Unified Parametric Approaches for Observer Design in Matrix Second-order Linear Systems

Yun-Li Wu and Guang-Ren Duan

Abstract: This paper designs observers for matrix second-order linear systems on the basis of generalized eigenstructure assignment via unified parametric approach. It is shown that the problem is closely related with a type of so-called generalized matrix second-order Sylvester matrix equations. Through establishing two general parametric solutions to this type of matrix equations, two unified complete parametric methods for the proposed observer design problem are presented. Both methods give simple complete parametric expressions for the observer gain matrices. The first one mainly depends on a series of singular value decompositions, and is thus numerically simple and reliable; the second one utilizes the right factorization of the system, and allows eigenvalues of the error system to be set undetermined and sought via certain optimization procedures. A spring-mass system is utilized to show the effect of the proposed approaches.

Keywords: Matrix second-order linear systems, generalized eigenstructure assignment, parametric methods, observer design.

1. INTRODUCTION

Matrix second-order linear (MSOL) systems capture the dynamic behavior of many natural phenomena, and have found applications in many fields, such as vibration and structural analysis, spacecraft control and robotics control, and hence have attracted much attention [1-11]. In this paper, we consider the control of the following matrix second-order dynamical linear system

$$\begin{cases} M\ddot{q} + D\dot{q} + Kq = Bu \\ y = R\dot{q} + Qq, \end{cases} \tag{1}$$

where $q \in \Re^n$, $u \in \Re^r$ and $y \in \Re^m$ are the state vector, the control vector and the output vector, respectively; M, D, K, B, Q and R are the system coefficient matrices of appropriate dimensions. In certain applications, the matrices M, D and K are usually called the mass matrix, the structural damping matrix and the stiffness matrix, respectively. These coefficient matrices satisfy the following assumption.

Assumption A1: $\text{rank}[R \quad Q] = m$.

Concerning the control of the MSOL system (1), most of the results are focused on stabilization [4,5], eigenstructure assignment [6,7] and the controllability and observability theory [8,9]. However, only a few scholars have paid attention to observer design for matrix second-order systems [10,11]. A type of observers for MSOL systems was proposed in [10] through taking different scenarios of available measurements into consideration. In [11], a class of observers for MSOL systems was constructed by utilizing unknown input observer methods and was used in a robust fault detection scheme and also used as an adaptive detection scheme for a certain class of actuator faults.

Many theoretical results for matrix second-order systems have been developed via the corresponding extended first-order state-space model

$$\begin{cases} \dot{z} = A_e z + B_e u \\ y = C_e z, \end{cases} \tag{2}$$

where

$$z = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad A_e = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \tag{3a}$$

$$B_e = \begin{bmatrix} 0 \\ M^{-1}B \end{bmatrix}, \quad C_e = [Q \quad R]. \tag{3b}$$

Therefore, these results inevitably involve manipulations on $2n$ dimensional matrices A_e, B_e and C_e. While in this paper, the approach developed

Manuscript received April 15, 2004; revised November 26, 2004; accepted February 7, 2005. Recommended by Editorial Board member Jae Weon Choi under the direction of Editor-in-Chief Myung Jin Chung. This work was supported by the Chinese National Natural Science Foundation under Grant No. 69925308.

Yun-Li Wu and Guang-Ren Duan are with the Center for Control Theory and Guidance Technology, Harbin Institute of Technology, Harbin, 150001, China (e-mails: wuyunli@hit.edu.cn, grduan@iee.org, grduan@cctgt.hit.edu.cn).

utilizes directly the original system data *M, D, K, B, Q* and *R*, and thus involves manipulations on *n*-dimensional matrices only.

This paper considers observer design for a class of MSOL systems on the basis of generalized eigenstructure assignment. Base on a series of singular value decompositions and the right factorization of the system, two complete parametric approaches are proposed. Very simple, complete parametric expressions for matrix gains of the observer system are established. These expressions contain a group of parameter vectors that represent the design degrees of freedom. Particularly, with the approach using right factorization, besides this group of parameters, the eigenvalues of the error system may also be treated as part of the design degrees of freedom since they appear as parameters in the expressions of the matrix gains of the observer system, and hence are not necessarily chosen a priori. This approach can provide all the degrees of design freedom that can be further utilized to achieve additional system specifications [12-16].

