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# INFLUENCE OF THERMAL CONDUCTIVITY AND VARIABLE VISCOSITY ON THE FLOW OF A MICROPOLAR FLUID PAST A CONTINUOUSLY MOVING PLATE WITH SUCTION OR INJECTION

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ABSTRACT This paper investigates the influence of thermal conductivity and variable viscosity on the problem of micropolar fluid in the presence of suction or injection. The fluid viscosity is assumed to vary as an exponential function of temperature and the thermal conductivity is assumed to vary as a linear function of temperature. The governing fundamental equations are approximated by a system of nonlinear ordinary differential equations and are solved numerically by using shooting method. Numerical results are presented for the distribution of velocity, microrotation and temperature profiles within the boundary layer. Results for the details of the velocity, angular velocity and temperature fields as well as the friction coefficient, couple stress and heat transfer rate have been presented.

## 1. INTRODUCTION

Recently, the study of the dynamics of micropolar fluids has received considerable interest, because of its wide applicability in energy, such as geothermal energy technology, petroleum recovery, glass fiber production, metal extrusion, hot rolling, the cooling and/or drying of paper and textiles, and wire drawing.

Most of the existing analytical studies for this problem are based on the constant physical properties of the ambient fluid [1,2,3]. However, it is known that these properties may change with temperature [4]. To accurately predict the flow and heat transfer rates it is necessary to take into account this variation of viscosity and thermal conductivity. The study of heat transfer and the flow field is necessary for determining the quality of the final products of these processes as explained by Karwe and Jaluria [5,6].

In studying the motion of such a fluid, the non-linearity of the basic equation and additional mathematical difficulties associated with it has led several investigators to explore the perturbation and numerical methods. Hydrodynamic flows of a viscous and incompressible fluid have been studied under different physical conditions with variable fluid properties by Hassanien [4] and Seddeek [7]. In many particle engineering system, both the plane surface and the ambient fluid are moving in parallel.

Hence, the aim of the present work is to study the effects of variable viscosity and variable thermal conductivity on heat transfer from moving plate in a steady, incompressible, micropolar fluid in the presence of suction or injection.

## 2. Mathematical Formulation :

We consider a steady two-dimensional flow of a micropolar incompressible fluid past a continuously moving plate with suction or injection. The origin is located at the spot through which the plate is drawn in the fluid medium, the x-axis is chosen along the plate and y-axis is taken normal to it. We assume that the fluid properties are isotropic and constant, except for the fluid viscosity $\mu$, which is assumed to vary as an exponential function of temperature T, in the form

$$\mu = \mu_o e^{-\beta_1 \Theta} \tag{1}$$

Also, we assume that, the fluid thermal conductivity k is assumed to vary as a linear function of temperature in the form [8]

$$k = k_o(1+\beta_2\Theta) \tag{2}$$

Where $\beta_1$ and $\beta_2$ are parameters depending on the nature of the fluid and $\mu_o$, $k_o$ are the thermal diffusivity and viscosity at temperature $T_w$, respectively.

Under the usual boundary layer approximation, the governing equation for this problem can be written as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})+k_1\frac{\partial \sigma}{\partial y} \tag{4}$$

$$G_1\frac{\partial^2 \sigma}{\partial y^2}-2\sigma-\frac{\partial u}{\partial y}=0 \tag{5}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}(k\frac{\partial u}{\partial y})+\frac{\nu}{c_p}(\frac{\partial u}{\partial y})^2 \tag{6}$$

Subject to the boundary conditions

$$\left.\begin{array}{llll} u=U_o, \quad v=V_w, \quad T=T_w, & \sigma=0 & \text{at} & y=0 \\ u\to 0, \quad T\to T_\infty, & \sigma\to 0 & \text{as} & y\to\infty \end{array}\right\} \tag{7}$$

where x and y are the coordinate direction, u, v, $\sigma$ and T are the fluid velocity components in the x and y directions, the component of microrotation and temperature, respectively. $k_1$, $G_1$ and $\rho$ are the coupling constant, the microrotation constant and density of the fluid, respectively. $c_p$, $U_o, V_m, T_w$ and $T_\infty$ are the specific heat of the fluid at constant pressure, the uniform velocity of the plate, a non-zero velocity component at the wall, the temperature of the plate and the temperature of the fluid far away from the plate.

