Design of Robust H_∞ Control for Interconnected Systems: A Homotopy Method

Ning Chen, Masao Ikeda, and Weihua Gui

Abstract: This paper considers a robust decentralized H_∞ control problem for uncertain large-scale interconnected systems. The uncertainties are assumed to be time-invariant, norm-bounded, and exist in subsystems. A design method based on the bounded real lemma is developed for a dynamic output feedback controller, which is reduced to a feasibility problem for a nonlinear matrix inequality (NMI). It is proposed to solve the NMI iteratively by the idea of homotopy, where some of the variables are fixed alternately on each iteration to reduce the NMI to a linear matrix inequality (LMI). A decentralized controller for the nominal system is computed first by imposing structural constraints on the coefficient matrices gradually. Then, the decentralized controller is modified again gradually to cope with the uncertainties. A given example shows the efficiency of this method.

Keywords: Decentralized control, H_∞ control, homotopy method, LMI, NMI, uncertainty.

1. INTRODUCTION

Decentralized H_∞ control problems for large-scale interconnected systems have been paid much attention. A homotopy method was presented using a matrix inequality for design of a decentralized H_∞ controller by Ikeda, *et al.* [4], where the system was deformed from the disconnected one to the connected one by increasing the interconnections between subsystems gradually. It was shown that the idea could be used to compute a controller which is robust to polytopic changes in interconnections. A design method for 2-channel systems using the stable factorization approach was proposed by Seo, *et al.* [8]. In [7], Scorletti and Duc considered dynamics in interconnections, and designed each local controller so that the corresponding closed-loop subsystem had a certain input-output disspative property to guarantee a specified H_∞ performance of the overall closed-loop system. An extension to decentralized H_∞ control for nonlinear interconnected system was given in [12].

Since system models always contain uncertainties, expected performances cannot be attained if the controller is designed only for the nominal model. This is especially true in the case of large-scale systems. Thus, it is desired that control system design be able to take into account modeling errors in the system. A few results on robust decentralized H_∞ control for interconnected systems with parameter uncertainty have been obtained. The robust decentralized H_∞ control problem was converted into a scaled H_∞ control problem for decoupled subsystems with no uncertainty by Wang, *et al.* [11]. A sufficient condition for solvability of the problem has been given. In [9], Shang and Sun applied the approach of Ikeda, *et al.* [4] to the case of norm-bounded uncertainties, which might exist in subsystems as well as interconnections. An extension to robust decentralized H_∞ control for nonlinear uncertain interconnected systems was given in [2].

It has been well known that LMI-based approaches [1,3,5] are very powerful for centralized controller design. However, it is not true in the decentralized case, where controller design problems cannot be reduced to a feasibility problem for LMIs because of the structural constraint on the controller, i.e., block-diagonal forms of coefficient matrices.

This paper considers a robust decentralized H_∞ control problem for uncertain large-scale interconnected systems. The uncertainties are assumed to be time-invariant, norm-bounded, and exist in subsystems. A design method based on the bounded real lemma is developed for a dynamic output feedback controller, which is reduced to a feasibility problem for a NMI. It is proposed to solve the NMI iteratively by the idea of homotopy, where some of the variables are fixed alternately on each iteration to

Manuscript received August 20, 2004; revised March 6, 2005, accepted March 24, 2005. Recommended by Editorial Board member Seung-Bok Choi under the direction of Editor-in-Chief Myung Jin Chung.

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reduce the NMI to a LMI. A decentralized controller for the nominal system is computed first by imposing structural constraints on the coefficient matrices gradually. Then, the decentralized controller is modified again gradually to cope with the uncertainties. A given example shows the efficiency of this method.

This paper is organized as follows. Section 2 is devoted to the formulation of the robust decentralized uncertain H_∞ control problem. Section 3 provides a sufficient condition for a decentralized H_∞ controller to exist without uncertainties. In Section 4, a computation algorithm is proposed using a homotopy method. The discussion is extended in Section 5 to the case with uncertainties in subsystem matrices. In Section 6, a computation algorithm for a robust controller is proposed using a homotopy method again. An example is presented in Section 7, which demonstrates the efficiency of the proposed algorithm.

2. PROBLEM DESCRIPTION

We consider an uncertain large-scale interconnected system composed of N subsystems

$$\begin{aligned} \dot{x}_i(t) &= (A_{ii} + \delta A_{ii})x_i(t) + B_{1i}w_i(t) \\ &\quad + (B_{2i} + \delta B_{2i})u_i(t) + \sum_{j=1, j\neq i}^{N} A_{ij}x_j(t), \\ z_i(t) &= C_{1i}x_i(t) + D_{12i}u_i(t), \\ y_i(t) &= C_{2i}x_i(t) + D_{21i}w_i(t), \quad i = 1, 2, \cdots, N, \end{aligned} \tag{1}$$

where $x_i(t) \in R^{n_i}$, $w_i(t) \in R^{r_i}$, $u_i(t) \in R^{m_i}$, $z_i(t) \in R^{l_i}$, and $y_i(t) \in R^{p_i}$ are the state, disturbance input, control input, controlled output, and measured output vectors, respectively. The matrices A_{ii}, B_{1i}, B_{2i}, C_{1i}, C_{2i}, D_{12i}, and D_{21i} are constant and of appropriate dimensions. The matrix A_{ij} describes the interconnection from subsystem j to subsystem i. The matrices δA_{ii} and δB_{2i} denote time-invariant uncertainties in the system and control input matrices. We suppose that

$$[\delta A_{ii} \;\; \delta B_{2i}] = E_i \Delta_i [G_{1i} \;\; G_{2i}], \tag{2}$$

where E_i, G_{1i}, G_{2i} are known constant matrices, and Δ_i is an unknown time-invariant matrix satisfying

$$\Delta_i^{\mathrm{T}} \Delta_i \le I. \tag{3}$$

We assume that the triplet (A_{ii}, B_{2i}, C_{2i}) is stabilizable and detectable.