The paper is composed of 6 sections. Section 2 gives the formulation of the observer design problem for the considered class of MSOL systems, whilst Section 3 presents some preliminary results. The general parametric approaches to a generalized eigenstructure assignment problem is developed in Section 4. In Section 5, an example is presented to demonstrate the utility and the effect of the proposed approaches. Concluding remarks follow in Section 6.

2. PROBLEM FORMULATION

For the MSOL dynamical system (1), consider the following full-order observer

$$(M + L_2R)\ddot{\hat{q}} +(D + L_1R + L_2Q)\dot{\hat{q}} +(K + L_1Q)\hat{q} = Bu + L_1y + L_2\dot{y}, \quad (4)$$

where $L_1, L_2 \in \Re^{n\times m}$ are observer gain matrices; the vectors $\hat{q}, \dot{\hat{q}} \in \Re^n$ are the estimated state and the corresponding estimated state derivative satisfying

$$\lim_{t\to\infty}(\hat{q}-q) = \lim_{t\to\infty}(\dot{\hat{q}}-\dot{q}) = 0\,. \quad (5)$$

Denote $e = \hat{q} - q$, then the error system from (1) and (4) is obtained as follows:

$$\overline{M}\,\ddot{e} + \overline{D}\dot{e} + \overline{K}e = 0\,, \quad (6)$$

where

$\overline{M} = M + L_2R\,, \overline{D} = D + L_1R + L_2Q\,, \overline{K} = K + L_1Q\,.$

Under the following constraint

Constraint C1: $\det\overline{M} \neq 0$,

the error system (6) can be written in the first-order state space form

$$\dot{\tilde{z}} = A_{eo}\tilde{z}\,, \quad (7)$$

where

$$\tilde{z} = \begin{bmatrix} e \\ \dot{e} \end{bmatrix},\ A_{eo} = \begin{bmatrix} 0 & I \\ -\overline{M}^{-1}\overline{K} & -\overline{M}^{-1}\overline{D} \end{bmatrix}. \quad (8)$$

Recall the fact that a non-defective matrix possesses eigenvalues which are less insensitive to the parameter perturbations in the matrix, we here require that the matrix A_{eo} to be non-defective, that is, the Jordan form of the matrix A_{eo} possesses a diagonal form:

$$\Lambda = \mathrm{diag}(s_1, s_2, \ldots, s_{2n})\,,\,. \quad (9)$$

where $s_i, i = 1,2,\ldots,2n$, are clearly the eigenvalues of the matrix A_{eo}.

Lemma 1: Let A_{eo} and Λ be given by (8) and (9), respectively. Then there exist matrices $T_e, T_{\dot{e}} \in C^{n\times 2n}$ satisfying

$$[T_e^T \quad T_{\dot{e}}^T]A_{eo} = \Lambda[T_e^T \quad T_{\dot{e}}^T]\,, \quad (10)$$

if and only if

$$\Lambda^2\overline{T}_{\dot{e}}^T M + \Lambda\overline{T}_{\dot{e}}^T D + \overline{T}_{\dot{e}}^T K + \Lambda Z^T R + Z^T Q = 0 \quad (11)$$

and

$$T_e^T = -\Lambda^{-1}\overline{T}_{\dot{e}}^T(K + L_1Q)\,, \quad (12)$$

where

$$\overline{T}_{\dot{e}}^T = T_{\dot{e}}^T\overline{M}^{-1},\ Z^T = \overline{T}_{\dot{e}}^T L_1 + \Lambda\overline{T}_{\dot{e}}^T L_2\,. \quad (13)$$

Proof: Since equation (10) can be divided into the following equations

$$\begin{cases} -T_{\dot{e}}^T\overline{M}^{-1}\overline{K} = \Lambda T_e^T & (14) \\ T_e^T - T_{\dot{e}}^T\overline{M}^{-1}\overline{D} = \Lambda T_{\dot{e}}^T\,. & (15) \end{cases}$$

Pre-multiplying (15) by Λ and substituting (14) into the obtained equation, yields

$$\Lambda^2 T_{\dot{e}}^T + \Lambda T_{\dot{e}}^T\overline{M}^{-1}\overline{D} + T_{\dot{e}}^T\overline{M}^{-1}\overline{K} = 0\,. \quad (16)$$

Substituting the representations of $\overline{D}$ and $\overline{K}$ into (16), we can obtain (11) after some manipulation by using (13). In addition, Pre-multiplying (14) by Λ^{-1}, the equation (12) is easily obtained. □

The above lemma states that the Jordan matrix of A_{eo} is Λ if and only if there exists $\overline{T}_{\dot{e}} \in C^{n\times 2n}$ satisfying (11) and (12), in this case the corresponding left eigenvector matrix of A_{eo} is given by

$$T_{eo}^T = [T_e^T \quad T_{\dot{e}}^T] = [-\Lambda^{-1}\overline{T}_{\dot{e}}^T\overline{K} \quad \overline{T}_{\dot{e}}^T\overline{M}]\,. \quad (17)$$

With the above understanding, the problem of observer design in the second-order dynamical system (1) can be stated as follows.