The governing equations (3)-(6) can be expressed in a simpler form by introducing the following transformation:

$$\left.\begin{aligned} &\eta = y\sqrt{\frac{U_o}{2\nu x}}, \quad \psi = \sqrt{2\nu U_o x}\, f(\eta)\ ,\ T = (T_w - T_\infty)\Theta(\eta) + T_\infty\ , \\ &\sigma = \sqrt{\frac{U_o^3}{2\nu x}}\, g(\eta)\ , \quad u = \frac{\partial\psi}{\partial y}\ , v = -\frac{\partial\psi}{\partial x} \end{aligned}\right\} \tag{8}$$

to obtain the ordinary differential equations for the function $f(\eta)$, $g(\eta)$ and $\Theta(\mu)$

$$f''' + e^{\beta_1\Theta}(f f'' + Kg') - \beta_1 f''\Theta = 0 \tag{9}$$

$$Gg'' - 2(2g + f'') = 0 \tag{10}$$

$$\Theta''(1 + \beta_2\Theta) + pr(Ec\, f''^2 + f\,\Theta' + \beta_2\Theta'^2) = 0 \tag{11}$$

and the boundary conditions (7) become

$$\left.\begin{aligned} &\text{at } \eta = 0:\ f = F_w\ ,\ f' = 1\ ,\ \Theta = 1\ ,\ g = 0 \\ &\text{as } \eta \to \infty:\ f' = \Theta = g = 0 \end{aligned}\right\} \tag{12}$$

In the above equations, a prime denotes differentiation with respect to $\eta$, and

$$\left.\begin{aligned} &K = \frac{k_1}{\nu}, \quad G = \frac{G_1 U_o}{\nu x}, \quad pr = \frac{\rho\nu c_p}{k_o}, \\ &Ec = \frac{U_o^2}{c_p(T_w - T_\infty)}, \quad F_w = -V_m\sqrt{\frac{2x}{\nu U_o}} \end{aligned}\right\} \tag{13}$$

are the Coupling constant parameter, Microrotation parameter, prandtl number, Eckert number and mass transfer parameter, respectively. Here $F_w$ is positive for suction and negative for injection.
For micropolar boundary layer flow, the wall skin friction $\tau_w$ is given by

$$\tau_w = [(\mu + k)\frac{\partial u}{\partial y} + k\sigma]_{y=0} \tag{14}$$

The skin friction coefficient $c_f$ can be defined as

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U_o^2} = -2Re_x^{-\frac{1}{2}} f''(0), \tag{15}$$

where $Re_x = \frac{U_o x}{\nu}$ the local Reynolds number.

The local heat flux coefficient t(or local Nusselt number) may be written as

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)} = -\frac{1}{2}Re_x^{\frac{1}{2}}\Theta'(0)\, , q_w = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{16}$$

## 3. RESULTS AND DISCUSSION

Equations (9), (10) and (11) with boundary conditions (12) can be integrated numerically by the Runge-kutta method with a systematic guessing of $f''(0)$, $g'(0)$ and $\Theta'(0)$ by the shooting technique. To assess the accuracy of the present method, comparisons between the present results and previously published data[9]. Table I presents the comparison of $\rightarrow f''(0)$, also Table II presents the comparison of the heat transfer rates $-\Theta'(0)$. In fact, this results show a close agreement, hence an encouragement for further study of the effects of other varies of parameters on the continuous moving surface.

Table I Comparison of the skin friction coefficient $f''(0)$.

| k | Soundalgekar et al.[9] | Present result |
|---|---|---|
| 0.1 | 0.6291 | 0.622588 |
| 0.2 | 0.6225 | 0.616853 |

Table II Comparison of the heat transfer rate $-\Theta'(0)$

| k | Soundalgekar et al.[9] | | Present result | |
|---|---|---|---|---|
| | G=2 | G=4 | G=2 | G=4 |
| 0.1 | 1.944 | 1.946 | 1.931308 | 1.931497 |
| 0.5 | 1.801 | - | 1.930250 | 1.931392 |

To study the behavior of the velocity, the angular velocity and the temperature profiles, curves are drawn for various values of the parameters that describe the flow in the case of air (pr=0.733) and water (pr=3) at $\eta_\infty = 7$, k=0.1,G=2 and Ec=0.02.