The whole large-scale system is written as

$$\begin{aligned} \dot{x} &= (A + \delta A)x + B_1 w + (B_2 + \delta B_2)u, \\ &= (A + E\Delta G_1)x + B_1 w + (B_2 + E\Delta G_2)u, \\ z &= C_1 x + D_{12}u, \\ y &= C_2 x + D_{21}w, \end{aligned} \tag{4}$$

where

$$\begin{aligned} &A = [A_{ij}]_{N\times N}, \\ &B_1 = \mathrm{diag}\{B_{11}, \cdots, B_{1N}\},\ B_2 = \mathrm{diag}\{B_{21}, \cdots, B_{2N}\}, \\ &C_1 = \mathrm{diag}\{C_{11}, \cdots, C_{1N}\},\ C_2 = \mathrm{diag}\{C_{21}, \cdots, C_{2N}\}, \\ &D_{12} = \mathrm{diag}\{D_{121}, \cdots, D_{12N}\}, \\ &D_{21} = \mathrm{diag}\{D_{211}, \cdots, D_{21N}\}, \\ &\delta A = \mathrm{diag}\{\delta A_{11}, \cdots, \delta A_{NN}\},\ \delta B_2 = \mathrm{diag}\{\delta B_{21}, \cdots, \delta B_{2N}\}, \\ &E = \mathrm{diag}\{E_1, \cdots, E_N\},\ \Delta = \mathrm{diag}\{\Delta_1, \cdots, \Delta_N\}, \\ &G_1 = \mathrm{diag}\{G_{11}, \cdots, G_{1N}\},\ G_2 = \mathrm{diag}\{G_{21}, \cdots, G_{2N}\}, \\ &x = \begin{bmatrix} x_1^{\mathrm{T}} & \cdots & x_N^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},\ w = \begin{bmatrix} w_1^{\mathrm{T}} & \cdots & w_N^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ &u = \begin{bmatrix} u_1^{\mathrm{T}} & \cdots & u_N^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},\ z = \begin{bmatrix} z_1^{\mathrm{T}} & \cdots & z_N^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ &y = \begin{bmatrix} y_1^{\mathrm{T}} & \cdots & y_N^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}. \end{aligned} \tag{5}$$

We adopt a strictly proper output feedback controller described by

$$\begin{aligned} \dot{x}_{ci} &= A_{ci}x_{ci} + B_{ci}y_i, \\ u_i &= C_{ci}x_{ci}, \quad i = 1, 2, \cdots, N, \end{aligned} \tag{6}$$

where $x_{ci} \in R^{n_i}$ is the state of the i-th local controller, and A_{ci}, B_{ci}, C_{ci}, are constant matrices to be determined. Then, the closed-loop system composed of (1) and (6) is written as

$$\begin{aligned} \dot{x}_{cl} &= A_{cl}x_{cl} + B_{cl}w, \\ z &= C_{cl}x_{cl}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} &x_{cl} = [x^{\mathrm{T}} \;\; x_c^{\mathrm{T}}]^{\mathrm{T}},\ x_c = \begin{bmatrix} x_{c1}^{\mathrm{T}} & \cdots & x_{cN}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}, \\ &A_{cl} = \begin{bmatrix} A + \delta A & (B_2 + \delta B_2)C_c \\ B_c C_2 & A_c \end{bmatrix},\ B_{cl} = \begin{bmatrix} B_1 \\ B_c D_{21} \end{bmatrix}, \\ &C_{cl} = \begin{bmatrix} C_1 & D_{12}C_c \end{bmatrix}, \\ &A_c = \mathrm{diag}\{A_{c1}, \cdots, A_{cN}\},\ B_c = \mathrm{diag}\{B_{c1}, \cdots, B_{cN}\}, \\ &C_c = \mathrm{diag}\{C_{c1}, \cdots, C_{cN}\}. \end{aligned}$$

The transfer function $T_{zw}(s)$ from the disturbance input w to the controlled output z is

$$T_{zw} = C_{cl}(sI - A_{cl})^{-1}B_{cl}.$$

We say that the system (1) is stabilizable with the disturbance attenuation level γ if the closed-loop system (7) is stable and satisfies $\|T_{zw}\|_\infty < \gamma$, where γ is a specified positive number. The control problem of this paper is to design a decentralized controller (6) realizing such a closed-loop system.

To solve the decentralized control problem, we employ the following lemmas.

Lemma 1 (Bounded Real Lemma, [3,5]): The following statements are equivalent:

(i) A_{cl} is a stable matrix and $\|T_{zw}\|_\infty < \gamma$.

(ii) There exists a positive definite matrix P which satisfies the LMI:

$$\begin{bmatrix} A_{cl}^{\mathrm{T}}P + PA_{cl} & PB_{cl} & C_{cl}^{\mathrm{T}} \\ B_{cl}^{\mathrm{T}}P & -\gamma I & 0 \\ C_{cl} & 0 & -\gamma I \end{bmatrix} < 0. \tag{8}$$

Lemma 2 [10]: Suppose that E, G, and Δ are matrices of appropriate dimensions and $\Delta^{\mathrm{T}}\Delta \le I$. Then, for any $\varepsilon > 0$, the inequality

$$E\Delta G + G^{\mathrm{T}}\Delta^{\mathrm{T}}E^{\mathrm{T}} \le \varepsilon EE^{\mathrm{T}} + \varepsilon^{-1}G^{\mathrm{T}}G$$

holds.

3. CONTROLLER DESIGN FOR NOMINAL CASE

First, we consider the case where no uncertainty exists, i.e., $\delta A_{ii} = 0$, $\delta B_i = 0$. A decentralized H_∞ output feedback controller is given as follows.

