J. KSIAM **Vol.9**, No.1, 31-53, 2005

ASSET MODEL INVESTED BY SHORT-SAMPLING INTERVALS

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ABSTRACT. We analyze some real data and, from the background of analysis of data, we define a multi-dimensional jump-type asset model which is derived from short-sampling asset prices. We study some basic properties of this asset model.

1. INTRODUCTION

The purpose of this paper is to introduce a new asset model which is derived from the investigation of asset price movements by short-sampling intervals. We analyze some real data by some computer programs and predict various types of asset models by changing sampling time intervals. Thus, we define an asset model which is useful to study short-term asset price movements, for example, intraday, half day, etc. For this model, we study option pricing mainly.

Since the stock market was started in 1531, Antwerp, Belgium, many mathematical asset model of stock markets was introduced for hundreds of years. The first attempt known to model the stock market using probability is due to L. Bachelier in Paris about 1900. His idea was developed by several persons(c.f. [3], [20]). A famous model in mathematical finance, Black-Scholes model was studied by F. Black and M. Scholes (1973) and R. Merton (1973). They used Itô stochastic calculus and Markov property of diffusion in key ways even if their main result was option pricing. Development of this area has been closely intertwined with that of the theory of stochastic integration. Much has happened in the intervening two decades. This area has become a branch of mathematics, and sometimes asset models are called financial models of stochastic processes defined by stochastic differential equations(SDE) in probability theory.

Date: 20 Feb. 2005.

2000 *Mathematics Subject Classification.* 60H10, 60J75, 62P20.

Key words and phrases. Asset model, SDE, European option, Lévy process.

*This research was accomplished with research fund provided by Kangwon National University, Korea, 2003.

Nowadays, we have many kinds of asset models which are represented and defined by the solution of SDE, and are distinguished as continuous-type, jump-type, etc.

Perhaps, some mathematicians who study mathematical finance may hope their results are used in real economic markets. But it is difficult to know the direct role of asset models for real economic markets. Many economists also study many types of asset models, for example, ARCH, GARCH, EWMA, EGARCH, etc., which are very strange to some mathematicians. Further, also there are many strange terms, for example, VaR, ISD, QMLE, NTM ATM, etc. In almost all of mathematical finance theory, one assume an asset model first and study its property. We analyze real market data, and make some tables and various types of figures. From these, we can predict many types of models by changing sampling intervals. From the background of analysis of data, we define an asset model by a SDE. Our analysis of data is not to prove our model, but to help understanding it(c.f. [4], [19]). For example, we would like to follow the development step of [16] than of [12].

From the analysis of data, we conclude it is possible to define a short-term asset model from short time data because we can get much information by using short-sampling time. Further, if we use the summation of return rates of another assets which are adjusted by influence levels, we can get more stable and realistic asset model. Thus, we define a short-term asset model as

$$dX_t^i = X_{t-}^i \sum_{j=1}^{m} \alpha_j^i(t-)(dW_t^j + dM_t^j), \quad d \le m, \quad i = 1, 2, \cdots, d, \tag{1}$$

where $\alpha_j^i(t-)$ denotes the level of influence to the ith asset price at time $t-$ from the jth source of uncertainty, W_t^j is a Brownian motion, and M_t^j is a jump-type martingale. Another special quality of this model is not have deterministic term which is an appreciation rate(c.f. [16]).

We define Q-price at time $t = 0$ of option $g(X_T)$ as

$$u^Q(0) = E^Q[e^{-rT} g(X_T)]. \tag{2}$$

Then, we get option price

$$\begin{aligned} u^Q(0) &= E^Q[BS(g(\cdot), \bar{x}_0, \rho, r, T)] \\ &= E^Q[E^*[g(\bar{x}_0 \exp\{\sum_{j=1}^{m}(-\frac{1}{2}\rho_j^2(T)) + T^{1/2}\sum_{j=1}^{m} \rho_j(T)\bar{W}_T^{j,Q})\})]], \end{aligned} \tag{3}$$

where E^* is an expectation with respect to equivalent martingale measure Q defined in $BS(g(\cdot), x_0, \rho, r, T)$ and $\bar{x}_0$ is an initial value. We hope that the trading strategy of our model and others are studied in the sequel.

In section 2, we analyze some real data by using some computer programs. Form these data, we predict many types of asset model by taking various sampling intervals. In section 3, we define an asset model and study option pricing for a fixed measure. In appendix, we include several figures which are implied by some sampling time.

2. ANALYSIS OF DATA

Table 1 is showing real trades of stock Samsung from January 20th, P.M. 1:26:56, 2005. As we see in Table 1, there are many trades by same price in short time, or at same time. For a little long time, there is no tick even if there are many trades. From this fact, we can predict asset model may not be represented by traditional asset models containing Brownian motion part and jump-part. Further, we can image that many different asset models may be obtained by taking different sampling time intervals.

Table 1
Real Trades of Stock Samsung
Jan.20th P.M. 1:26:56 - P.M. 1:28:30, 2005

Time hh mm ss	Conclude Price	Ratio	Ratio	Ups and Downs	Sell Price	Call Price	Conclude Number
13 26 56	481000	2	1000	0.21	481500	481000	23
13 26 57	481000	2	1000	0.21	481500	481000	69
13 26 58	481000	2	1000	0.21	481500	481000	14
13 26 58	481000	2	1000	0.21	481500	481000	56
13 27 06	481000	2	1000	0.21	481500	481000	47
13 27 09	481000	2	1000	0.21	481500	481000	100
13 27 17	481000	2	1000	0.21	481500	481000	5
13 27 17	481000	2	1000	0.21	481500	481000	3
13 27 21	481000	2	1000	0.21	481500	481000	14
13 27 22	481000	2	1000	0.21	481500	481000	486
13 27 28	481000	2	1500	0.21	481500	481000	1
13 27 37	481000	2	1000	0.21	481500	481000	64
13 27 37	481000	2	1000	0.21	481500	481000	156
13 27 58	481000	2	1000	0.21	481500	481000	10
13 28 03	481000	2	1000	0.21	481500	481000	1
13 28 13	481000	2	1000	0.21	481500	481000	5
13 28 24	481000	2	1000	0.21	481500	481000	14
13 28 24	481000	2	1000	0.21	481500	481000	46
13 28 30	481000	2	1000	0.21	481500	481000	34
....	....	...	.....	......	......	......	...

