## Recursive Design of Nonlinear Disturbance Attenuation Control for STATCOM

#### Feng Liu, Shengwei Mei, Qiang Lu, and Masuo Goto

Abstract: In this paper, a nonlinear robust control approach is applied to design a controller for the Static Synchronous Compensator (STATCOM). A robust control dynamic model of STATCOM in a one-machine, infinite-bus system is established with consideration of the torque disturbance acting on the rotating shaft of the generator set and the disturbance to the output voltage of STATCOM. A novel recursive approach is utilized to construct the energy storage function of the system such that the solution to the disturbance attenuation control problem is acquired, which avoids the difficulty involved in solving the Hamilton-Jacobi-Issacs (HJI) inequality. Sequentially, the nonlinear disturbance attenuation control strategy of STATCOM is obtained. Simulation results demonstrate that STATCOM with the proposed controller can more effectively improve the voltage stability, damp the oscillation, and enhance the transient stability of power systems compared to the conventional PI+PSS controller.

**Keywords:** Disturbance attenuation, nonlinear robust control, recursive design, STATCOM.

#### 1. INTRODUCTION

Power system control mainly includes three parts: generator control, transmission control and load control. Before FACTS (Flexible AC Transmission Systems) technology was brought forward, power system control focused on the generator part and the load part, as the controllability in electric transmission networks was very weak. The situation has been changed since the development of FACTS technology, which can be utilized to flexibly control the power flow in the electric transmission network. STATCOM is one of the most important devices of FACTS. The ability of STATCOM to support the voltage on the connection point as well as improve system voltage stability has been well documented [1-7]. In fact, with the proper control strategies, STATCOM has numerous benefits on systems, such as enhancing the

transmission ability of lines, improving the transient stability, and damping the oscillation.

Many large-capability STATCOM equipments have been developed and put into operation in the U.S. and Japan [1,5,7]. Conventional PID [8] and PSS supplementary control [3] are employed in the majority of cases. Although they have simple structures and are easy to design, their dynamic responses are comparatively slow, and performance is not very satisfying. Mitsubishi, Hitachi and Toshiba adopted the VQ control method, which is based on instantaneous active and reactive power [2]. Since the time constant of the DC voltage loop and that of the reactive power control loop have the same order of magnitude, and because the system is strongly nonlinear, this decoupling method may stray from what is desired. Additionally, most control methods have not considered the external disturbances to systems. As complex and large-scale nonlinear systems, power systems always suffer various types of external disturbances. So it is important to take account of these disturbances and try to reduce their ill effects on the system. For the sake of endowing the system strong robustness under large disturbances, a robust control model of STATCOM has been established based on the 3rd-order dynamic equations of STATCOM with the consideration of the generator's dynamics and external disturbances.

In this paper, we manage to apply the nonlinear robust control in the sense of  $L_2$ -gain [9] to STATCOM control. Usually, approaches to solve nonlinear robust control depend on the resolution of the Hamilton-Jacobi-Isaacs (HJI) inequality. Since HJI

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inequality is too difficult to solve analytically, it seems unfeasible to obtain nonlinear robust control strategies of STATOM in this way. In this paper, a recursive approach [11] is applied to construct the energy storage function of the system so that the solution of the control problem is acquired, which avoids the difficulty caused by solving HJI inequality.

This paper is organized as follows. The problem description of nonlinear disturbance attenuation control and a brief review of the recursive design method for constructing Lyapunov function are given in Section 2. A robust control model of STATCOM considering the generator's dynamics has been established in Sections 3 and 4. In Section 5, a recursive approach is applied to construct the energy storage function of the system so that the nonlinear disturbance attenuation control strategy of STATCOM is obtained. In Section 6, in order to ensure the effects of the proposed control strategy, a simulation has been carried out in a one-machine, infinite-bus system with a set of STATCOM located on the mid-point of the long distance transmission line. Finally, some conclusions are given.