Theorem 1: For a given constant $\gamma > 0$, the system (1) with no uncertainty is stabilizable with the disturbance attenuation level γ by a decentralized controller (6), if there exist positive definite block-diagonal matrices X, Y and block-diagonal matrices F, L, Q as a solution set to the inequalities

$$T(X,Y,F,L,Q) = \begin{bmatrix} J_{11} & J_{21}^{\mathrm{T}} & B_1 & XC_1^{\mathrm{T}} + F^{\mathrm{T}}D_{12}^{\mathrm{T}} \\ J_{21} & J_{22} & YB_1 + LD_{21} & C_1^{\mathrm{T}} \\ B_1^{\mathrm{T}} & B_1^TY + D_{21}^{\mathrm{T}}L^{\mathrm{T}} & -\gamma I & 0 \\ C_1X + D_{12}F & C_1 & 0 & -\gamma I \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0, \tag{10}$$

where

$$\begin{aligned} J_{11} &= AX + XA^{\mathrm{T}} + B_2F + F^{\mathrm{T}}B_2^{\mathrm{T}}, \\ J_{21} &= A^{\mathrm{T}} + YAX + L\,C_2X + YB_2F + Q, \\ J_{22} &= YA + A^TY + LC_2 + C_2^{\mathrm{T}}L^{\mathrm{T}}, \end{aligned} \tag{11}$$

and the sizes of submatrices in the block-diagonal matrices are compatible with the dimensions of the state, input, and output vectors of subsystems. Then, the coefficient matrices of a decentralized H_∞ output feedback controller (6) are given by

$$A_c = V^{-1}QU^{-\mathrm{T}},\ B_c = V^{-1}L,\ C_c = FU^{-\mathrm{T}}, \tag{12}$$

where the matrices U and V are block-diagonal and determined by the equation

$$UV^{\mathrm{T}} = I - XY. \tag{13}$$

Proof: Using the solutions to (9), (10), and (13), we define a symmetric matrix

$$P = \begin{bmatrix} Y & V \\ V^{\mathrm{T}} & U^{-1}XYXU^{-\mathrm{T}} - U^{-1}XU^{-\mathrm{T}} \end{bmatrix}, \tag{14}$$

which is positive definite. The positive definiteness of P can be shown by the Schur complement and the fact that $Y > 0$ and

$$\begin{aligned} &U^{-1}XYXU^{-\mathrm{T}} - U^{-1}XU^{-\mathrm{T}} - V^{\mathrm{T}}Y^{-1}V \\ &= U^{-1}(XYX - X - (I - XY)Y^{-1}(I - YX))U^{-\mathrm{T}} \\ &= U^{-1}(X - Y^{-1})U^{-\mathrm{T}} \\ &> 0, \end{aligned} \tag{15}$$

which is equivalent to (10). We then substitute $Q = VA_cU^{\mathrm{T}}$, $L = VB_c$, $F = C_cU^{\mathrm{T}}$ into (9), and rewrite it to obtain

$$\begin{bmatrix} \Pi_1^{\mathrm{T}}(A_{cl}^{\mathrm{T}}P + PA_{cl})\Pi_1 & \Pi_1^{\mathrm{T}}PB_{cl} & \Pi_1^{\mathrm{T}}C_{cl}^{\mathrm{T}} \\ B_{cl}^{\mathrm{T}}P\Pi_1 & -\gamma I & 0 \\ C_{cl}\Pi_1 & 0 & -\gamma I \end{bmatrix} < 0, \tag{16}$$

where

$$\Pi_1 = \begin{bmatrix} X & I \\ U^{\mathrm{T}} & 0 \end{bmatrix}. \tag{17}$$

By pre- and post-multiply (16) by

$$\begin{bmatrix} \Pi_1^{-\mathrm{T}} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},\ \begin{bmatrix} \Pi_1^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \tag{18}$$

respectively, we obtain (8) and conclude by Lemma 1 that the closed-loop system (7) is stable with the disturbance attenuation level γ. □

A simple choice of U and V satisfying (13) is $U = I - XY$ and $V=I$. We take this choice in the example of Section 7.

The idea of Theorem 1 is standard for centralized H_∞ controller design [6], where we do not impose any structural constraint on the matrices X, Y, F, L, Q, and by choosing Q suitably, we can eliminate the bilinear term J_{21} so that BMI (9) becomes an LMI. Thus, in the centralized control case, the H_∞ control problem can be transformed to feasibility of LMIs. However, in the decentralized control case, the solution matrices have to be block-diagonal and BMI (9) cannot be converted to an LMI.

4. A SOLUTION ALGORITHM FOR NOMINAL CASE

In order to solve BMI (9), we fix a group of variables to make it an LMI. First, we fix the variables Y and L, then BMI (9) becomes an LMI in X, F, and Q. Next, we fix the variables X and F, then BMI (9) becomes another LMI. By solving these two problems alternately, we expect that we obtain a solution of the BMI.

This idea is successful if the values of fixed variables are chosen suitably. However, such a choice is equivalent to finding a solution of the BMI, and there is no obvious way. To find a suitable choice, we introduce an iterative method [13]. That is, we use a homotopy method with a real number λ varying from 0 to 1, which defines a matrix function

$$H(X,Y,F,L,Q,\lambda) = T(X,Y,F,L,(1-\lambda)Q_F + \lambda Q) < 0, \tag{19}$$

where J_{21} in (9) is replaced by

$$J_{21} = A^{\mathrm{T}} + YAX + L\,C_2X + YB_2F + (1-\lambda)Q_F + \lambda Q, \tag{20}$$

and Q_F is a fixed full matrix. Then,

$$H(X,Y,F,L,Q,\lambda) = \begin{cases} T(X,Y,F,L,Q_F), \lambda = 0 \\ T(X,Y,F,L,Q), \lambda = 1 \end{cases} \tag{21}$$

and the problem of finding a solution to (9) is embedded in the parametrized family of problems

$$H(X,Y,F,L,Q,\lambda) < 0, \quad \lambda \in [0,1]. \tag{22}$$

We start computing the solution to (22) and (10) with $\lambda = 0$, which we denote by X_0, Y_0, F_0, L_0 and Q_F. These initial values are computed by solving (9) and (10) without the structural constraint on Q. Then, we can choose a full matrix

$$Q_F = -(A^{\mathrm{T}} + Y_0AX_0 + L_0CX_0 + Y_0B_2F_0), \tag{23}$$

to eliminate the bilinear term J_{21} in (9). We solve the resultant LMI (9) with (10) to obtain block-diagonal X_0, Y_0, F_0, and L_0.