Problem OD (Observer Design)**:** Given system (1) satisfying Assumption A1, and the matrix Λ given in (9), with s_i, $i=1,2,\ldots,2n$, being a group of self-conjugate complex numbers (not necessarily distinct), find a general parametric form for the matrices $L_1, L_2 \in \Re^{n\times m}$ and $T_{eo} \in C^{2n\times 2n}$ such that the matrix equation (11) and $\det T_{eo} \neq 0$ hold.

On the other hand, after transpose on the both sides of (11), we can obtain

$$M^T\overline{T}_{\dot{e}}\Lambda^2 + D^T\overline{T}_{\dot{e}}\Lambda + K^T\overline{T}_{\dot{e}} + R^T Z\Lambda + Q^T Z = 0 .(18)$$

Clearly, equation (18) becomes the type of generalized Sylvester matrix equation investigated in [14,15] when $M=0$ and $R=0$. Due to this fact, the equation (18) is called the second-order generalized Sylvester matrix equation.

It follows from the above deduction that, once a pair of matrices $\overline{T}_{\dot{e}}$ and Z satisfying the second-order generalized Sylvester matrix equation (18) and the condition $\det T_{eo} \neq 0$ are obtained, a pair of observer gain matrices can be easily obtained from the second equation in (13) as follows:

$$[L_1^T \quad L_2^T]^T = [\overline{T}_{\dot{e}}^T \quad \Lambda\overline{T}_{\dot{e}}^T]^{-1} Z^T . \tag{19}$$

Therefore, to solve Problem OD, the key step is to find a solution to the following problem.

Problem SGSE (Second-order Generalized Sylvester Equation)**:** Given the matrices, $M, D, K \in \Re^{n\times n}$, $Q, R \in \Re^{m\times n}$ satisfying Assumption A1, and a diagonal matrix (9) find a parameterization for all the matrices $\overline{T}_{\dot{e}} \in C^{n\times 2n}$ and $Z \in C^{m\times 2n}$ satisfying the matrix equation (18).

3. SOLUTION TO PROBLEM SGSE

Denote

$$\overline{T}_{\dot{e}} = [t_1 \quad t_2 \quad \cdots \quad t_{2n}], \tag{20}$$

$$Z = [z_1 \quad z_2 \quad \cdots \quad z_{2n}], \tag{21}$$

then, in view of (9), we can convert the second-order generalized Sylvester matrix equation (18) into the following column form

$$(s_i^2 M^T + s_i D^T + K^T)t_i + (s_i R^T + Q^T)z_i = 0,\quad i=1,2,\ldots,2n . \tag{22}$$

3.1. Case of prescribed s_i, $i=1,2,\ldots,2n$

The equations in (22) can be further written in the following form

$$\Phi_i\begin{bmatrix} t_i \\ z_i \end{bmatrix} = 0,\quad i=1,2,\ldots,2n, \tag{23}$$

where

$$\Phi_i = [s_i^2 M^T + s_i D^T + K^T \quad s_i R^T + Q^T],\quad i=1,2,\ldots,2n . \tag{24}$$

This states that

$$\begin{bmatrix} t_i \\ z_i \end{bmatrix} \in Ker\Phi_i,\quad i=1,2,\ldots,2n . \tag{25}$$

The following algorithm produces two sets of constant matrices N_i and D_i, $i=1,2,\ldots,2n$, to be used in the representation of the solution to the matrix equation.