## 2. NONLINEAR DISTURBANCE ATTENUATION CONTROL

2.1. The problem of nonlinear disturbance attenuation control

In this section, we briefly state the essential concept of the nonlinear disturbance attenuation problem.

Consider a nonlinear affine system with external disturbances in the form of

$$\begin{cases} \dot{x} = f(x) + g_1(x)w + g_2(x)u \\ y = h(x) \end{cases}$$
 (1)

where  $x \in R^n$  is the state vector,  $u \in R^m$  is the control input,  $w \in R^r$  is the exogenous disturbance, and  $y \in R^s$  is the regulation output. The functions f,  $g_1$ ,  $g_2$  are smooth vector fields with corresponding dimensions, and f(0)=0, h(0)=0.

**Problem Statement:** For system (1), the nonlinear disturbance attenuation problem (NDAP) is to construct  $C^1$  state feedback controller u=u(x) such that for a given positive constant  $\gamma$ , the corresponding closed-loop system can satisfy the following  $L_2$  gain dissipative inequality

$$\int_{0}^{T} \|y\|^{2} \le \gamma^{2} \int_{0}^{T} \|w\|^{2} + V(x(0))$$

$$\forall w \in L_{2}(0, T), \quad \forall T > 0$$
(2)

where

$$L_2(0,T) = \{ w \, | \, w \, ; [0,T) \to R^{n \times k}, \; \int_0^T \left\| \mathbf{w} \right\|^2 dt < + \infty \} \; ;$$

 $V(\cdot)$  is a nonnegative storage function to be constructed, x(0) is the initial state, and w is the exogenous disturbance.  $\| \cdot \|$  denotes the usual Euclidean norm for vectors.

The dissipation inequality states that the inputoutput maps have  $L_2$ -gain  $\leq \gamma$  for every initial condition  $x(0)=x_0$ . The  $L_2$ -gain of a system reflects essentially the amplification between the amplitude of the input-output of the system. We can try to enhance robustness and dynamic performance of the system under various disturbances by design of the so-called disturbance attenuation controller. The ability of disturbance attenuation can be valued by the predestined positive number  $\gamma$ . The smaller  $\gamma$  is, the stronger the effect of disturbance attenuation will be. If  $\gamma$  reaches its minimum, then we can acquire the socalled optimal robust control. Obviously, if the value of  $\gamma$  prescribed is too small, which would be less than its existing minimum value, the robust control issue has no solution. To design a control strategy in the sense of disturbance attenuation, we often set a proper positive  $\gamma$  firstly, and then solve the dissipative inequality. From the point of view of engineering design, we do not always seek for the "optimal" robust control law, but seek for an appropriate or satisfactory one.

As well known, the nonlinear disturbance problem can be transformed into that of finding a nonnegative solution of HJI inequality, which is a nonlinear partial differential one. Because it is very difficult to acquire an analytic nonnegative solution of dissipative inequality in a large area from the view of modern mathematics, to seek a new treatment to avoid the obstacle of solving HJI inequality is imperative for design disturbance attenuation controllers of power systems. In reference [11], a novel recursive design approach is proposed to directly obtain the solution of the attenuation control problem by constructing the storage function of the dynamic system instead of solving the HJI inequality, which is suitable to a vast class of engineering control systems.

#### 2.2. A brief review of the recursive design method

This section introduces the basic approach of recursive design in control design [11].

Consider the nonlinear cascade system

$$\begin{cases} \dot{x}_1 = f(x_1) + \phi(x_1, x_2) \\ \dot{x}_2 = u \end{cases}$$
 (3)

where  $x_1,x_2 \in \mathbb{R}$  are state variables,  $u \in \mathbb{R}^m$  is control input,  $f(\bullet)$  and  $\phi(\cdot,\cdot)$  are  $C^1$  mappings with f(0)=0,  $\phi(0,0)=0$ . A basic process of recursive design technique for the control Lyapunov function is described as follows:

**Step 1:** Choose a positive-definite function  $W(x_1)$ 

such that

$$\frac{\partial W}{\partial x_1} f(x_1) < 0 \qquad \forall x_1 \neq 0 \tag{4}$$

holds

**Step 2:** Select functions  $\varphi(x_1, x_2)$  that satisfy

$$\phi(x_1, x_2) = \varphi(x_1, x_2)x_2. \tag{5}$$

**Step 3:** Construct the following positive-definite function

$$V(x) = W(x_1) + \frac{1}{2}x_2^T x_2,$$
(6)

where  $x = [x_1, x_2]^T$ .