Now, we consider a homotopy path to transform this initial solution at $\lambda = 0$ to a solution at $\lambda = 1$. Let M be a positive integer and consider $(M+1)$ points $\lambda_k = k/M$ $(k = 0, 1, \cdots, M)$ in the interval [0,1] to generate a family of problems

$$H(X,Y,F,L,Q,\lambda_k) < 0. \tag{24}$$

If the problem at the k-th point is feasible, we denote the obtained solution by $(X_k, Y_k, F_k, L_k, Q_k)$. Then, we compute a solution $(X_{k+1}, F_{k+1}, Q_{k+1})$ or $(Y_{k+1}, L_{k+1}, Q_{k+1})$ of $H(X,Y,F,L,Q,\lambda_{k+1}) < 0$ by alternately solving it as LMIs with two of the five variables being fixed as $Y = Y_k, L = L_k$ or $X = X_k, F = F_k$. If the family of problems $H(X,Y,F,L,Q,\lambda_k) < 0$, $k = 0,1,\cdots,M$, are all feasible a solution of (9) and (10) is obtained at $k = M$ $(\lambda = 1)$. If it is not the case, that is, $H(X,F,Y,L,Q,\lambda_{k+1}) < 0$ is not feasible for some k when we set $Y = Y_k, L = L_k$ and when $X = X_k$, $F = F_k$, we consider more points in the interval $[\lambda_k, 1]$ by increasing M, and repeat the procedure from the solution $(X_k, Y_k, F_k, L_k, Q_k)$ at $\lambda = \lambda_k$.

The above idea is summarized as a computation algorithm for a decentralized H_∞ output feedback controller.

Step 1: Compute block-diagonal solutions X_0, Y_0, F_0, L_0 of (9) and (10) with $J_{21} = J_{12}^T = 0$. If it is feasible, set Q_F as (23). Initialize M to a certain positive integer, and set a certain upper bound $M_{\max}$ for M. Set $k = 0$.

Step 2: Set $k := k+1$ and $\lambda_k = k/M$. Compute block-diagonal solutions X, F, Q of $H(X, Y_{k-1}, F, L_{k-1}, Q, \lambda_k) < 0$ and (10) with $Y = Y_{k-1}$. If it is not feasible, go to Step 3. If it is feasible, set $X_k = X$, $F_k = F$, and compute block-diagonal solutions Y, L, Q of $H(X_k, Y, F_k, L, Q, \lambda_k) < 0$ and (10) with $X = X_k$. Then, set $Y_k = Y$, $L_k = L$, $Q_k = Q$, and go to Step 5.

Step 3: Compute block-diagonal solutions Y, L, Q of $H(X_{k-1}, Y, F_{k-1}, L, Q, \lambda_k) < 0$ and (10) with $X = X_{k-1}$. If it is not feasible, go to Step 4. If it is feasible, set $Y_k = Y$, $L_k = L$, and compute block-diagonal solutions X, F, Q of $H(X, Y_k, F, L_k, Q, \lambda_k) < 0$ and (10) with $Y = Y_k$. Then, set $X_k = X$, $F_k = F$, $Q_k = Q$, and go to Step 5.

Step 4: Set $M := 2M$ under the constraint $M \le M_{\max}$, set $X_{2(k-1)} = X_{k-1}$, $Y_{2(k-1)} = Y_{k-1}$, $F_{2(k-1)} = F_{k-1}$, $L_{2(k-1)} = L_{k-1}$, $k := 2(k\text{-}1)$ and go to Step 2. If we cannot increase M any more, we conclude that this algorithm does not converge.

Step 5: If $k<M$, go to Step 2. If $k=M$, the obtained matrices X_M, Y_M, F_M, L_M, and Q_M are solutions of (9) and (10).

Step 6: Compute block-diagonal matrices U and V such that $UV^{\mathrm{T}} = I - X_M Y_M$ and define the coefficient matrices of a decentralized H$_\infty$ controller as

$$A_c = V^{-1} Q_M^{\mathrm{T}} U^{-\mathrm{T}},\ B_c = V^{-1} L_M,\ C_c = F_M U^{-\mathrm{T}}.$$

Remark 1: At each of Steps 2 and 3, we suggest solving two LMIs obtained by fixing some of the variables in BMI (22). It is theoretically not necessary to deal with the second one, but according to authors' experiences, it improves the convergence of the algorithm.

Remark 2: In Step 4, we may simply set $M := 2M$, k=0, and go back to Step 2. This means that we compute a different homotopy path from the beginning.

5. ROBUST CONTROL FOR UNCERTAIN CASE

We now consider the interconnected system (1) with uncertainties δA_{ii} and δB_{2i} of the form (2). A robust decentralized H$_\infty$ controller is given as follows.

Theorem 2: For a given constant $\gamma > 0$, the uncertain system (1) is robustly stabilizable with the disturbance attenuation level γ by a decentralized controller (6), if there exist positive definite block-diagonal matrices $\tilde{X}$, $\tilde{Y}$, block-diagonal matrices $\tilde{F}$, $\tilde{L}$, $\tilde{Q}$, and a positive constant ε as solutions to the inequalities

$$\tilde{T}(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon) = \begin{bmatrix} \tilde{J}_{11} & \tilde{J}_{21}^{\mathrm{T}} & B_1 & \tilde{X}C_1^{\mathrm{T}} + \tilde{F}^{\mathrm{T}} D_{12}^{\mathrm{T}} \\ \tilde{J}_{21} & \tilde{J}_{22} & \tilde{Y}B_1 + \tilde{L}D_{21} & \tilde{Y}C_1^{\mathrm{T}} \\ B_1^{\mathrm{T}} & B_1^{\mathrm{T}}\tilde{Y} + D_{21}^{\mathrm{T}}\tilde{L}^{\mathrm{T}} & -\gamma I & 0 \\ C_1\tilde{X} + D_{12}\tilde{F} & C_1\tilde{Y} & 0 & -\gamma I \end{bmatrix} < 0, \quad (25)$$

