study of [1]. However, in this case major parameter uncertainties are considered for Generator #1. Particularly, while the actual machine parameters are $M_1 = 4$ sec and $D_1 = 3$ p.u., the controller is designed with $M_1^* = 8$ sec and $D_1^* = 5$ p.u. The controllers' parameters are considered to be:

$$c_{11} = 5,\quad c_{12} = 15,\quad c_{13} = 100,\quad \gamma_1 = 10,$$
$$c_{21} = 5,\quad c_{22} = 15,\quad c_{23} = 100,\quad \gamma_2 = 10,$$
$$\lambda_1 = 100,\ \delta_1 = 0.0005,\ \lambda_2 = 100,\ \delta_2 = 0.001$$

Case 2: Unknown step increase of the mechanical power: A step increase of the mechanical input power of Generator #1 by 0.1p.u. is considered (a variation of 10% of the nominal value Pm10 = 1p.u.) at t = 30sec. The controllers' parameters are the same as in Case 1. From these controller parameter values the variation in the power angle for generator #1 is calculated through (68) at 3.05 deg.

The simulation results are given in Figs. 1-6 for Case 1 and in Figs. 7-12 for Case 2. In both cases the design parameters are selected in accordance to the theoretical results obtained from the previous analysis. As expected, the response characteristics are very satisfactory and the whole system performance appears to be clearly robust under both parameter uncertainties and external disturbances.

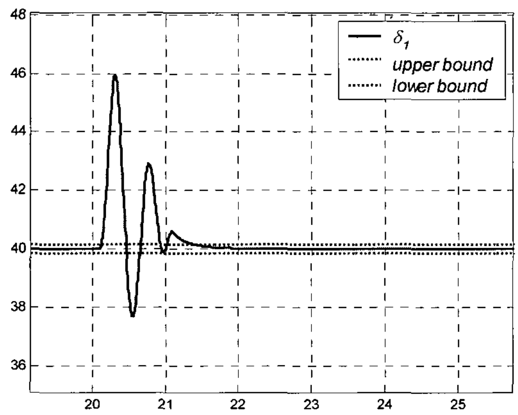

Fig. 1. Power angle response (in deg) for generator #1 (Case 1).

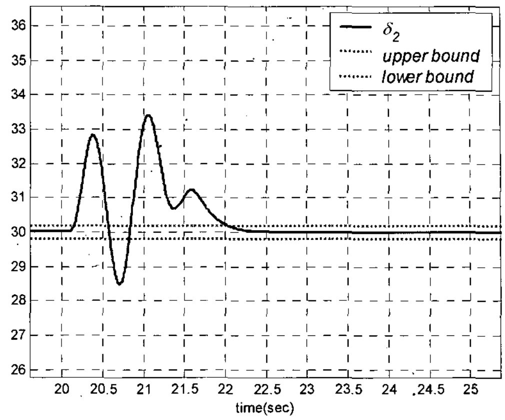

Fig. 2. Power angle response (in deg) for generator #2 (Case 1).

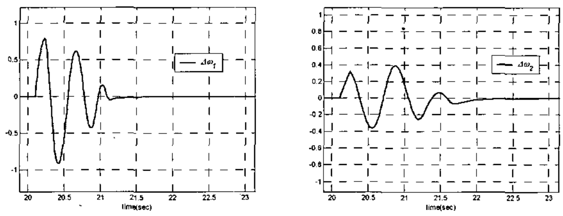

Fig. 3. Rotor speed deviations for machines #1 and #2 respectively (Case 1).

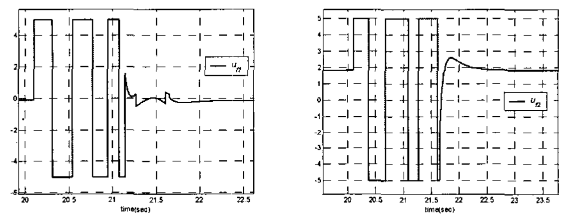

Fig. 4. Excitation voltages for machines #1 and #2 respectively (Case 1).

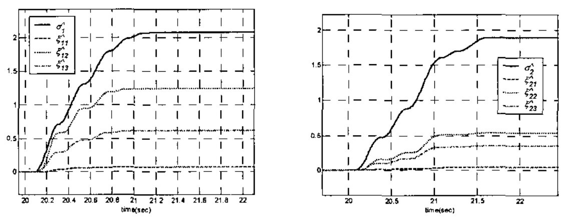

Fig. 5. Estimated parameters for machines #1 and #2 respectively (Case 1).