Consider that

$$\dot{V}(x) = \frac{\partial W}{\partial x} \dot{x}_1 + x_2^T \dot{x}_2$$

$$= \frac{\partial W}{\partial x} f(x_1) + \frac{\partial W}{\partial x} \varphi(x_1, x_2) + x_2^T u$$

$$= \frac{\partial W}{\partial x} f(x_1) + x_2 \{ \varphi^T(x_1, x_2) \frac{\partial^T W}{\partial x} + u \}.$$
(7)

Let

$$u = -\varphi^{T}(x_{1}, x_{2}) \frac{\partial^{T} W}{\partial x_{1}} - \varepsilon x_{2}, \tag{8}$$

where  $\varepsilon$  is a positive number. Then there is

$$\dot{V}(x) = \frac{\partial W}{\partial x_1} f(x_1) - \varepsilon x_2^2 < 0.$$
 (9)

So it can be seen that the designed control law (8) can asymptotically stabilize system (3), and the positive definite function V(x) is a control Lyapunov function of system (3).

The structure of system (7) is depicted in Fig. 1. In fact, system (7) is composed of two series sub systems: one is the driving system, that is, the integrator  $\dot{x}_2 = u$ ; another is the driven system  $\dot{x}_1 = f(x_1) + \phi(x_1, x_2)$ . If the driven system is not asymptotically stable, then  $x_2$  can be considered as a control input of the driving system firstly, so that a suitable function  $x_2=a_1(x_1)$  could be selected so as to stabilize the driven system. Moreover, we can construct the actual control u such that  $x_2$  converges to  $a_1(x_1)$ . Thus the whole system (3) can be stabilized.

Mentioned above is the recursive design method for constructing the Lyapunov function, which can be extended to a class of more general nonlinear systems with series integrators. It is also called the back-stepping method and has been widely used to design the nonlinear disturbance attenuation controller for STATCOM in the following sections.

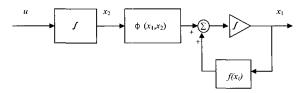


Fig. 1. Aseriescontrolsystem.

# 3. DYNAMIC MODEL OF STATCOM INCLUDING DYNAMICS OF THE GENERATOR

In a transmission system with a long-distance tieline, we usually locate STATCOM at the mid-point of the transmission line. A one-machine, infinite-bus system shown in Fig. 2 is considered in our paper. Fig. 3 presents the equivalent circuit of the system.

According to the realistic operation, we know that the control of the output voltage of STATCOM can be implemented by either regulating the firing angel  $\delta_{svg}$  or conductive angel  $\theta$ . In an actual system, usually only one of these alternative values is regulated. Assuming the energy consumed by the capacity on the DC side of STATCOM only is provided by the AC system, if  $\theta$  is taken as constant, and resistant r is ignored, the differential equation will be predigested to be

$$\dot{V}_{svg} = -\frac{K^2 \sin^2 \frac{\theta}{2}}{Cx} V_s \sin(\delta_{svg} - \alpha). \tag{10}$$

If the 2nd-order classical model of generators is adopted, and the real power loss in the transmission lines is ignored, the dynamic model of a one-machine, infinite-bus system with STATCOM can be expressed by the differential equations as in the following

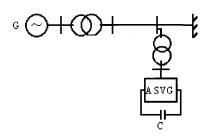


Fig. 2. The one-machine, infinite-bus system with STATCOM.