$$\begin{bmatrix} \tilde{X} & I \\ I & \tilde{Y} \end{bmatrix} > 0, \quad (26)$$

where

$$\begin{aligned} \tilde{J}_{11} &= A\tilde{X} + \tilde{X}A^{\mathrm{T}} + B_2\tilde{F} + \tilde{F}^{\mathrm{T}}B_2^{\mathrm{T}} + \varepsilon EE^{\mathrm{T}} \\ &\quad + \varepsilon^{-1}(\tilde{X}G_1^{\mathrm{T}} + \tilde{F}^{\mathrm{T}}G_2^{\mathrm{T}})(G_1\tilde{X} + G_2\tilde{F}), \\ \tilde{J}_{21} &= A^{\mathrm{T}} + \tilde{Y}A\tilde{X} + \tilde{L}C_2\tilde{X} + \tilde{Y}B_2\tilde{F} + \tilde{Q} + \varepsilon\tilde{Y}EE^{\mathrm{T}} \\ &\quad + \varepsilon^{-1}G_1^{\mathrm{T}}(G_1\tilde{X} + G_2\tilde{F}), \\ \tilde{J}_{22} &= \tilde{Y}A + A^{\mathrm{T}}\tilde{Y} + \tilde{L}C_2 + C_2^{\mathrm{T}}\tilde{L}^{\mathrm{T}} + \varepsilon\tilde{Y}EE^{\mathrm{T}}\tilde{Y} \\ &\quad + \varepsilon^{-1}G_1^{\mathrm{T}}G_1, \end{aligned} \quad (27)$$

and the sizes of submatrices in the block-diagonal matrices are compatible with the dimensions of the state, input, and output vectors of subsystems. Then, coefficient matrices of a robust decentralized H$_\infty$ output feedback controller (6) are given by

$$A_c = \tilde{V}^{-1}\tilde{Q}\tilde{U}^{-\mathrm{T}},\ B_c = \tilde{V}^{-1}\tilde{L},\ C_c = \tilde{F}\tilde{U}^{-\mathrm{T}}, \quad (28)$$

where the matrices $\tilde{U}$ and $\tilde{V}$ are block-diagonal and determined by the equation

$$\tilde{U}\tilde{V}^{\mathrm{T}} = I - \tilde{X}\tilde{Y}. \quad (29)$$

Proof: From Theorem 1, we note that if a matrix inequality, which is similar to (9), but contains uncertain matrices δA_{ii} and δB_{2i} in the upper left 2×2 blocks, has a solution with (10) independently of the uncertainties, then the obtained controller is robust. Based on Lemma 2, the uncertain term satisfies

$$\begin{aligned} &\begin{bmatrix} \delta A\tilde{X} + \tilde{X}\delta A^{\mathrm{T}} + \delta B_2\tilde{F} + \tilde{F}^{\mathrm{T}}\delta B_2^{\mathrm{T}} & (\delta A^{\mathrm{T}} + \tilde{Y}\delta A\tilde{X} + \tilde{Y}\delta B_2\tilde{F})^{\mathrm{T}} \\ \delta A^{\mathrm{T}} + \tilde{Y}\delta A\tilde{X} + \tilde{Y}\delta B_2\tilde{F} & \tilde{Y}\delta A + \delta A^{\mathrm{T}}\tilde{Y} \end{bmatrix} \\ &= \begin{bmatrix} E \\ \tilde{Y}E \end{bmatrix} \Delta \begin{bmatrix} G_1\tilde{X} + G_2F & G_1 \end{bmatrix} + \begin{bmatrix} \tilde{X}G_1^{\mathrm{T}} + \tilde{F}^{\mathrm{T}}G_2^{\mathrm{T}} \\ G_1^{\mathrm{T}} \end{bmatrix} \Delta^{\mathrm{T}} \begin{bmatrix} E^{\mathrm{T}} & E^{\mathrm{T}}\tilde{Y} \end{bmatrix} \\ &\le \varepsilon \begin{bmatrix} E \\ \tilde{Y}E \end{bmatrix} \begin{bmatrix} E^{\mathrm{T}} & E^{\mathrm{T}}\tilde{Y} \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} \tilde{X}G_1^{\mathrm{T}} + \tilde{F}^{\mathrm{T}}G_2^{\mathrm{T}} \\ G_1^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} G_1\tilde{X} + G_2F & G_1 \end{bmatrix} \end{aligned} \quad (30)$$

and we can conclude that (25) and (26) give such a robust solution. □

6. A SOLUTION ALGORISM FOR UNCERTAIN CASE

To compute a solution of the matrix inequalities (25) and (26), we again employ a homotopy method in a different way. For this purpose, we first decompose $\tilde{T}(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon)$ of (25) into the nominal part $T(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q})$ defined by (9) and the term $\tilde{T}_u(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon)$ generated by the uncertainties as

$$\begin{aligned} &\tilde{T}(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon) \\ &\quad = T(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q}) + \tilde{T}_u(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon), \end{aligned} \quad (31)$$

where

$$\tilde{T}_u(\tilde{Y},\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon) = \begin{bmatrix} \tilde{J}_{u11} & \tilde{J}_{u21}^{\mathrm{T}} & 0 & 0 \\ \tilde{J}_{u21} & \tilde{J}_{u22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (32)$$

$$\begin{aligned} \tilde{J}_{u11} &= \varepsilon EE^{\mathrm{T}} + \varepsilon^{-1}(\tilde{X}G_1^{\mathrm{T}} + \tilde{F}^{\mathrm{T}}G_2^{\mathrm{T}})(G_1\tilde{X} + G_2\tilde{F}), \\ \tilde{J}_{u21} &= \varepsilon\tilde{Y}EE^{\mathrm{T}} + \varepsilon^{-1}G_1^{\mathrm{T}}(G_1\tilde{X} + G_2\tilde{F}), \\ \tilde{J}_{u22} &= \varepsilon\tilde{Y}EE^{\mathrm{T}}\tilde{Y} + \varepsilon^{-1}G_1^{\mathrm{T}}G_1. \end{aligned}$$