Algorithm P1 (Solving N_i and D_i, $i=1,2,\ldots,2n$):

Step 1: Applying SVD to the matrix Φ_i, i=1,2…, $2n$ gives

$$P_i\Phi_i Q_i = \begin{bmatrix} \mathrm{diag}[\sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_{n_i}] & 0 \\ 0 & 0 \end{bmatrix}, \tag{26}$$

where $P_i \in C^{n\times n}$ and $Q_i \in C^{(n+m)\times(n+m)}$ are two orthogonal matrices; $\sigma_j > 0$, $j=1,2,\ldots,n_i$, are the singular values of Φ_i, and

$$n_i = \mathrm{rank}[s_i^2 M^T + s_i D^T + K^T \quad s_i R^T + Q^T],\quad i=1,2,\ldots,2n . \tag{27}$$

Step 2: Obtain the matrices $N_i \in \Re^{n\times(n+m-n_i)}$ and $D_i \in \Re^{m\times(n+m-n_i)}$, $i=1,2,\ldots,2n$, by partitioning the matrix Q_i as follows:

$$Q_i = \begin{bmatrix} * & N_i \\ * & D_i \end{bmatrix},\quad i=1,2,\ldots,2n . \tag{28}$$

As a result of (26), the matrices $N_i \in \Re^{n\times(n+m-n_i)}$ and $D_i \in \Re^{m\times(n+m-n_i)}$, $i=1,2,\ldots,2n$, obtained through the above Algorithm P1 satisfy

$$\Phi_i\begin{bmatrix} N_i \\ D_i \end{bmatrix} = 0,\quad i=1,2,\ldots,2n . \tag{29}$$

This indicates that the columns of $[N_i^T \quad D_i^T]^T$ form a set of basis for $Ker\Phi_i$.

The above deduction clearly yields the following result.

Theorem 1: Let n_i, $i=1,2,\ldots,2n$, be defined by (27), and $N_i \in \Re^{n\times(n+m-n_i)}$ and $D_i \in \Re^{m\times(n+m-n_i)}$, $i=1,2,\ldots,2n$, be obtained via Algorithm P1. Then all the matrices $\overline{T}_{\dot{e}}$ and Z satisfying the second-order generalized Sylvester matrix equation (18) can be parameterized by columns as follows:

$$\begin{bmatrix} t_i \\ z_i \end{bmatrix} = \begin{bmatrix} N_i \\ D_i \end{bmatrix} f_i, \quad i = 1,2,\ldots,2n, \tag{30}$$

where $f_i \in C^{n+m-n_i}$, $i = 1,2,\ldots,2n$, are a set of arbitrary parameter vectors.

Regarding the observability of system (1), we have the following lemma.

Lemma 2 [9]: System (1) is observable if and only if

$$\text{rank}\begin{bmatrix} sR+Q \\ s^2M+sD+K \end{bmatrix} = n, \quad \forall s \in C. \tag{31}$$

Based on the above lemma, the following corollary of Theorem 1 can be immediately derived.

Corollary 1: Let system (1) be observable, and Λ be given by (9), then the degrees of freedom existing in the general solution to the second-order generalized Sylvester matrix equation (18) is $2n \times m$.

Proof: Due to the observability of system (1), we have from Lemma 2 $n_i = n$, $i = 1,2,\ldots,2n$. Thus the conclusion immediately follows from Theorem 1. □

3.2. Case of undetermined s_i, $i = 1,2,\ldots,2n$

By performing the right factorization of

$$G(s) = (s^2M^T + D^Ts + K^T)^{-1}(R^Ts + Q^T),$$

we can obtain a pair of real coefficient polynomial matrices $N(s) \in \Re^{n\times m}[s]$ and $D(s) \in \Re^{m\times m}[s]$ satisfying

$$(s^2M^T + D^Ts + K^T)^{-1}(R^Ts + Q^T) = -N(s)D^{-1}(s). \tag{32}$$

Theorem 2: Let system (1) be observable, and $N(s) \in \Re^{n\times m}[s]$ and $D(s) \in \Re^{m\times m}[s]$ satisfying the right factorization (32). Then

1) The matrices $\overline{T}_{\dot{e}}$ and Z given by (20), (21) and

$$\begin{bmatrix} t_i \\ z_i \end{bmatrix} = \begin{bmatrix} N(s_i) \\ D(s_i) \end{bmatrix} f_i, \quad i = 1,2,\ldots,2n \tag{33}$$

satisfy the second-order generalized Sylvester matrix equation (18) for all $f_i \in C^m$.

2) When

$$\text{rank}\begin{bmatrix} N(s_i) \\ D(s_i) \end{bmatrix} = m, \quad i = 1,2,\ldots,2n \tag{34}$$

hold, (33) gives all the solutions to Problem SGSE.