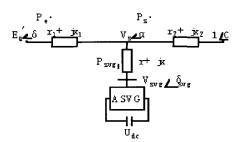


Fig. 3. The equivalent circuit.

$$\dot{\delta}(t) = \omega(t) - \omega_0,$$

$$\dot{\omega}(t) = \frac{\omega_0}{H} (P_m - P_e) - \frac{D}{H} (\omega(t) - \omega_0),$$

$$\dot{V}_{svg} = -\frac{K^2 \sin^2 \frac{\theta}{2}}{Cx} V_s \sin(\delta_{svg} - \alpha).$$
(11)

In this model, the following notations are used:

 $\delta$ : rotor angle

 $\omega$ : rotor speed

H: inertia coefficient of a generator set

D: damping constant  $P_m$ : mechanical power

 $P_e$ : active electric power

 $V_{svg}$ : magnitude of the fundamental frequency component of STATCOM AC voltage

 $\delta_{\text{svg}}$ : firing angel of STATCOM

 $V_s$ : magnitude of the bus voltage on the connection point.

 $\alpha$ : angle of the bus voltage on the connection point

 $\theta$ : conductive angel of STATCOM

K: a proportional constant determined by the linking type of the inverting bridge of STATCOM

C: capacity of STATCOM

x: reactances between the system and STATCOM, including the reactance of the transformer and that of STATCOM

#### 4. THE ROBUST CONTROL MODEL

Like most engineering systems, power systems are always influenced more or less by external disturbances in their operations. Because it is difficult to precisely describe these disturbances by differential equations, an essential task in robust controller design is to attenuate the influence of disturbances on system outputs to a possible extent under the prerequisite condition of system stability, to endow the system with strong robustness [11-13].

In order to apply the approach of nonlinear disturbances attenuation control to STATCOM control, the disturbances to the system should be considered. In fact, the effects of most defaults and actions on power systems are caused by the disturbances to the mechanical power and electric variables of power systems. Although such disturbances have too many types to count, it is not necessary to exactly describe them but only to use a set of generalized disturbances to denote them.

In the model of a one-machine, infinite-bus system with STATCOM, which is mentioned above, all the disturbances can be classified into two kinds: One is  $\varepsilon_1$ , which denotes torque disturbance acting on rotating shaft of the generator; the other is  $\varepsilon_2$ , which denotes the output voltage disturbance of STATCOM.

Then the robust control model of STATCOM includes the dynamics of the generator and can be rewritten as follows

$$\dot{\delta}(t) = \omega(t) - \omega_0,$$

$$\dot{\omega}(t) = \frac{\omega_0}{H} (P_m - P_e) - \frac{D}{H} (\omega(t) - \omega_0) + \frac{\omega_0}{H} \varepsilon_1, \quad (12)$$

$$\dot{V}_{svg} = -\frac{K^2 \sin^2 \frac{\theta}{2}}{Cx} V_s \sin(\delta_{svg} - \alpha) + \frac{K^2 \sin^2 \frac{\theta}{2}}{Cx} \varepsilon_2.$$

The feedback regulation output is chosen as

$$z = \begin{bmatrix} q_1(\delta(t) - \delta_0) \\ q_2(\omega(t) - \omega_0) \end{bmatrix},$$

where  $q_1$  and  $q_2$  are positive weighting constants determined by the designer. They do not influence the level of disturbance attenuation, which is mainly determined by the value of  $\gamma$ .

Now, we shall resolve the control strategy of the firing angel  $\delta_{svg}$  of STATCOM such that the dissipative inequality is satisfied, and sequentially obtain the nonlinear disturbance attenuation control strategy in the sense of  $L_2$ -gain of STATCOM under the disturbances  $\varepsilon_1$  and  $\varepsilon_2$ .