Figs. 1-3 show the effect of the variable viscosity parameter $\beta_1$ on the velocity, the angular velocity and the temperature distribution, respectively. As shown, the velocity and the angular velocity are decreasing with increasing $\beta_1$, but the temperature increases as $\beta_1$ increases. So, we conclude that for both air and water, the consequence of having a significant temperature dependent viscosity is to produce a marked effect on the temperature field in these convection flows. The temperature variation for both air and water is shown in Fig. 4, for various values of the thermal conductivity $\beta_2$ parameter. Clearly the temperature profiles increases as the thermal conductivity parameter $\beta_2$ increases in two cases.

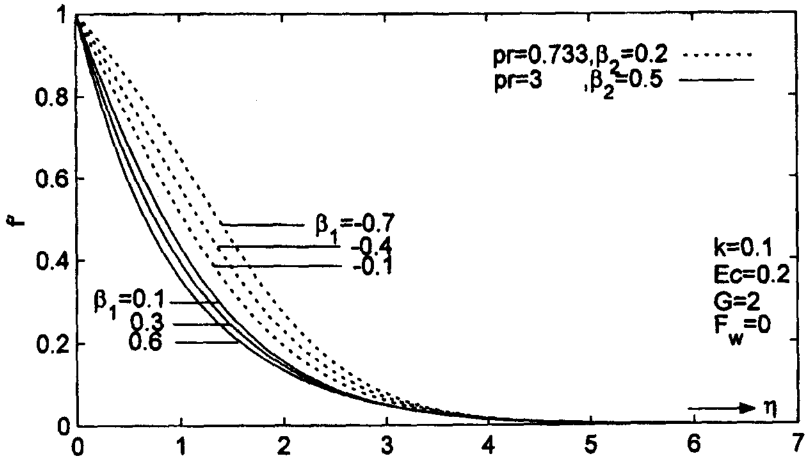


Fig. 1. Effects of viscosity parameter $\beta_1$ on velocity distribution $f'$

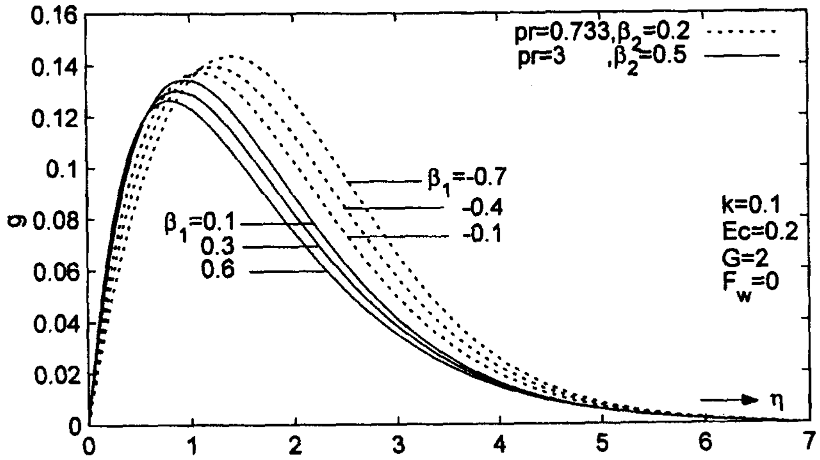


Fig. 2. Effects of viscosity parameter $\beta_1$ on angular velocity g

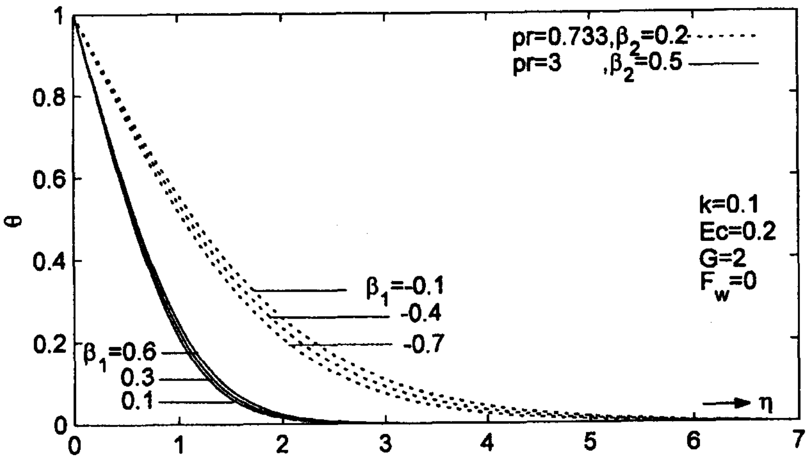


Fig. 3. Effects of viscosity parameter $\beta_1$ on temperature distribution $\theta$