2.1. **Real tick data.** To predict many different type of asset models, we use the US Treasury Bond Futures Chicago contract with a maturity of December 1990. Our sample begins on the 1st of October, 1990, A.M. 07:20:31. The first part of several minutes of real tick data is shown in Table 2. This stream of data resembles what might be seen on a real-time data-feed offered by many vendors.

Table 2
US Treasury Bond Futures
Price Ticks 1990 October 1

Time of Day hh mm ss	Contract Code Month Year	Futures Price US Dollar
07 20 31	Z 1990	89.59375
07 20 32	Z 1990	89.56250
07 20 33	Z 1990	89.59375
07 20 34	H 1991	89.18750
07 20 38	H 1991	89.15625
07 20 38	Z 1990	89.56250
07 20 45	Z 1990	89.59375
07 20 54	Z 1990	89.56250
07 21 00	Z 1990	89.59375
07 21 11	Z 1990	89.56250
07 21 18	Z 1990	89.59375
07 21 21	Z 1990	89.56250
07 21 41	Z 1990	89.59375
07 21 55	Z 1990	89.53125
07 21 59	Z 1990	89.56250
07 21 59	Z 1990	89.53125
07 22 10	Z 1990	89.56250
07 22 12	Z 1990	89.53125
07 22 18	Z 1990	89.56250
....	....	....

If we read Figure 1, almost all ticks are one unit up(+) or down(-). Further, we notice that, so-called, big jumps are not occur. We can find a little big jump at a little before 4000th tick in Figure 1. Figure 2 shows the movements of actual price till the time which occurs the 4000th tick. Following Table 3 shows the number of each Δt for 500 seconds, 500 seconds, 600 seconds, and 240 seconds. Figure 3 also shows that the distribution of the length of time interval Δt between a tick and next tick. Thus, from this Figure 3, we know that many ticks occur in short time, i.e., many next ticks occur within several seconds.

Table 3
Time(seconds) between ticks(Δt)
Oct. 1st A.M. 07:20:31 - 07:51:11(30min.40sec.)

Size of Δt (seconds)	20:31 -28:51 500sec.	28:51 -37:11 500sec.	37:11 -47:11 600sec.	46:35 -50:35 240sec.
40				1
38			1	
33		1	1	
30				1
28		1		
27			1	
26				1
24		1		
23			1	
22			2	1
20	2			
19	1		3	1
18	1	1		
17		1	1	
16	1	2		1
15	1	1		
14	4	1	3	2
13		1	1	
12		2	1	1
11	4	1	4	
10	1	4	2	
9	5	2	5	
8	3	3	3	1
7	7	3	5	1
6	6	5	4	2
5	4	4	2	1
4	2	7	1	3
3	6	7	2	3
2	8	11	3	6
1	16	6	5	
0	2	3	2	1

2.2. **Return rates.** As we see in Table 4, if we consider ticks only, then the number of returns are not symmetric in short time interval: 500 seconds, 500 seconds, 600 seconds, and 240 seconds. If we cumulate all ticks for one month, then as we see in Table 5, we notice that returns may converge to some almost symmetric curve.

Table 4
Real Total Ticks for 500, 500,600,240,1840 Seconds
Oct. 1st A.M. 07:20:31 - 07:51:11(30min.40sec.)

Tick size $\times 10^{-4}$	20:31 -28:51 500sec.	28:51 -37:11 500sec.	37:11 -47:11 600sec.	46:35 -50:35 240sec.	20:31 -51:11 1840sec.
3	37	33	28	14	112
0	0	0	0	0	0
-3	36	35	27	14	112
-7	1	0	0	0	1
sum.	74	68	55	28	225

Table 5
Real Total Ticks Number for One Month
Oct. 1st A.M. 07:20:31 - One Month

Tick size $\times 10^{-4}$	15	11	7	3	0	-3	-7	-11	-15	sum.
Number of Ticks	7	0	11	19621	16	19544	22	0	10	39210

Let us think 10 seconds investigation interval, for 1840 seconds, as we see in Table 6 and 7, the return rates are not distributed as normal symmetric density and there is no big jump in this short time, 1840 seconds. Figure 4 is a graph of the first vertical of Table 6 and Table 7 for the first 500 seconds. Figures 5 shows the cumulated ticks of 10 seconds sampling interval for 40,000 seconds(11hours, 6minutes and 40seconds). But, they show also that the return rate have not balanced curve in short-term asset price movements. In Figure 5, we get a little big jump at just before of 20,000 seconds.

Table 6 10 Second Interval Investigation Tick Numbers Oct. 1st A.M. 07:20:31 - 07:51:11(30min.40sec.)					
Tick size $\times 10^{-4}$	20:31 -28:51 500sec.	28:51 -37:11 500sec.	37:11 -47:11 600sec.	47:11 -51:11 240sec.	20:31 -51:11 1840sec.
7	2	5	0	0	7
3	11	9	12	5	37
0	23	20	37	16	96
-3	10	16	10	3	39
-7	4	0	1	0	5
sum.	50	50	60	24	184

Table 7 10 Second Interval Cumulated Tick Numbers Oct. 1st A.M. 07:20:31 - 07:51:11(1840second)				
Tick size $\times 10^{-4}$	20:31 -28:51 500sec.	20:31 -37:11 1000sec.	20:31 -47:11 1600sec.	20:31 -51:11 1840sec.
7	2	7	7	7
3	11	20	32	37
0	23	43	80	96
-3	10	26	36	39
-7	4	4	5	5
sum.	50	100	160	184

As we meet in Figure 3 of [10], micro structure of asset price movements is jump type perfectly. From this, if we use a difference equation $X_{t+\Delta t} - X_t = X_t \cdot Y_t$, for $\Delta t = 10$ second, we may derive some asset models which are usual in mathematical finance area, and are represented by using Brownian motion and jump-type Lévy process. But, in our short-sampling data in short time, almost all of return rates are concentrated at around 0, and it is not easy to predict common models from our Tables and Figures.