### 5. RECURSIVE DESIGN OF DISTURBANCE ATTENUATION CONTROLLER

Firstly, we set the pre-feedback such as

$$v = -\frac{K^2 \sin^2 \frac{\theta}{2}}{Cx} V_s \sin(\delta_{svg} - \alpha). \tag{13}$$

Namely

$$\delta_{svg} = \alpha + \arcsin(-\frac{Cxv}{K^2 \sin^2 \frac{\theta}{2} V_s}), \tag{14}$$

which is denoted by

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \delta - \delta_0 \\ \omega - \omega_0 \\ V_{svg} - V_{svg0} \end{bmatrix}, a_1 = \frac{\omega_0}{H}, a_2 = -\frac{D}{H},$$

$$a_3 = \frac{K^2 \sin^2 \frac{\theta}{2}}{Cx}, \quad P_m = P_{mo}, \quad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}.$$
 (15)

By selecting the regulation output as

$$z = \begin{bmatrix} q_1 x_1 \\ q_2 x_2 \end{bmatrix} \quad q_1 \ge 0, \quad q_2 \ge 0, \quad q_1^2 + q_2^2 \le 1, \tag{16}$$

the system can be rewritten as:

$$\dot{x}_1 = x_2, 
\dot{x}_2 = a_1(P_{mo} - P_e) + a_2 x_2 + a_1 \varepsilon_1,$$
(17)

$$\dot{x}_3 = v + a_3 \varepsilon_2,$$

$$z = \begin{bmatrix} q_1 x_1 \\ q_2 x_2 \end{bmatrix}.$$

In this system, v can be regarded as a function of x supposed as

$$v = \beta(x_1, x_2, x_3). \tag{18}$$

A coordinate transformation is chosen as

$$\begin{cases} \hat{x}_1 = x_1 \\ \hat{x}_2 = kx_1 + x_2 \\ \hat{x}_3 = \phi(x_1, x_2, x_3) \end{cases}$$
 (19)

where k>0 is a given positive number and  $\phi$  is a smooth function to be designed. If we find  $v=\beta(x_1,x_2,x_3)$  and  $\phi(x_1,x_2,x_3)$  such that the following inequality

$$\int_{0}^{T} \|z\|^{2} dt \le \gamma^{2} \int_{0}^{T} \|\varepsilon\|^{2} dt + V(x_{0}), \quad T > 0$$
 (20)

holds, namely the  $L_2$ -gain is less than or equal to  $\gamma$ , and the robust controller is solvable. Here,  $V(\cdot)$  is the storage function to be constructed, x0 the initial state of the system, and  $\gamma$  a prescribed positive number.

From the coordinate transformation (9), we have

$$\dot{\hat{x}}_{1} = -kx_{1} + \hat{x}_{2},$$

$$\dot{\hat{x}}_{2} = k\dot{x}_{1} + \dot{x}_{2} = (k + a_{2})x_{2} + a_{1}\overline{P}_{e} + a_{1}\varepsilon_{1},$$

$$\dot{\hat{x}}_{3} = \frac{\partial \phi}{\partial x_{1}}\dot{x}_{1} + \frac{\partial \phi}{\partial x_{2}}\dot{x}_{2} + \frac{\partial \phi}{\partial x_{3}}\dot{x}_{3},$$
(21)

where  $\overline{P}_e = P_{mo} - P_e$ .

At first, we let

$$V_1(\hat{x}_1, \hat{x}_2) = \frac{\sigma}{2} \hat{x}_1^2 + \frac{1}{2} \hat{x}_2^2,$$
 (22)

where  $\sigma > 0$  is a positive number to be selected.

Now, let us introduce a function denoted by  $H_1$  defined as

$$H_1 = \dot{V_1} \Big|_{(14)} + \frac{1}{2} \{ \|z\|^2 - \gamma^2 \|\varepsilon_1\|^2 \},$$
 (23)

where  $\dot{V}_1|_{(21)}$  denotes the differential of function  $V_1(\hat{x}_1, \hat{x}_2)$  with respect to time t along the state equation of system (21). Let

$$e = \sigma k - \frac{1}{2}q_1^2 - \frac{1}{2}q_2^2k^2,$$

$$\mu_1 = \frac{1}{\gamma^2}ka_1^2 + \sigma - \frac{1}{2}kq_2^2,$$

$$\mu_2 = \frac{a_1^2}{\gamma^2} + a_2 + k + \frac{1}{2}q_2^2,$$
(24)