Proof: It follows from (32) that

$$(s_i^2M^T + s_iD^T + K^T)N(s_i) + (s_iR^T + Q^T)D(s_i) = 0. \tag{35}$$

Using (33) and (35) yields

$$\begin{aligned}&(s_i^2M^T + s_iD^T + K^T)t_i + (s_iR^T + Q^T)z_i \\ &= [(s_i^2M^T + s_iD^T + K^T)N(s_i) + (s_iR^T + Q^T)D(s_i)]f_i \cdot \\ &= 0, \ i = 1,2,\ldots,2n.\end{aligned}$$

This states that the equations in (22) hold. Therefore, the first conclusion of the theorem is true. □

It follows from Corollary 1 that, under the observability of system (1), the degrees of freedom existing in the general solution to the matrix equation (18) with Λ given by (9) is $2nm$, while in the solution (33), the number of free parameters just equal to $2nm$. Further, it is clear that all the parameters in the solution (33) have contributions when condition (34) holds. With this we complete the proof.

4. SOLUTION TO PROBLEM OD

Regarding solutions to Problem OD, we have the following results based on the discussion in Section 2 and the results in Section 3.

Theorem 3: Let n_i, $i = 1,2,\ldots,2n$, be defined by (27), and $N_i \in \Re^{n\times(n+m-n_i)}$ and $D_i \in \Re^{m\times(n+m-n_i)}$, i=1,2…,2n, be given by Algorithm P1. Then

1) Problem OD has solutions if and only if there exist a group of parameters $f_i \in C^{n+m-n_i}$, i=1,2…,2n, satisfying Constraint C1 and the following constraints:

Constraint C2: $f_i = \bar{f}_j$ if $s_i = \bar{s}_j$,

Constraint C3: $\det T_{eo} \neq 0$,

Constraint C4a: $\det \overline{T}_{oa} \neq 0$ with

$$\overline{T}_{oa} = \begin{bmatrix} N_1f_1 & N_2f_2 & \cdots & N_{2n}f_{2n} \\ s_1N_1f_1 & s_2N_2f_2 & \cdots & s_{2n}N_{2n}f_{2n} \end{bmatrix}.$$

2) When the above conditions are met, all the solutions to the Problem OD are given by

$$[L_1^T \quad L_2^T]^T = \overline{T}_{oa}^{-T}[D_1f_1 \quad D_2f_2 \quad \cdots \quad D_{2n}f_{2n}]^T \tag{36}$$

and

$$T_{eo} = [T_e^T \quad T_{\dot{e}}^T]^T = F(f_i, s_i), \tag{37}$$

where $F(*)$ represents a nonlinear function matrix with respect to the variable $*$ and its expression is shown in (17).

Theorem 4: Let system (1) be observable, and $N(s) \in \Re^{n\times m}[s]$ and $D(s) \in \Re^{m\times m}[s]$ be a pair of polynomial matrices satisfying the right factorization (32). Then

1) Problem OD has solutions if and only if there exist a group of parameters $f_i \in C^m$, $i = 1,2,\ldots,2n$, satisfying Constraint C1-C3 and

Constraint C4b: $\det \overline{T}_{ob} \neq 0$ with

$$\overline{T}_{ob} = \begin{bmatrix} N(s_1)f_1 & N(s_2)f_2 & \cdots & N(s_{2n})f_{2n} \\ s_1N(s_1)f_1 & s_2N(s_2)f_2 & \cdots & s_{2n}N(s_{2n})f_{2n} \end{bmatrix}.$$

2) When the above conditions are met, all the solutions to the Problem OD are given by

$$[L_1^T \quad L_2^T]^T = \overline{T}_{ob}^{-T}[D(s_1)f_1 \quad D(s_2)f_2 \quad \cdots \quad D(s_{2n})f_{2n}]^T \tag{38}$$

and

$$T_{eo} = [T_e^T \quad T_{\dot{e}}^T]^T = F(\ f_i,\ s_i), \tag{39}$$

where $F(*)$ represents a nonlinear function matrix with respect to the variable $*$ and its expression is shown in (17).

The proof of the above two theorems can be easily carried out based on the discussion in Section 2 and the results in Section 3. The only thing which needs to be mentioned is that Constraint C2 is required because it is a necessary and sufficient condition for the matrices L_1 and L_2 given by (36) or (38) to be real.