2.3. **Long-sampling interval.** Figure 6 is that of the sampling interval is $\Delta t = 100$ seconds (one minute and 40 seconds) for 400,000 seconds (111 hours, 6 minutes and 40 seconds). Figure 7 is that of the sampling interval is $\Delta t = 1,000$ seconds (16 minutes and 40 seconds) for 500,000 seconds (138 hours, 53 minutes and 20 seconds). Figure 8 is that of the sampling interval is $\Delta t = 10,000$ seconds (2 hours, 46 minutes and 40 seconds) for 600,000 seconds (166 hours, and 40 minutes). Finally, Figure 9 is that of the sampling interval is $\Delta t = 100,000$ seconds (27 hours, 46 minutes and 40 seconds) for 1,500,000 seconds (416 hours, 40 minutes).

From these long-sampling interval data, we notice that the distributions of return rates may have almost normal density functions. For these asset models, we also have some articles. Particularly, in [1], we meet many figures for the sampling interval $\Delta t =$ one day, one month, three month, and one year in Figure 6 in page 41 of [1], and meet an asset model: Disentangled diffusion from jumps model.

2.4. **Statistical method to convolution simulation.** If we want to get m-fold convolution density for short-term asset model, we can use statistical data to get more stable density. The distribution(density) of summed return is represented as more stable bell shape than each unbalanced figure.

Simulation of three stocks tick frequency				
Tick-size	S-asset	L-asset	H-asset	S/2+L/4+H/4
6×10^{-4}	1	4	3	2.25
	1	1	3	1.5
4×10^{-4}	6	3	2	4.25
	10	3	1	6
2×10^{-4}	11	1	2	6.25
	96	183	94	117.25
0	340	270	360	327.5
	95	123	125	109.5
-2×10^{-4}	20	2	3	11.25
	10	0	0	5
-4×10^{-4}	2	1	2	1.75
	5	4	2	4
-6×10^{-4}	3	5	3	3.5
sum.	600	600	600	600

2.5. **Conclution.** As we see Table 4,6, and 7, in short-sampling asset price movements of short time, the mean of return rates are not 0 and also density is not symmetric. We notice from Table 5, if we cumulate for long time, the density of distribution of return rates may converge to some heavy tail with jump model. Even if we can get much information for short time data by using short-sampling intervals, the distributions of short-term asset models may not be stable.

The short-term asset price movements are flexible figures relatively. We think it comes from volition and the intension of each investor (writer). Many writer's will influence to the prices of assets. Their decision comes from many information, for example, auxiliary data, another asset prices, waiting number of stocks, etc. Thus, if we think an short-sampling asset model, we must take into account above many factors.

From many tables and figures, a short-term asset model, it is difficult to represent by using a fixed stochastic process because derived figures are changeable and data are not many(c.f. Table 6). Thus, if we use the summation of return rates of another assets which are adjusted by influence levels, we can get more stable realistic asset model than one represented by a return rate process. Further, in short-term (for example, intraday, half day, 500 seconds, etc) asset models, the deterministic terms of return rate processes are not needed because they are stable increasing rates of assets derived from (arising in) a little long time in general.

3. ASSET MODELS

In mathematical finance side, asset model started from Black-Scholes model(c.f., [6], [18]). As we know, asset models were developed by many mathematicians. Nowadays, we can meet many types of asset models represented by the solutions of SDEs.

In general, many asset models were started form one of assumptions which are of two kinds of return rates. One is Z_t defined by $Z_t = \log X_t - \log X_{t-1}$, and the other is Z_t defined by $Z_t = (X_t - X_{t-1})/X_{t-1}$. For the prior assumption, if we think the returns over n periods, then the sum is

$$Z_t + Z_{t+1} + \cdots + Z_{t+n-1} = \log X_{t+n-1} - \log X_{t-1}.$$

But many traditional models which are derived from above lognormality(prior assumption) deviate in systematic ways from empirical observation(c.f., [4], [9]). From the second assumption of return rate Z_t defined by $Z_t = (X_t - X_{t-1})/X_{t-1}$ for an asset price X_t, we can derive a stochastic difference equation;

$$\begin{aligned} X_k - X_{k-1} &= X_{k-1}U_k, \quad k = 1, 2, \cdots, \\ &:= X_{k-1}(\xi_k - a), \end{aligned}$$

where ξ_k are some random variables, a is some real number. From the Figure 1(c.f., [10], Figure 3), as we predicted in section 2.2, if we take unit time as $\Delta t = 10$ seconds, we can derive stochastic difference equations on a probability space (Ω, F, P),

$$\begin{aligned} X_k^n - X_{k-1}^n &= C(X_k^n)(\xi_k^n - a_k^n), \quad k \in N \\ X_0^n &= x_0 \in R^d, \end{aligned} \tag{4}$$

where $a_k^n = E[\xi_k^n I_{[0,1]}(|\xi_k^n|)|F_{k-1}^n]$ and C is a Lipschitz continuous function from R^d to $R^d \times R^m$, and define an interpolating process X_t^n of $\{X_k^n\}_k$ by

$$X_t^n = X_{[nt]}^n, \quad for \quad t \in [0, \infty). \tag{5}$$

Then, under some assumptions on $\{\xi_k^n\}$ for the weak converges of $\{X^n\}_n$ to a jump-diffusion, we can get that the X_t^n of (5) is a solution of

$$X_t^n = x_0 + \int_0^{t+} v(X_{s-}^n) dZ_s^n,$$

where

$$\begin{aligned} Z_t^n &= \int_0^{t+} \int_{|z|\leq 1} z^n \tilde{N}_p(dz, ds) + \int_0^{t+} \int_{|z|>1} z^n N_p(dz, ds) \\ &= \sum_{k=1}^{[nt]} \{\xi_k^n - E[\xi_{k,1}^n | F_{k-1}^n]\}, \end{aligned}$$

and that the law of $\{X_t^n\}_n$ of (5) converges weakly to the law of unique solution of SDE

$$X_t = x_0 + \int_0^{t+} C(X_{s-}) dY_s, \tag{6}$$

where Y_t is a Lévy process

$$Y_t = W_t + \int_0^{t+} \int_{|z|\leq 1} z\tilde{N}_p(dz, ds) + \int_0^{t+} \int_{|z|>1} zN_p(dz, ds), \tag{7}$$