and

$$\hat{x}_3 = \phi(x_1, x_2, x_3) = \mu_1 x_1 + \mu_2 x_2 - x_3. \tag{25}$$

Then we know that

$$H_{1} = -e\hat{x}_{1}^{2} - \frac{1}{4}(\gamma \varepsilon_{1} - \frac{2}{\gamma}a_{1}\hat{x}_{2})^{2} - \frac{\gamma^{2}}{4}\varepsilon_{1}^{2}$$

$$+\hat{x}_{2}(a_{1}\overline{P}_{e} + x_{3} + \hat{x}_{3})$$

$$\leq -e\hat{x}_{1}^{2} + \hat{x}_{2}\hat{x}_{3} - \frac{\gamma^{2}}{4}\varepsilon_{1}^{2}$$

$$+\hat{x}_{2}(a_{1}\overline{P}_{e} + x_{3}).$$
(26)

Considering (22), we construct the following nonnegative function

$$V_2(\hat{x}_1, \hat{x}_2, \hat{x}_3) = \frac{\sigma}{2} \hat{x}_1^2 + \frac{1}{2} \hat{x}_2^2 + \frac{1}{2} \hat{x}_3^2$$
 (27)

and define

$$H_{2} = \dot{V}_{2} \Big|_{(21)} + \frac{1}{2} (\|z\|^{2} - \gamma^{2} \|\varepsilon\|^{2})$$

$$= \{\dot{V}_{1} \Big|_{(21)} + \frac{1}{2} (\|z\|^{2} - \gamma^{2} \|\varepsilon_{1}\|^{2})\} - \frac{1}{2} \gamma^{2} \varepsilon_{2}^{2} + \hat{x}_{3} \dot{x}_{3},$$
(28)

where  $|\vec{v}_2|_{(21)}$  denotes the differential of function  $V_2(\hat{x}_1, \hat{x}_2, \hat{x}_3)$  with respect to time t along the state equation of system (21). Then we have

$$H_{2} \leq -e_{1}\hat{x}_{1}^{2} - \frac{1}{4}(\gamma\varepsilon_{1} - \frac{\hat{x}_{3}\mu_{2}a_{1}}{\gamma})^{2} - \frac{1}{2}(\gamma\varepsilon_{2} + \frac{a_{3}\hat{x}_{3}}{\gamma})^{2} + \hat{x}_{2}(a_{1}\overline{P}_{e} + x_{3}) + \hat{x}_{3}\{kx_{1} + (1 + \mu_{1} + \mu_{2}a_{2})x_{2} + \frac{\hat{x}_{3}}{\gamma^{2}}(\mu_{2}^{2}a_{1}^{2} + \frac{a_{3}^{2}}{2}) + \mu_{2}a_{1}\overline{P}_{e} - v\}.$$

$$(29)$$

According to inequality (29), if there is

$$v = kx_1 + (1 + \mu_1 + \mu_2 a_2)x_2 + \mu_2 a_1 \overline{P}_e$$

$$+ \frac{\hat{x}_3}{\gamma^2} (\mu_2^2 a_1^2 + \frac{a_3^2}{2}) + \frac{\hat{x}_2}{\hat{x}_3} (a_1 \overline{P}_e + x_3)$$
(30)

then the following equation will hold

$$H_2 \le -e_1 \hat{x}_1^2. \tag{31}$$

In view of (24), let

$$\sigma \ge \frac{1}{2k} (q_1^2 + q_2^2 k^2). \tag{32}$$

Then there is

 $e_1 \geq 0$ .