We introduce a real number $\lambda \in [0,1]$ and define the matrix function

$$\tilde{H}(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon,\lambda)=T(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q})+\lambda\tilde{T}_u(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon). \tag{33}$$

Then,

$$\tilde{H}(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon,\lambda)=\begin{cases} T(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q}),\ \lambda=0 \\ \tilde{T}(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon),\ \lambda=1 \end{cases} \tag{34}$$

and the problem of finding a solution to (25) is embedded in the parametrized family of problems

$$\tilde{H}(\tilde{X},\tilde{F},\tilde{Y},\tilde{L},\tilde{Q},\varepsilon,\lambda)<0,\quad \lambda\in[0,1]. \tag{35}$$

To solve (35) from $\lambda=0$ to $\lambda=1$, we apply the Schur complement and consider two matrix inequalities shown as

$$\tilde{H}_1(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon,\lambda)=\begin{bmatrix} \tilde{J}_{11} & \tilde{J}_{21}^{\mathrm{T}} & B_1 & \tilde{X}C_1^{\mathrm{T}}+\tilde{F}^{\mathrm{T}}D_{12}^{\mathrm{T}} & \tilde{X}G_1^{\mathrm{T}}+\tilde{F}^{\mathrm{T}}G_2^{\mathrm{T}} \\ \tilde{J}_{21} & \tilde{J}_{22} & \tilde{Y}B_1+\tilde{L}D_{21} & \tilde{Y}C_1^{\mathrm{T}} & G_1^{\mathrm{T}} \\ B_1^{\mathrm{T}} & B_1^{\mathrm{T}}\tilde{Y}+D_{21}^{\mathrm{T}}\tilde{L}^{\mathrm{T}} & -\gamma I & 0 & 0 \\ C_1\tilde{X}+D_{12}\tilde{F} & C_1\tilde{Y} & 0 & -\gamma I & 0 \\ G_1\tilde{X}+G_2\tilde{F} & G_1 & 0 & 0 & -\varepsilon\lambda^{-1}I \end{bmatrix}<0, \tag{36}$$

where

$$\begin{aligned} \tilde{J}_{11}&=A\tilde{X}+\tilde{X}A^{\mathrm{T}}+B_2\tilde{F}+\tilde{F}^{\mathrm{T}}B_2^{\mathrm{T}}+\varepsilon\lambda EE^{\mathrm{T}}, \\ \tilde{J}_{21}&=A^{\mathrm{T}}+\tilde{Y}A\tilde{X}+\tilde{L}C_2\tilde{X}+\tilde{Y}B_2\tilde{F}+\tilde{Q}+\varepsilon\lambda\tilde{Y}EE^{\mathrm{T}}, \\ \tilde{J}_{22}&=\tilde{Y}A+A^{\mathrm{T}}\tilde{Y}+\tilde{L}C_2+C_2^{\mathrm{T}}\tilde{L}^{\mathrm{T}}+\varepsilon\lambda\tilde{Y}EE^{\mathrm{T}}\tilde{Y}, \end{aligned} \tag{37}$$

and

$$\tilde{H}_2(\tilde{X},\tilde{Y},\tilde{F},\tilde{L},\tilde{Q},\varepsilon,\lambda)=\begin{bmatrix} \tilde{J}_{11} & \tilde{J}_{21}^{\mathrm{T}} & B_1 & \tilde{X}C_1^{\mathrm{T}}+\tilde{F}^{\mathrm{T}}D_{12}^{\mathrm{T}} & E \\ \tilde{J}_{21} & \tilde{J}_{22} & \tilde{Y}B_1+\tilde{L}D_{21} & \tilde{Y}C_1^{\mathrm{T}} & \tilde{Y}E \\ B_1^{\mathrm{T}} & B_1^{\mathrm{T}}\tilde{Y}+D_{21}^{\mathrm{T}}\tilde{L}^{\mathrm{T}} & -\gamma I & 0 & 0 \\ C_1\tilde{X}+D_{12}\tilde{F} & C_1\tilde{Y} & 0 & -\gamma I & 0 \\ G_1\tilde{X}+G_2\tilde{F} & G_1 & 0 & 0 & -\varepsilon^{-1}\lambda^{-1}I \end{bmatrix}<0, \tag{38}$$

where

$$\begin{aligned} \tilde{J}_{11}&=A\tilde{X}+\tilde{X}A^{\mathrm{T}}+B_2\tilde{F}+\tilde{F}^{\mathrm{T}}B_2^{\mathrm{T}} \\ &\quad+\varepsilon^{-1}\lambda(\tilde{X}G_1^{\mathrm{T}}+\tilde{F}^{\mathrm{T}}G_2^{\mathrm{T}})(G_1\tilde{X}+G_2\tilde{F}), \\ \tilde{J}_{21}&=A^{\mathrm{T}}+\tilde{Y}A\tilde{X}+\tilde{L}C_2\tilde{X}+\tilde{Y}B_2\tilde{F}+\tilde{Q} \\ &\quad+\varepsilon^{-1}\lambda G_1^{\mathrm{T}}(G_1\tilde{X}+G_2\tilde{F}), \\ \tilde{J}_{22}&=\tilde{Y}A+A^{\mathrm{T}}\tilde{Y}+\tilde{L}C_2+C_2^{\mathrm{T}}\tilde{L}^{\mathrm{T}}+\varepsilon^{-1}\lambda G_1^{\mathrm{T}}G_1. \end{aligned} \tag{39}$$

Both inequalities (36) and (38) are equivalent to (35).

We note that if we fix the variables $\tilde{Y}$ and $\tilde{L}$, (36) becomes an LMI in $\tilde{X}$, $\tilde{F}$, $\tilde{Q}$ and ε. Also, if we fix the variables $\tilde{X}$ and $\tilde{F}$, (38) becomes an LMI in $\tilde{Y}$, $\tilde{L}$, $\tilde{Q}$ and ε^{-1}. We solve (35) and (26) increasing λ gradually by computing such LMIs alternately.