W_t is a centered Brownian motion with a covariance matrix V, N_p is a stationary Poisson process with the intensity measure $\nu(dz)ds$.

This result looks like very useful one. But when we want to buy or sell some given stock in real stock market, first, we check the movements of prices of the past to given fixed stock, and then we compare present price with some another stock prices which influence to given fixed stock by using electric bulletin board or internet. Thus, we assume that the price movements are influenced from some another stock prices. Our model is similar as that of [16] when we don't think jumps, and is influenced from [4] which described the empirically observed distribution more closely than the traditionally used lognormal model. From Table 4, and 5, we also know that the distribution of X_t is concentrated in 0. Thus the form of density function is very high at mean 0, and the variance is very small.

For $dX_t = (dX_t^1, dX_t^2, \cdots, dX_t^d)^*$ representing d asset prices and $\alpha_j^i(t-) > 0$ which denotes the levels of influence to the ith asset price from the jth source of uncertainty (weight coefficients), we define an asset model as a SDE

$$dX_t^i = X_{t-}^i \sum_{j=1}^m \alpha_j^i(t-) dY_t^j, \quad d \le m, \quad i = 1, 2, \cdots, d, \tag{8}$$

where

$$Y_t^j = W_t^j + \int_0^{t+} \int_{|z|<1} z^j \tilde{N}_p(dz, ds), \tag{9}$$

and $\alpha_j^i(t-)$ denotes the level of influence to the ith asset price at time $t-$ from the jth source of uncertainty, and W_t^j is a Brownian motion having mean μ and variance V and the density function which is heavy tail form. If we put

$$M_t^j := \int_0^t \int_{0<|z|<1} z^j \tilde{N}_p(dz, ds),$$

then, we denote our asset model by the form

$$dX_t^i = X_{t-}^i \sum_{j=1}^m \alpha_j^i(t-)(dW_t^j + dM_t^j), \tag{10}$$

where M_t^j is the martingale given by stochastic integrals which is integrated by compensated counting measure $\tilde{N}_p(dz, ds)$ denoted from stationary Poisson point process $\{p_t\}$ on $R^m - \{0\}$ with intensity measure ν (Lévy measure) satisfying

(A.1). $\int_{|x|\ge 1} z^2 \nu(dz) < \infty$,

(A.2). $\int \ln(1+x)^2 \nu(dz) < \infty$,

(A.3). The support of ν is contained in $(-1, \infty)$,

(A.4). $\int \min\{1, x^2\} \nu(dz) < \infty$.

Further, if we think our model as a volatility model, then we can represent it, by using a volatility σ_t process, as the following form:

$$dX_t^i = X_{t-}^i \sum_{j=1}^m \alpha_j^i(t-)(\sigma_t^j d\bar{W}_t^j + dM_t^j), \tag{11}$$

where $\bar{W}_t^j$ are standard Brownian motions.

In the following, we think the option pricing of asset price X_t^i expiring at time $T > 0$, and whose payoff at time T is equal to $g(X_T^i)$. We assume that payoff function $g(\cdot)$ is a real, positive, measurable function and that $g(X_T^i)$ has a finite expectation. In incomplete markets, it is almost impossible to get the exact value of option because we can't find an appropriate new probability measure. Thus, we assume that there is a probability measure Q equivalent to the given probability P, and denote E^Q as the expectation relative to new equivalent measure Q. In any case, we define Q-price at time $t=0$ of option $g(X_T^i)$ as following

$$u^{Q,i}(0) = E^{Q,i}[e^{-rT}g(X_T^i)]. \tag{12}$$

This Q-price of option is useful from a theoretical and a practical point of view. In fact, under any meaningful definition of feasible self-financing replicating strategy, the Q-price is a price that prevents arbitrage opportunities. Further, in short-term asset model, option pricing of jump-type asset model under a fixed equivalent martingale measure Q has some meaning because almost all of circumstance of price movements are not changed in short time (not influenced from interest rate, etc).

From the previous, ith asset prices(ith coordinate) is represented as following; for deterministic functions $\alpha_j^i(s-)$,

$$X_t^i = x_0^i + \int_0^t X_{s-}^i \sum_{j=1}^m \alpha_j^i(s-)(\sigma_t^j d\bar{W}_s^j + dM_s^j), \quad i,j = 1,2,\cdots,m. \tag{13}$$

In the following, we will omit the superscript i. We will get the Q-price of option $g(X_T)$. Thus, we assume that Q is an equivalent martingale measure, and get an SDE:

$$\begin{aligned} dX_t &= X_{t-}\sum_{j=1}^m[\alpha_j(t-)\sigma_t^j d\bar{W}_t^{Q,j} + \alpha_j(t-)dM_t^{Q,j}] \\ &= X_{t-}\sum_{j=1}^m[\rho_j(t)d\bar{W}_t^{Q,j} + \alpha_j(t-)dM_t^{Q,j}], \end{aligned} \tag{14}$$

where $\rho_j(t) := \alpha_j(t-)\sigma_t^j$ are coefficients of standard Brownian motions named by influence volatility,

$$\begin{aligned} \bar{W}_t^{Q,j} &= \bar{W}_t^j - \int_0^t \Phi_s^j ds, \\ M_t^{Q,j} &= N_t - \int_0^t (1+\Psi_s^j)\lambda ds, \end{aligned}$$

and where Φ_s^j and Ψ_s^j are given by same terminology as [13]. We get a solution as a closed form of X_t,

$$\begin{aligned} X_t &= x_0 \exp\{\sum_{j=1}^m[-\frac{1}{2}\rho_j^2(t) + \rho_j(t)\bar{W}_t^{Q,j} - \alpha_j(t-)\int_0^t \lambda^j(1+\Psi_s^j)ds]\} \\ &\quad \times \Pi_{0\le s\le T}[\Pi_{j=1}^m(1+\alpha_j(s-)\Delta N_s^j)]. \end{aligned} \tag{15}$$