(32) and (31) yield that

$$|\dot{V}_2|_{(21)} + \frac{1}{2} \{ ||z||^2 - \gamma^2 ||\varepsilon||^2 \} \le -e_1 \hat{x}_1^2 \le 0.$$
 (33)

That is:

$$\int_{0}^{T} \|z\|^{2} dt \le \gamma^{2} \int_{0}^{T} \|\varepsilon\|^{2} dt + 2V_{2}(x_{0}) - V_{2}(x_{T})$$

$$\le \gamma^{2} \int_{0}^{T} \|\varepsilon\|^{2} dt + 2V_{2}(x_{0}).$$
(34)

Setting

$$V(x) = 2V_2(x),$$

we have

$$\int_0^T \|z\|^2 dt \le \gamma^2 \int_0^T \|\varepsilon\|^2 dt + V(x_0).$$

Now, we have successfully solved the problem of nonlinear disturbance attenuation control in the sense of  $L_2$ -gain for system (21), and the control strategy is

$$\begin{cases} \delta_{svg} = \alpha + \arcsin(-\frac{Cxv}{K^2 \sin^2 \frac{\theta}{2} V_s}) \\ v = kx_1 + (1 + \mu_1 + \mu_2 a_2) x_2 + \mu_2 a_1 \overline{P}_e \\ + \frac{\hat{x}_3}{\gamma^2} (\mu_2^2 a_1^2 + \frac{a_3^2}{2}) + \frac{\hat{x}_2}{\hat{x}_3} (a_1 \overline{P}_e + x_3) \end{cases}$$
(35)

Note that the regulating area of the firing angle  $\delta_{svg}$  is limited by  $\left|\delta_{svg} - \alpha\right| \leq \Delta$ . From the coordinate transformation (19), we have

$$\delta_{svg} = \begin{cases} \alpha + \Delta, & \beta \ge \Delta \\ \alpha + \beta, & -\Delta \le \beta \le \Delta \\ \alpha - \Delta, & \beta \le -\Delta \end{cases}$$
 (36)

where

$$\beta = \arcsin(\frac{Cxv}{K^2 \sin^2 \frac{\theta}{2} V_s}).$$

Finally, the control strategy can be written in the form of

$$v = k \int_{0}^{t} \Delta \omega d\tau + (1 + \mu_{1} + \mu_{2}a_{2})\Delta \omega + \mu_{2}a_{1}(P_{m0} - P_{e})$$

$$+ \frac{\mu_{1} \int_{0}^{t} \Delta \omega d\tau + \mu_{2}\Delta \omega - \Delta V_{svg}}{\gamma^{2}} (\mu_{2}^{2}a_{1}^{2} + \frac{a_{3}^{2}}{2})$$

$$+ \frac{k \int_{0}^{t} \Delta \omega d\tau + \Delta \omega}{\mu_{1} \int_{0}^{t} \Delta \omega d\tau + \mu_{2}\Delta \omega - \Delta V_{svg}} (a_{1}(P_{m0} - P_{e}) + \Delta V_{svg}).$$
(37)

**Remark 1:** In this control strategy,  $\Delta\omega = \omega - \omega_0$ ,  $\Delta V_{svg} = V_{svg} - V_{svg0}$ . Note that the rotor speed is a variable of the generator and cannot directly be acquired locally by STATCOM. Fortunately, the development of PMU and WAMS technology make it feasible in practical operation. The power angel deviation  $\Delta\delta$  can be calculated by  $\Delta\delta = \int_0^t \Delta\omega d\tau$  or directly acquired from PMU. The parameters  $k, \mu_1, \mu_2, a_1, a_2, a_3$ , are determined by equalities (15), (19) and (24).

#### 6. SIMULATION RESULTS

- 1) Description of a one-machine infinite-bus system with STATCOM (see Fig. 4)
  - 2) Parameters of the Generator:

$$H = 12.922, V_s = 1.0, T'_{d0} = 6.55,$$

$$x_d = 0.8285, \ x_d = 0.1045, \ x_T = 0.0292,$$

$$x_{L1} = x_{L2} = x_{L3} = x_{L4} = 0.18$$

3) Parameters of STATCOM:

$$C = 15000uF$$
,  $x_L = 0.9101846\Omega$ ,  $r = 0.1177\Omega$ ,

$$K = 2\sqrt{6} / \pi, \theta = 120^{\circ}, \quad x_T = 0.1 \ (p.u.)$$

4) Parameters of the  $L_2$ -gain Disturbances Attenuation Controller:

$$q1 = 0.04$$
,  $q2 = 0.06$ ,  $k = 4$ ,  $\sigma = 80$ ,  $\gamma = 0.5$ 

- 5) The control strategy of conventional controller is shown in Fig. 5
  - 6) Fault Setting:

To analyze the performance of a power system under large disturbances, we assume that a balanced three-phase short-circuit fault occurs at the position near to the connection point of STATCOM at t=0.1s, and is cleared at t=0.3s.