In the rest of this section, let us give some remarks to the above results.

Remark 1: It follows from well-known pole assignment result that Problem SGSE has a solution when the system (1) is observable and the eigenvalues of the error system (6) s_i, $i = 1,2,\ldots,2n$, are distinct. In this case, there exist parameter vectors f_i, $i = 1,2,\ldots,2n$, satisfying Constraint C4a or C4b. As a matter of fact, it can be reasoned that, in this case, "almost all" parameter vectors f_i, $i = 1,2,\ldots,2n$, satisfy Constraint C4a or C4b. Therefore, in such applications Constraint C4a or C4b can be often neglected.

Remark 2: The above two theorems give complete parametric solutions to the Problem SGSE. The free parameter vectors f_i, $i = 1,2,\ldots,2n$, represent the degrees of freedom in the eigenstructure assignment design, and can be sought to meet certain desired system performances.

Remark 3: The solution given in Theorem 3 utilizes only a series of singular value decompositions, and hence is numerically very simple and reliable. As for the solution given in Theorem 4, it has the advantage that the eigenvalues of the error system (6) s_i, $i = 1,2,\ldots,2n$, can be set undetermined and used as a part of extra design degrees of freedom to be sought with f_i, i=1,2…,2n, by certain optimization procedures.

Remark 4: Regarding solutions to the right coprime factorization (32), several methods have been given in [13] (see also [14]). Numerical methods can be found in [15,16].

Remark 5: The eigenstructure assignment results can be easily extended into the defective case, that is, the case that the error system possesses a general Jordan form. However, from the control system design point of view, this is not desired since the eigenvalues of defective matrices are more sensitive to parameter perturbations than those of non-defective matrices.

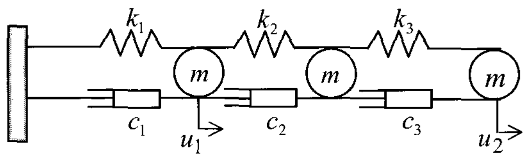

Fig. 1. The mass-spring-dashpots system.

5. AN EXAMPLE

Consider a simple linear dynamical system consisting of three lumped mass-spring-dashpots, connected in series and fixed at one end as shown in Fig. 1. [6]. When $m = 1$, $k_1 = k_2 = 5$, $k_3 = 20$ and $c_1 = c_3 = 2$, $c_2 = 0.5$, the equation of motion can be written in the form of (1) with

$$M = \mathrm{diag}(1, 1, 1),$$

$$D = \begin{bmatrix} 2.5 & -0.5 & 0 \\ -0.5 & 2.5 & -2 \\ 0 & -2 & 2 \end{bmatrix},\ K = \begin{bmatrix} 10 & -5 & 0 \\ -5 & 25 & -20 \\ 0 & -20 & 20 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},\ R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

It is verified according to Lemma 2 that this system is observable.

5.1. Case of prescribed s_i, $i = 1,2,\ldots,6$

Given the eigenvalues of the error system in the form of (6)

$$s_1 = -3 - \mathrm{i},\ s_2 = -3 + \mathrm{i},\ s_3 = -1 - 0.5\mathrm{i},$$
$$s_4 = -1 + 0.5\mathrm{i},\ s_5 = -2 - 1.5\mathrm{i},\ s_6 = -2 + 1.5\mathrm{i}.$$

After the SVD decomposition to the matrix Φ_i in the form of (24), we can obtain N_i and D_i, $i = 1,2,\ldots,6$, as follows:

$$N_{1,2} = \begin{bmatrix} 0.2818 \pm 0.0118\mathrm{i} & -0.0027 \mp 0.0095\mathrm{i} \\ 0.0435 \pm 0.0290\mathrm{i} & -0.0243 \mp 0.0238\mathrm{i} \\ 0.0202 \pm 0.0261\mathrm{i} & -0.0537 \mp 0.0271\mathrm{i} \end{bmatrix},$$

$$N_{3,4} = \begin{bmatrix} 0.1887 \mp 0.0598\mathrm{i} & -0.0954 \mp 0.0583\mathrm{i} \\ 0.1368 \pm 0.0022\mathrm{i} & -0.1882 \mp 0.1055\mathrm{i} \\ 0.1292 \pm 0.0099\mathrm{i} & -0.2249 \mp 0.1106\mathrm{i} \end{bmatrix},$$