To get the option price $u^Q(0)$, we introduce the Black-Scholes model (c.f. [13]). Let $BS(g(\cdot), x_0, \rho, r, T)$ be the no-arbitrage price of option of Black-Scholes model $X_t, 0 \leq t \leq T$ which is the solution of SDE:

$$dX_t = X_{t-} \sum_{j=1}^{m} \rho_j(t) d\bar{W}_t^{Q,j},$$

and whose solution is

$$X_t = x_0 \exp\{\sum_{j=1}^{m}(-\frac{1}{2}\rho_j^2(t) + \rho_j(t)\bar{W}_t^{Q,j})\},$$

where r is the risk-free interest rate, ρ is influence volatility and T is the maturity. Then we get

$$\begin{aligned} &BS(g(\cdot), x_0, \rho, r, T) \\ &:= e^{-rT} E^*[g(x_0 \exp\{\sum_{j=1}^{m}(-\frac{1}{2}\rho_j^2(T)) + T^{1/2}\sum_{j=1}^{m}\rho_j(T)\bar{W}_T^{Q,j}\})] \\ &= e^{-rT}\int_{-\infty}^{\infty}[g(x_0 \exp\{\sum_{j=1}^{m}(-\frac{1}{2}\rho_j^2(T)) + T^{1/2}y\})]f(y)dy, \end{aligned} \tag{16}$$

where $f(y)$ is a m-fold convolution function defined by

$$f(y) = \frac{d}{dy}Q(\sum_{j=1}^{m}\rho_j(T)\bar{W}_T^{Q,j} \leq y), \quad j = 1, 2, \cdots, m.$$

Thus, we get option price of our model;

Theorem 3.1 Let M^Q be a Q-local martingale, that is Q-independent of W^Q. Then we get

$$\begin{aligned} u^Q(0) &= E^Q[BS(g(\cdot), \bar{x}_0, \rho, r, T)] \\ &= E^Q[E^*[g(\bar{x}_0 \exp\{\sum_{j=1}^{m}(-\frac{1}{2}\rho_j^2(T)) + T^{1/2}\sum_{j=1}^{m}\rho_j(T)\bar{W}_T^{Q,j})\})]], \end{aligned} \tag{17}$$

where E^* is the expectation w.r.t. equivalent measure Q defined in $BS(g(\cdot), x_0, \rho, r, T)$ and

$$\begin{aligned} \bar{x}_0 &= x_0 \exp\{-\sum_{j=1}^{m}\alpha_j(T)\int_0^T \lambda^j(1+\Psi_s^j)ds\} \\ &\quad \Pi_{0\leq s\leq T}[\Pi_{j=1}^{m}(1+\alpha_j(s-)\Delta N_s^j)]. \end{aligned}$$