We note also that the solution to (35) and (26) with $\lambda=0$ is the same as the solution to (9) and (10). Therefore, we choose the solution of the nominal case as the initial values $\tilde{X}_0,\tilde{Y}_0,\tilde{F}_0,\tilde{L}_0$ in the homotopy method for the uncertain case.

We formulate this idea in an algorithm as follows.

Step 1: Compute block-diagonal solutions X,Y,F,L of (9) and (10) using the algorithm given in Section 4, which we denote by $\tilde{X}_0,\tilde{Y}_0,\tilde{F}_0,\tilde{L}_0$. Initialize M to a certain positive integer, and set a certain upper bound $M_{\max}$ for M. Set $k=0$.

Step 2: Set $k:=k+1$ and $\lambda_k=k/M$. Compute block-diagonal solutions $\tilde{X},\tilde{F},\tilde{Q}$ and a positive number ε of $\tilde{H}_1(\tilde{X},\tilde{Y}_{k-1},\tilde{F},\tilde{L}_{k-1},\tilde{Q},\varepsilon,\lambda_k)<0$ and (26) with $\tilde{Y}=\tilde{Y}_{k-1}$. If it is not feasible, go to Step 3. If it is feasible, set $\tilde{X}_k=\tilde{X},\tilde{F}_k=\tilde{F}$, and compute block-diagonal solutions $\tilde{Y},\tilde{L},\tilde{Q}$ and a positive number ε^{-1} of $\tilde{H}_2(\tilde{X}_k,\tilde{Y},\tilde{F}_k,\tilde{L},\tilde{Q},\varepsilon^{-1},\lambda_k)<0$ and (26) with $\tilde{X}=\tilde{X}_k$. Then, set $\tilde{Y}_k=\tilde{Y}$, $\tilde{L}_k=\tilde{L}$, $\tilde{Q}_k=\tilde{Q}$, and go to Step 5.

Step 3: Compute block-diagonal solutions $\tilde{Y}$, $\tilde{L}$, $\tilde{Q}$ and a positive number ε^{-1} of $\tilde{H}_2(\tilde{X}_{k-1},\tilde{Y},\tilde{F}_{k-1},\tilde{L},\tilde{Q},\varepsilon^{-1},\lambda_k)<0$ and (26) with $\tilde{X}=\tilde{X}_{k-1}$. If it is not feasible, go to Step 4. If it is feasible, set $\tilde{Y}_k=\tilde{Y}$, $\tilde{L}_k=\tilde{L}$, and compute block-diagonal solutions $\tilde{X},\tilde{F},\tilde{Q}$ and a positive number ε of $\tilde{H}_1(\tilde{X},\tilde{Y}_k,\tilde{F},\tilde{L}_k,\tilde{Q},\varepsilon,\lambda_k)<0$ and (26) with $\tilde{Y}=\tilde{Y}_k$. Then, set $\tilde{X}_k=\tilde{X},\tilde{F}_k=\tilde{F}$, $\tilde{Q}_k=\tilde{Q}$, and go to Step 5.

Step 4: Set $M:=2M$ under the constraint $M\le M_{\max}$, set $\tilde{X}_{2(k-1)}=\tilde{X}_{k-1}$, $\tilde{Y}_{2(k-1)}=\tilde{Y}_{k-1}$,

$\tilde{F}_{2(k-1)} = \tilde{F}_{k-1}$, $\tilde{L}_{2(k-1)} = \tilde{L}_{k-1}$, $k := 2(k-1)$ and go to Step 2. If we cannot increase M any more, we conclude that this algorithm does not converge.

Step 5: If $k < M$, go to Step 2. If $k = M$, the obtained matrices $\tilde{X}_M, \tilde{Y}_M, \tilde{F}_M, \tilde{L}_M, \tilde{Q}_M$, and constant ε are solutions of the (25) and (26).

Step 6: Compute block-diagonal matrices $\tilde{U}$ and $\tilde{V}$ such that $\tilde{U}\tilde{V}^{\mathrm{T}} = I - \tilde{X}_M\tilde{Y}_M$ and define the coefficient matrices of a robust decentralized H_∞ controller as

$$A_c = \tilde{V}^{-1}\tilde{Q}_M\tilde{U}^{-\mathrm{T}},\quad B_c = \tilde{V}^{-1}\tilde{L}_M,\quad C_c = \tilde{F}_M\tilde{U}^{-\mathrm{T}}.$$

7. AN EXAMPLE

In this section, we present a simple example. The interconnected system (1) we consider is composed of two subsystems, the matrices of which are

$$A_{11} = \begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix}, B_{11} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, B_{21} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, C_{21} = \begin{bmatrix} 1 & 2 \end{bmatrix},$$

$$D_{121} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, D_{211} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix},$$

$$A_{22} = \begin{bmatrix} -2 & -3 \\ 2 & -1 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_{12} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, C_{22} = \begin{bmatrix} -1 & -1 \end{bmatrix},$$

$$D_{122} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, D_{212} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} -1 & 0 \\ -2 & -1 \end{bmatrix},$$

and the uncertainty matrices are defined by

$$E_1 = \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix}, \quad G_{11} = \begin{bmatrix} -0.06 & 0.04 \end{bmatrix}, \quad G_{21} = -0.06,$$

$$E_2 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, \quad G_{12} = \begin{bmatrix} 0.04 & -0.02 \end{bmatrix}, \quad G_{22} = -0.1.$$

We specify the disturbance attenuation level as $\gamma = 8.75$.