$$N_{5,6} = \begin{bmatrix} 0.3220 \mp 0.0576\mathrm{i} & 0.0114 \mp 0.0320\mathrm{i} \\ 0.0671 \pm 0.0729\mathrm{i} & -0.0161 \mp 0.0641\mathrm{i} \\ 0.0337 \pm 0.0827\mathrm{i} & -0.0481 \mp 0.0685\mathrm{i} \end{bmatrix}$$

$$D_{1,2}=\begin{bmatrix}0.9574\mp 0.0094i & 0.0028\pm 0.0027i\\ 0.0018\pm 0.0005i & 0.9976\mp 0.0006i\end{bmatrix},$$
$$D_{3,4}=\begin{bmatrix}0.9611\mp 0.0092i & 0.0371\pm 0.0361i\\ 0.0375\mp 0.0088i & 0.9356\mp 0.0150i\end{bmatrix},$$
$$D_{5,6}=\begin{bmatrix}0.9351\mp 0.0264i & 0.0004\pm 0.0138i\\ 0.0076\pm 0.0018i & 0.9936\mp 0.0035i\end{bmatrix}.$$

A group of parameter vectors f_i, $i=1,2,\ldots,6$, are arbitrarily chosen to satisfy Constraints C1-C4a and the condition $\det T_{eo}\neq 0$,

$$f_{1,2}=\begin{bmatrix}1\\ 0.5\end{bmatrix},\quad f_{3,4}=\begin{bmatrix}-0.2\\ 1\end{bmatrix},\quad f_{5,6}=\begin{bmatrix}-2\\ 1\end{bmatrix}.$$

Then we can obtain the gain matrices

$$L_1=\begin{bmatrix}5.9632 & 1.8581\\ -33.3582 & -16.2385\\ 26.7726 & 12.4177\end{bmatrix}$$

and

$$L_2=\begin{bmatrix}1.9705 & 2.1215\\ -18.5733 & -20.0546\\ 15.7585 & 19.0313\end{bmatrix}.$$

5.2. Case of undetermined s_i, $i=1,2,\ldots,6$

The right coprime polynomial matrices $N(s)$ and $D(s)$ can be easily solved as

$$N(s)=\begin{bmatrix}0 & -2s^2-20s\\ -2s^2-20s & 0\\ -s^3-2.5s^2-25s & 0.5s^2+5s\end{bmatrix}$$

and

$$D(s)=\begin{bmatrix}-s^2-20s-100 & 2s^3+25s^2+70s+200\\ s^5+4.5s^4+46s^3+20s^2+100s & -0.5s^4-6s^3-20s^2-100s\end{bmatrix}.$$

The same eigenvalues and parameter vectors f_i, $i=1,2,\ldots,6$, as Case 1 are selected, which can meet Constraints C1-C4b and the condition $\det T_{eo}\neq 0$. So the gain matrices are obtained as:

$$L_1=\begin{bmatrix}56.43 & 3.28\\ -123.69 & 12.52\\ 135.6 & -9.20\end{bmatrix},\quad L_2=\begin{bmatrix}15.1503 & 1.7042\\ 11.1146 & -2.0408\\ 15.5561 & 5.1488\end{bmatrix}.$$

6. CONCLUDING REMARKS

The type of proportional-differential observers for a class of MSOL systems is proposed directly in the matrix second-order framework and based on generalized eigenstructure assignment via unified parametric approaches. Through establishing two general parametric solutions to this type of matrix equations, two unified complete parametric methods for the proposed observer design problem are presented. The first one mainly depends on a series of singular value decompositions, and is thus numerically simple and reliable; the second one utilizes the right factorization of the system, and allows eigenvalues of the error system to be set undetermined and sought via certain optimization procedures. Also the two approaches can provide all the degrees of design freedom that can be further utilized to achieve additional system specifications. The example shows the effect of the two approaches.

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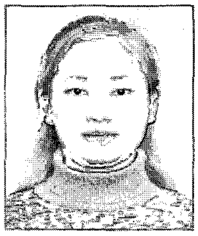

Yun-Li Wu received the B.Sc. and M.Sc. degrees in Control Theory and Control Engineering from Harbin University of Science and Technology in 1999 and 2002, respectively. Her interest includes eigenstructure assignment, robust control and descriptor systems.

Guang-Ren Duan for photograph and biography, see p. 116 of the March 2005 issue (vol. 3, no. 1) of this journal.