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4. Appendix: FIGURES

Figure 1. Tick Returns on US T-Bound Future

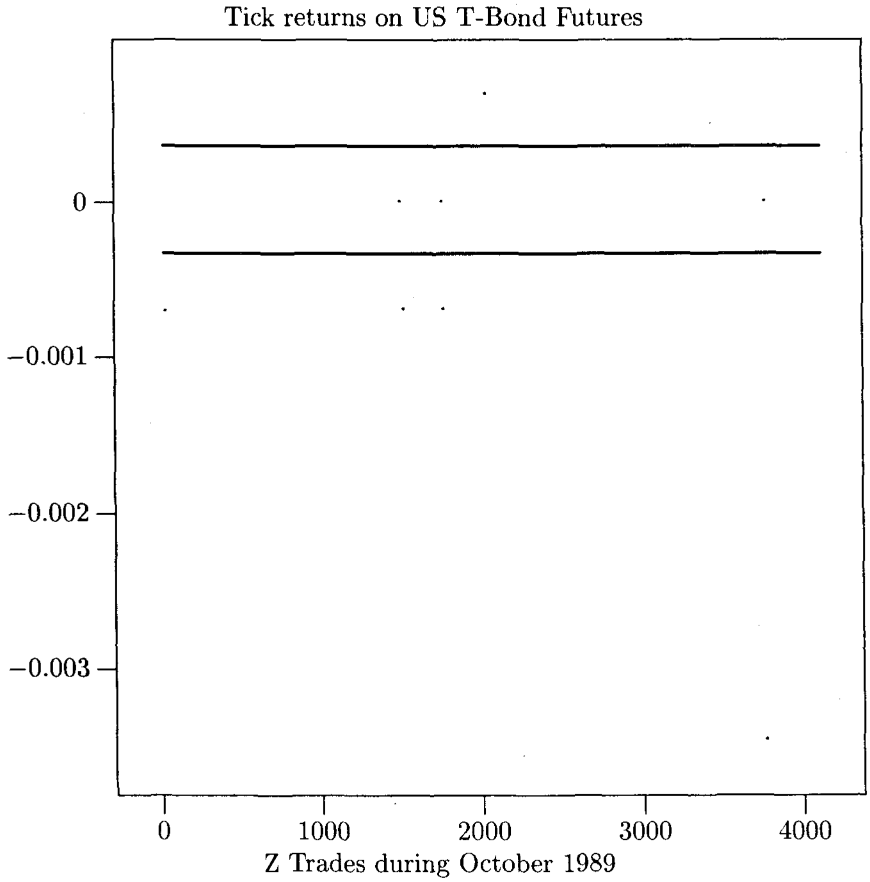

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Figure 2. Actual Prices

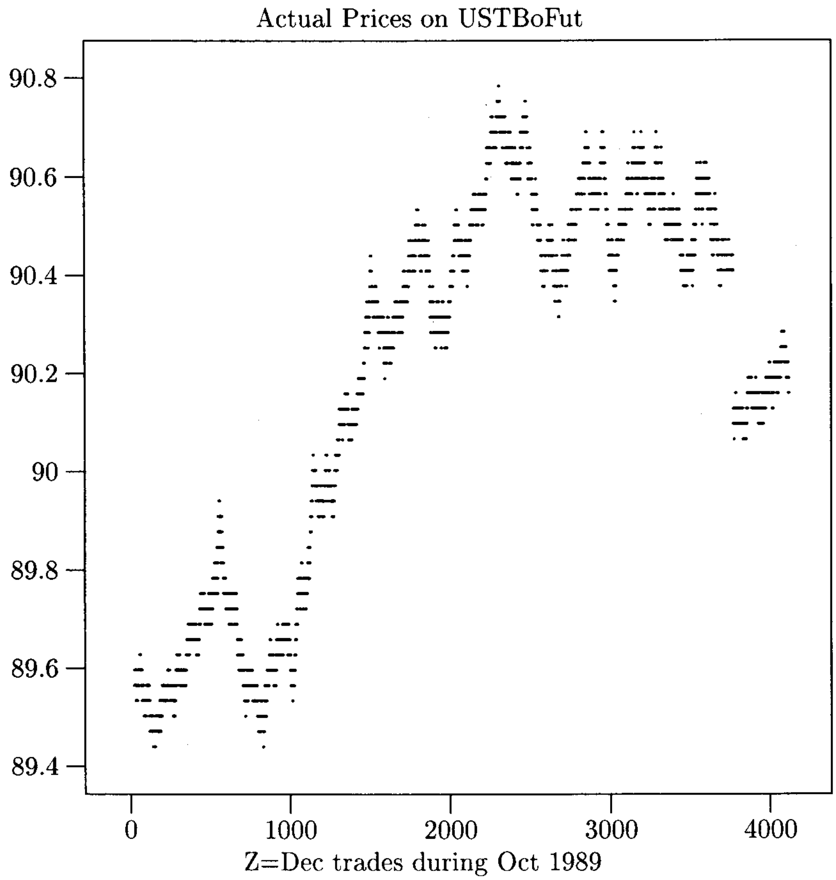

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Figure 3. dt Change Time

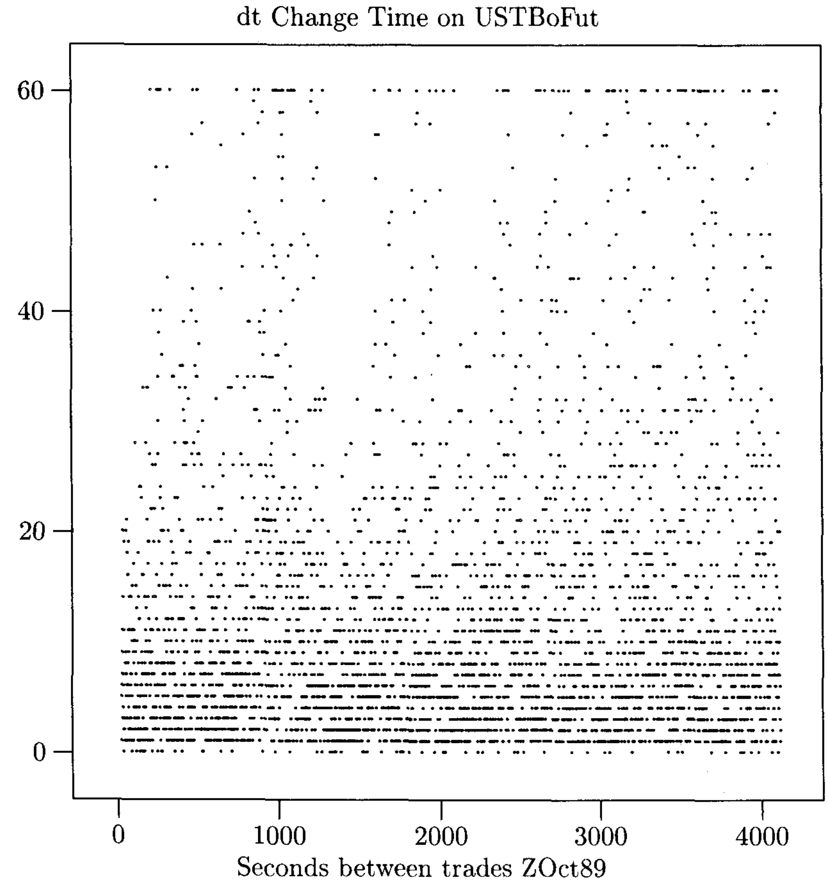

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Figure 4. 10 seconds Returns