7) Analysis of Simulation Results:

Considering the prescribed system and fault, we study the different responses of several indices between conventional PID+PSS control as well as the nonlinear disturbance attenuation. The rotor angle, rotor speed, AC voltage of STATCOM, voltage on the connection point, and transmission active and reactive power on the lines are monitored. The following figures indicate the different response curves of those indices.

Figs. 6 and 7 show the responses of rotor angle  $\delta$  and rotor speed  $\omega$ , which are the targets of the distur-

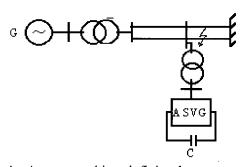


Fig. 4. A one-machine infinite bus system with STATCOM.

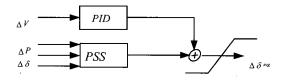


Fig. 5. Block of the conventional controller.

bance attenuation control. With the proposed control method, the system resumes its stability faster than the conventional one. Except for rotor angle  $\delta$  and rotor speed  $\omega$ , voltage is another index of the dynamic performance. So we inspect the responses of the AC voltage of STATCOM and that on the connection point. Figs. 8 and 9 reflect that the two control methods have almost identical voltage responses. The last two figures demonstrate the responses of the active power

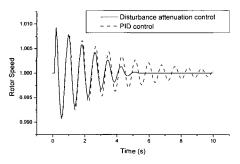


Fig. 6. Rotor response.

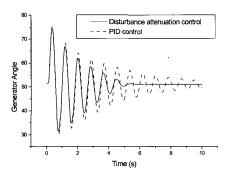


Fig. 7. Angle response.

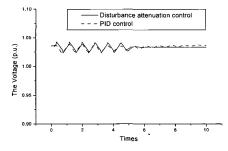


Fig. 8. AC voltage response on the ASVG.

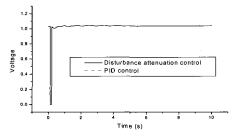


Fig. 9. Voltage response on the point where the fault occurs.

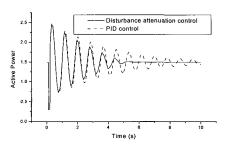


Fig. 10. Active power response of generator.

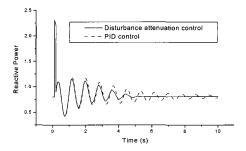


Fig. 11. Reactive power response of generator.

and reactive power. According to the simulation results, it is obvious that, compared to the conventional control method, nonlinear disturbance attenuation control is more effective in damping the oscillation, maintaining the speedy recovery of the system voltage, and limiting the fluctuation of transmission power of the power system under large disturbances.

#### 7. CONCLUSIONS

Numerous approaches of nonlinear robust control have been developed in recent years. However, it is seldom applied to the control of FACTS devices. We introduce the theory of nonlinear disturbance attenuation control in the sense of  $L_2$ -gain into the design of the STATCOM controller. A recursive approach is applied to construct the energy storage function of the system such that the solution of the control problem is acquired, which avoids the difficulty from solving the HJI inequality. Sequentially. the nonlinear disturbances attenuation control in sense of  $L_2$ -gain for STATCOM is obtained. Since the impact of the extend disturbance has been considered and attenuated by control to keep the  $L_2$ -gain less than or equal to a given constant  $\gamma$ , the proposed control strategy has a strong robustness. Simulation results demonstrate the excellent damping and dynamic performance of the OMIB system with the proposed controller.

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