We consider the first stage of the solution method, that is, the case of no uncertainty. We obtain the initial value for the homotopy path by relaxing the structural constraint on Q in (9) as

$$X_0 = \begin{bmatrix} 6.96 & -5.95 & 0 & 0 \\ -5.95 & 8.96 & 0 & 0 \\ 0 & 0 & 4.23 & -0.62 \\ 0 & 0 & -0.62 & 2.15 \end{bmatrix},$$

$$Y_0 = \begin{bmatrix} 15.19 & -4.81 & 0 & 0 \\ -4.81 & 1.97 & 0 & 0 \\ 0 & 0 & 0.90 & -0.09 \\ 0 & 0 & -0.09 & 0.56 \end{bmatrix},$$

$$F_0 = \begin{bmatrix} -7.74 & -0.31 & 0 & 0 \\ 0 & 0 & 4.94 & -0.48 \end{bmatrix},$$

$$L_0 = \begin{bmatrix} -7.74 & 0 \\ -0.31 & 0 \\ 0 & -4.94 \\ 0 & -0.48 \end{bmatrix},$$

and compute the matrix Q_F as follows.

$$Q_F = \begin{bmatrix} 420.73 & -453.07 & -59.84 & -0.57 \\ -120.33 & 134.27 & 19.83 & -0.07 \\ 4.60 & -5.12 & 29.23 & -0.67 \\ 2.89 & -3.14 & 2.11 & 2.43 \end{bmatrix}.$$

Then, we compute the solutions of (9) and (10) by using the homotopy method proposed in Section 4 with M=4 as

$$X = \begin{bmatrix} 11.28 & -10.44 & 0 & 0 \\ -10.44 & 13.68 & 0 & 0 \\ 0 & 0 & 1.17 & 0.32 \\ 0 & 0 & 0.32 & 2.12 \end{bmatrix},$$

$$Y = \begin{bmatrix} 13.96 & -4.30 & 0 & 0 \\ -4.30 & 1.76 & 0 & 0 \\ 0 & 0 & 1.10 & -0.08 \\ 0 & 0 & -0.08 & 0.56 \end{bmatrix},$$

$$F = \begin{bmatrix} 0 & -8.75 & 0 & 0 \\ 0 & 0 & -0.0050 & -8.76 \end{bmatrix},$$

$$L = \begin{bmatrix} -8.70 & 0 \\ -0.0145 & 0 \\ 0 & -8.74 \\ 0 & 0.0003 \end{bmatrix},$$

$$Q = \begin{bmatrix} 741.44 & -789.22 & 0 & 0 \\ -213.24 & 231.56 & 0 & 0 \\ 0 & 0 & 11.56 & 9.98 \\ 0 & 0 & 0.45 & 5.72 \end{bmatrix}.$$

The corresponding coefficient matrices of a decentralized H_∞ controller are

$$A_c = \begin{bmatrix} 621.0 & 1883.1 & 0 & 0 \\ -257.7 & -780.1 & 0 & 0 \\ 0 & 0 & -37.9 & -17.0 \\ 0 & 0 & 17.6 & -56.4 \end{bmatrix},$$

$$B_c = \begin{bmatrix} -8.70 & 0 \\ -0.0145 & 0 \\ 0 & -8.74 \\ 0 & 0.0003 \end{bmatrix},$$

$$C_c = \begin{bmatrix} 151.28 & 456.00 & 0 & 0 \\ 0 & 0 & -31.23 & 91.42 \end{bmatrix}.$$

The disturbance attenuation level achieved by this controller is 8.53.

Next, we consider the second stage of the solution method, that is, the uncertain case. Taking the above solutions as the initial values $\tilde{X}_0, \tilde{Y}_0, \tilde{F}_0, \tilde{L}_0$, we compute the solution to (25) and (26) by the homotopy method proposed in the previous section with M=8, to obtain

$$\tilde{X} = \begin{bmatrix} 10.33 & -9.27 & 0 & 0 \\ -9.27 & 12.20 & 0 & 0 \\ 0 & 0 & 1.11 & 0.34 \\ 0 & 0 & 0.34 & 2.01 \end{bmatrix},$$

$$\tilde{Y} = \begin{bmatrix} 13.61 & -4.14 & 0 & 0 \\ -4.14 & 1.69 & 0 & 0 \\ 0 & 0 & 1.06 & -0.10 \\ 0 & 0 & -0.10 & 0.56 \end{bmatrix},$$

$$\tilde{F} = \begin{bmatrix} -0.37 & -8.16 & 0 & 0 \\ 0 & 0 & 0.02 & -8.24 \end{bmatrix},$$

$$\tilde{L} = \begin{bmatrix} -8.75 & 0 \\ 0 & 0 \\ 0 & -8.75 \\ 0 & -0.0001 \end{bmatrix},$$

$$\tilde{Q} = \begin{bmatrix} 662.37 & -692.47 & 0 & 0 \\ -187.67 & 200.61 & 0 & 0 \\ 0 & 0 & 10.44 & 11.30 \\ 0 & 0 & 0.47 & 5.21 \end{bmatrix}.$$

Then, the coefficient matrices of a robust decentralized H_∞ controller are

$$A_c = \begin{bmatrix} 12836 & 39131 & 0 & 0 \\ -5247 & -15994 & 0 & 0 \\ 0 & 0 & -203 & 220 \\ 0 & 0 & 855 & -1485 \end{bmatrix},$$

$$B_c = \begin{bmatrix} -8.75 & 0 \\ 0 & 0 \\ 0 & -8.75 \\ 0 & -0.0001 \end{bmatrix},$$

$$C_c = \begin{bmatrix} 3114.1 & 9491.0 & 0 & 0 \\ 0 & 0 & -1518.2 & 2626.4 \end{bmatrix}.$$

The disturbance attenuation level achieved by this controller is 8.65.

8. CONCLUSIONS

This paper has considered a robust decentralized H_∞ control problem for uncertain large-scale interconnected systems. The uncertainties are assumed to be time-invariant, norm-bounded, and exist in subsystems. A design method based on the bounded real lemma and the idea of homotopy has been developed for a dynamic output feedback controller. A decentralized controller for the nominal system is computed first by imposing structural constraints on the coefficient matrices gradually. Then, the decentralized controller is modified again gradually to cope with the uncertainties.

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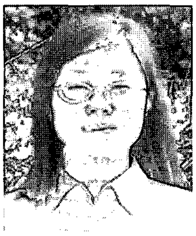

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