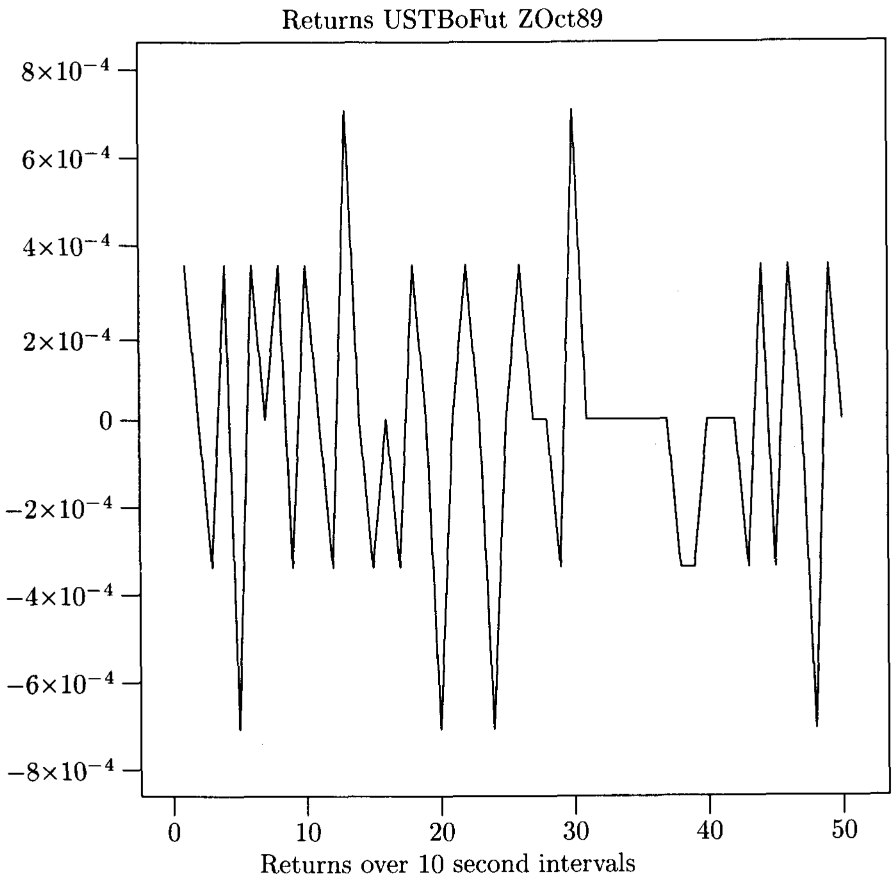

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Figure 5. 10 Seconds Returns

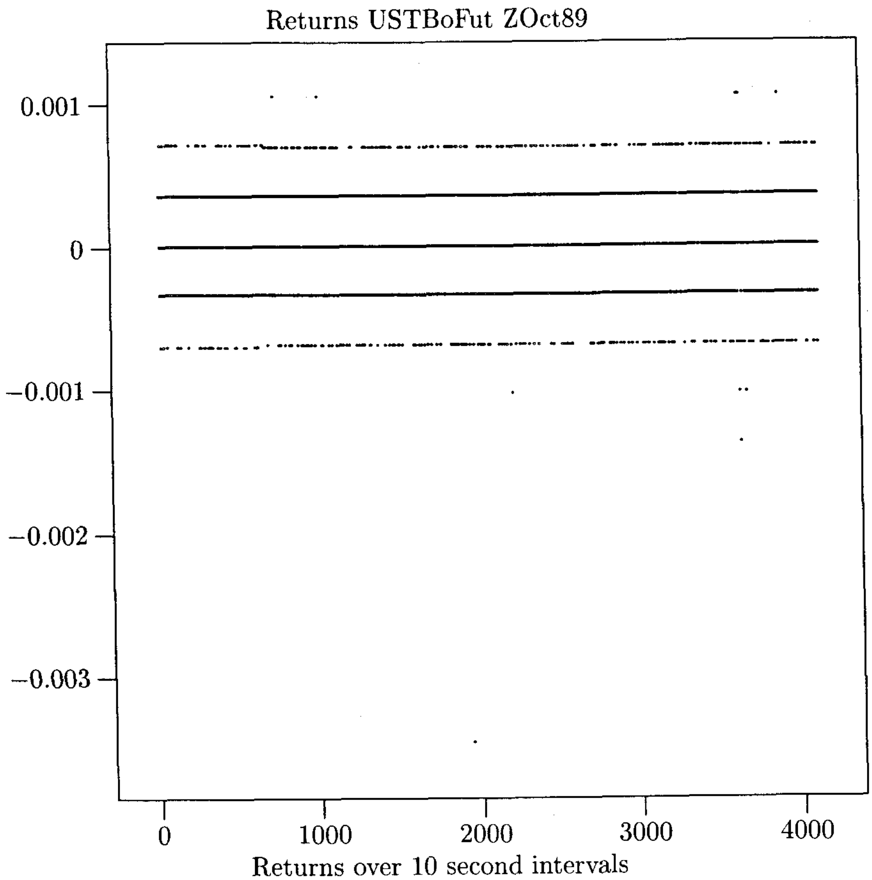

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Figure 6. 100 Second Returns