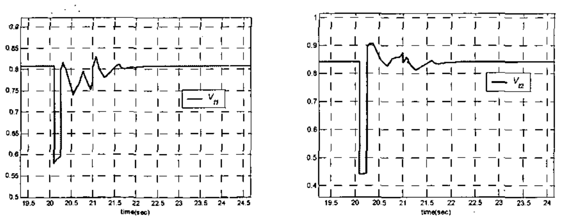

Fig. 6. Terminal voltages for machines #1 and #2 respectively (Case 1).

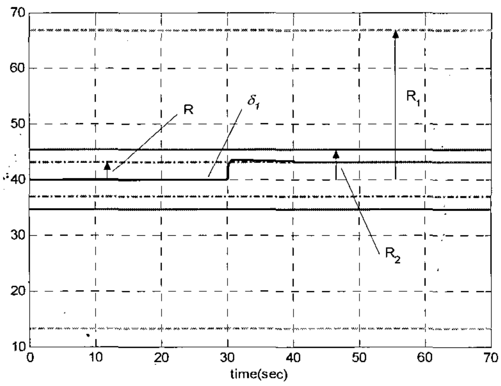

Fig. 7. Power angle response and calculated bounds for generator #1 (Case 2).

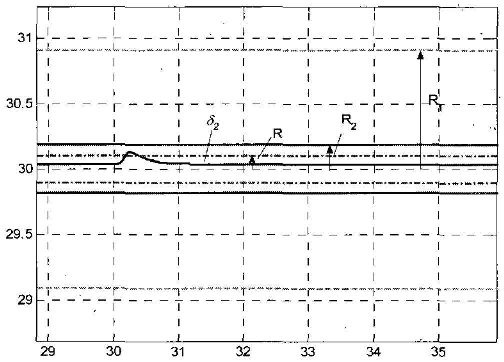

Fig. 8. Power angle response and calculated bounds for generator #2 (Case 2).

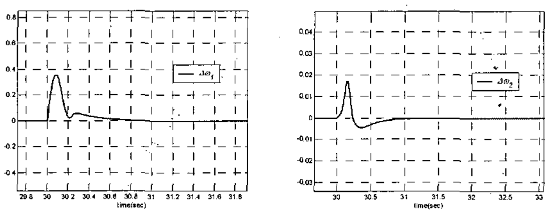

Fig. 9. Rotor speed deviations in machines #1 and #2 respectively (Case 2).

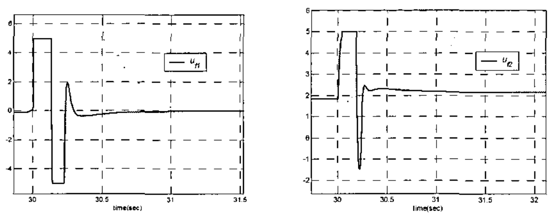

Fig. 10. Excitation voltages for generators #1 and #2 respectively (Case 2).

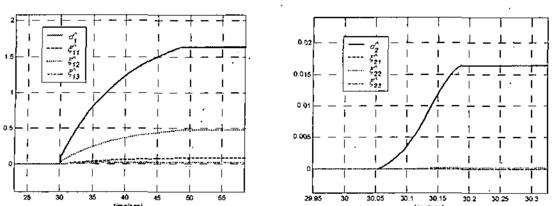

Fig. 11. Estimated parameters for machines #1 and #2 respectively (Case 2).

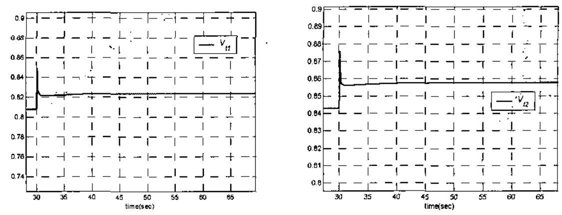

Fig. 12. Terminal voltages for machines #1 and #2 respectively (Case 2).

6. CONCLUSIONS

An adaptive nonlinear excitation controller as proposed in [1] is analysed and tested under large system uncertainties and external disturbances. Theoretical results are proved that constitute the frame for the design of such a controller. This frame actually is a set of simple design rules, associated to the selection of some parameters that ensure robustness and disturbance attenuation. All the simulation results confirm our theoretical outcomes providing a very satisfactory system performance.

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Haris E. Psillakis for photograph and biography, see p. 287 of the June issue (vol. 3, no. 2) of this journal.

Antonio T. Alexandridis for photograph and biography, see p. 287 of the June issue (vol. 3, no. 2) of this journal.