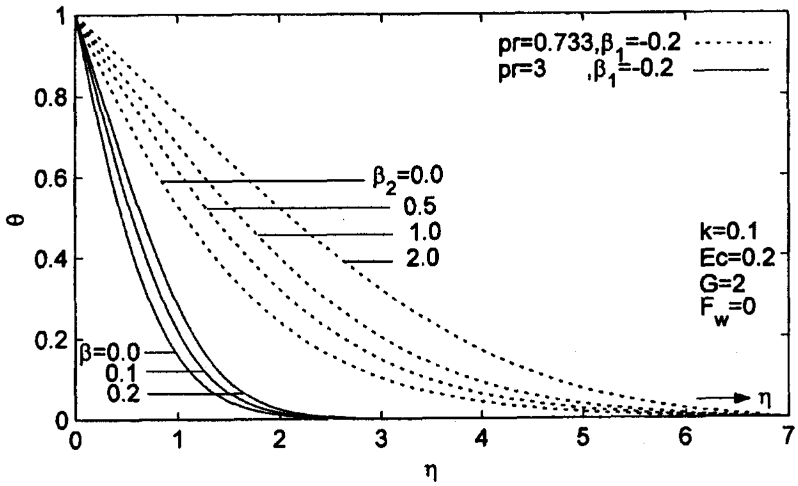


Fig. 4. Effects of the thermal conductivity parameter $\beta_2$ on temperature distribution $\theta$

Figs. 5-7 illustrate the velocity, angular velocity and temperature fields, respectively, for different values of porosity parameter $F_W$. This figures indicate that, all of this quantities decreases as $F_W$ increases. It can be seen that the velocity increases monotonically with injection $(F_W < 0)$ and decreases with increases in suction $(F_W > 0)$. Also, increasing values of the injection parameter move the location of the maximum value of the microrotation away from the surface.

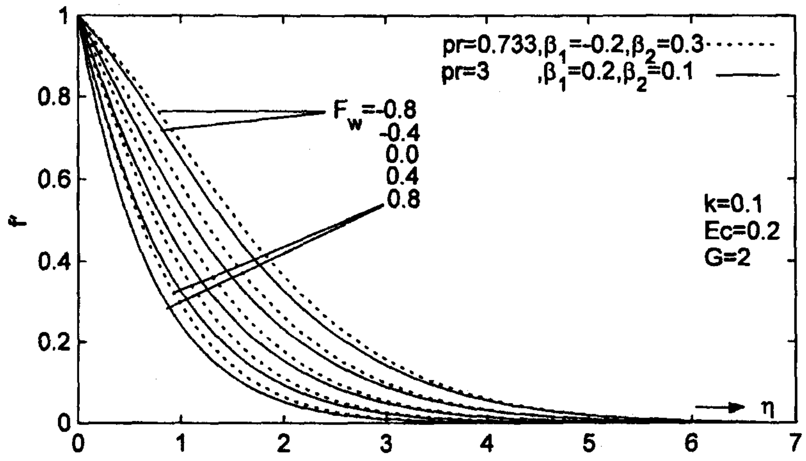


Fig. 5. Effects of porosity parameter $F_w$ on velocity distribution $f'$

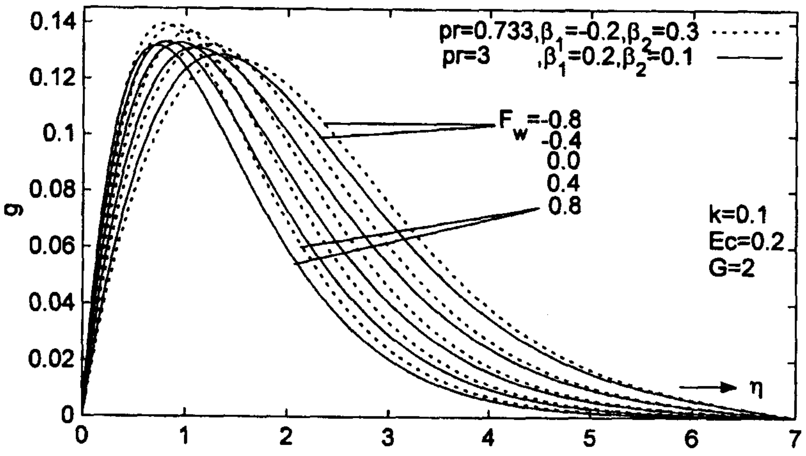


Fig. 6. Effects of porosity parameter $F_w$ on angular velocity g

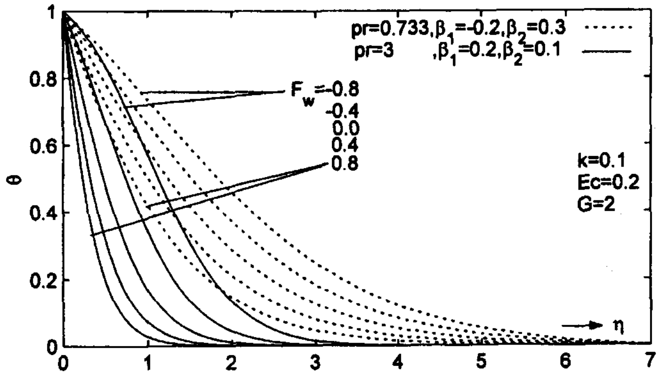


Fig. 7. Effects of porosity parameter $F_w$ on the temperature distribution $\theta$

Table III represents values of the skin friction coefficient $f''(0)$, plate couple stress $g'(0)$ and the heat transfer rates $-\theta'(0)$ for various values of the variable viscosity parameter $\beta_1$, the variable thermal conductivity parameter $\beta_2$ and the porosity parameter $F_W$. It is clear that, with increasing $\beta_1$, $f''(0)$ and $-\theta'(0)$ decrease and $g'(0)$ increases in two cases (air and water), whereas with increasing $\beta_2$, $f''(0)$, $g'(0)$ and $-\theta'(0)$ decrease the case of air, but $f''(0)$ and $g'(0)$ increase and $-\theta'(0)$ decreases as $\beta_2$ increases in the case of water. $g'(0)$ and $-\theta'(0)$ increase as $F_W$ increases whereas $f''(0)$ decreases. We conclude that for the case of injection $(F_W < 0)$, the heat transfer rate was reported to decrease with increased injection.

Table III Values of $f''(0)$, $g'(0)$ and $-\theta'(0)$ for k=0.1, Ec=0.02 and G=2

| pr | $\beta_1$ | $\beta_2$ | $F_w$ | f''(0) | g'(0) | $-\theta'(0)$ |
|---|---|---|---|---|---|---|
| 0.733 | -0.1 | 0.2 | 0.0 | -0.485592 | 0.312835 | 0.464397 |
| | -0.4 | 0.2 | 0.0 | -0.372869 | 0.275746 | 0.485617 |
| | -0.7 | 0.2 | 0.0 | -0.262136 | 0.234901 | 0.507664 |
| | -0.2 | 0.5 | 0.0 | -0.552239 | 0.332948 | 0.383020 |
| | -0.2 | 2.0 | 0.0 | -0.556309 | 0.331002 | 0.228236 |
| | -0.2 | 4 | 0.0 | -0.558481 | 0.330390 | 0.158079 |
| | -0.2 | 0.2 | 0.8 | -0.241789 | 0.211229 | 0.221376 |
| | -0.2 | 0.2 | 0.0 | -0.551329 | 0.332630 | 0.426182 |
| | -0.2 | 0.2 | -0.8 | -1.003199 | 0.452500 | 0.721355 |
| 3.0 | 0.1 | 0.5 | 0.0 | -0.666476 | 0.362577 | 0.903471 |
| | 0.3 | 0.5 | 0.0 | -0.813429 | 0.390704 | 0.879574 |
| | 0.6 | 0.5 | 0.0 | -0.982137 | 0.420909 | 0.852131 |
| | 0.2 | 0.0 | 0.0 | -0.717061 | 0.370385 | 1.191196 |
| | 0.2 | 0.1 | 0.0 | -0.713081 | 0.371713 | 0.895878 |
| | 0.2 | 0.2 | 0.0 | -0.710075 | 0.372670 | 0.735516 |
| | 0.2 | 0.1 | -0.8 | -0.259679 | 0.230134 | 0.203541 |
| | 0.2 | 0.1 | 0.8 | -1.409252 | 0.499884 | 2.743576 |

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