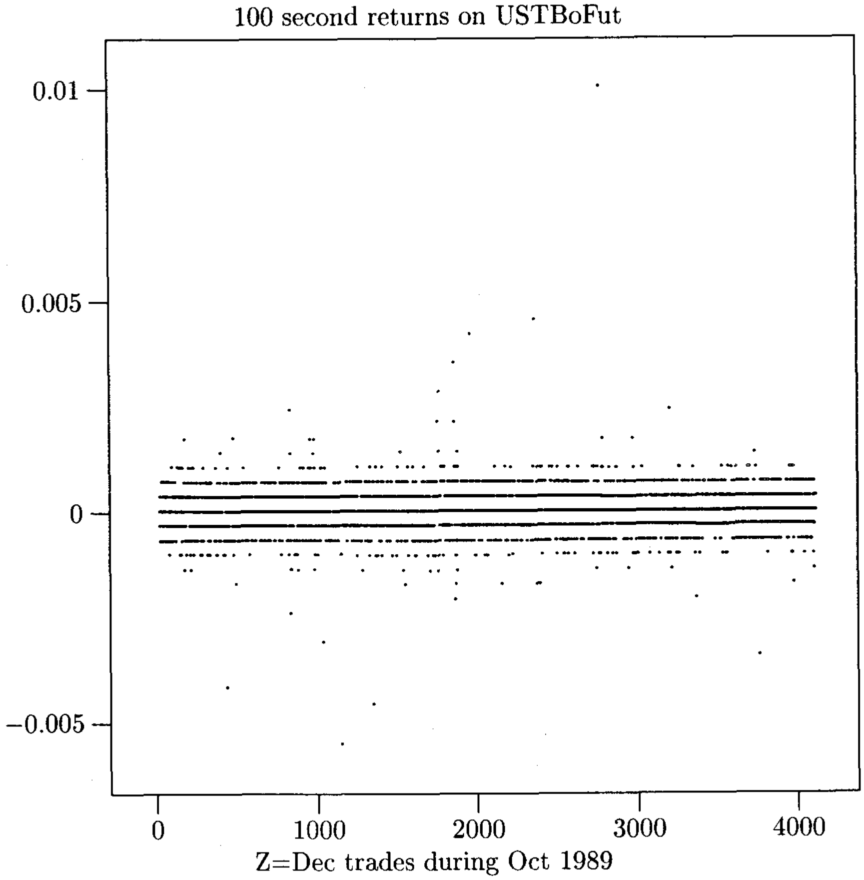

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Figure 7. 1,000 Second Returns

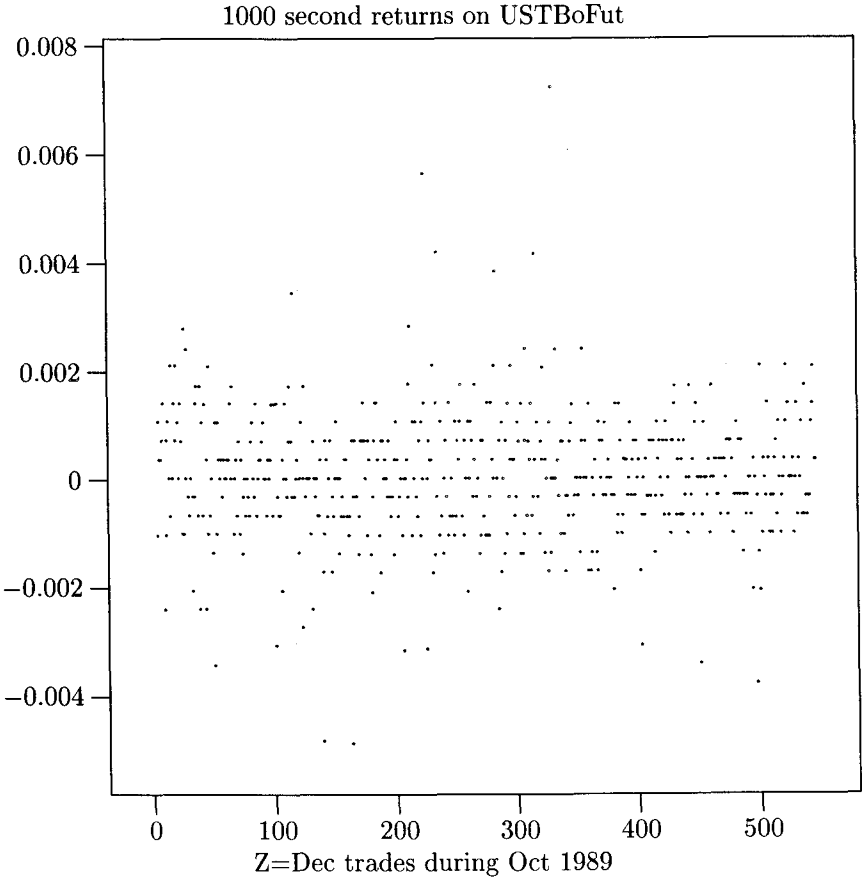

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Figure 8. 10,000 Seconds Returns

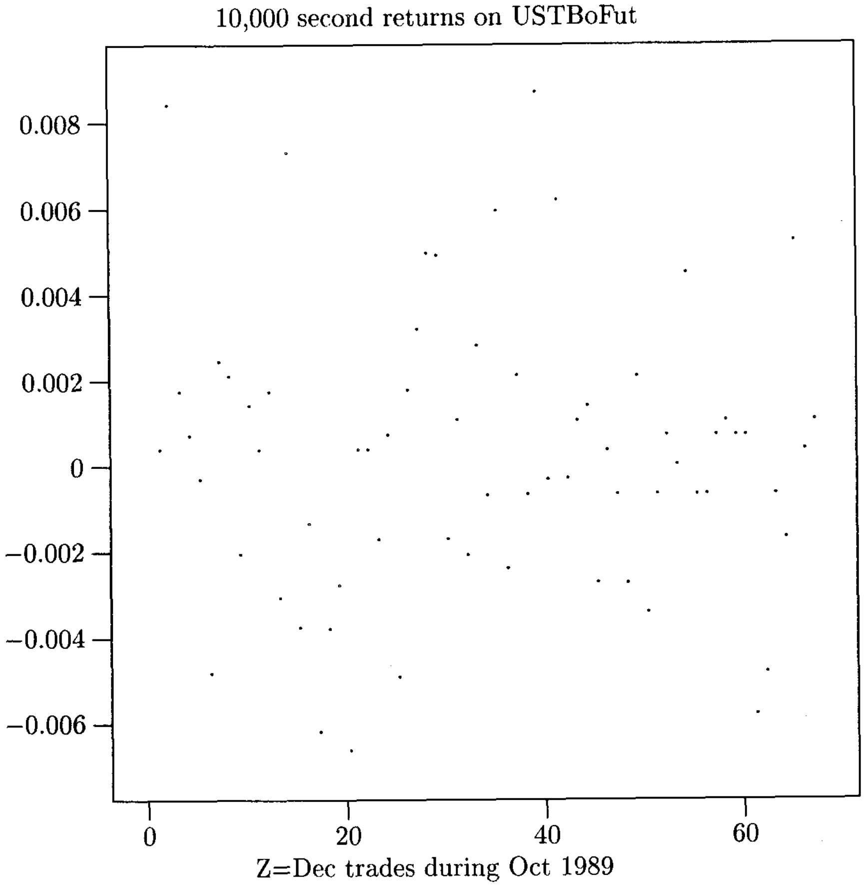

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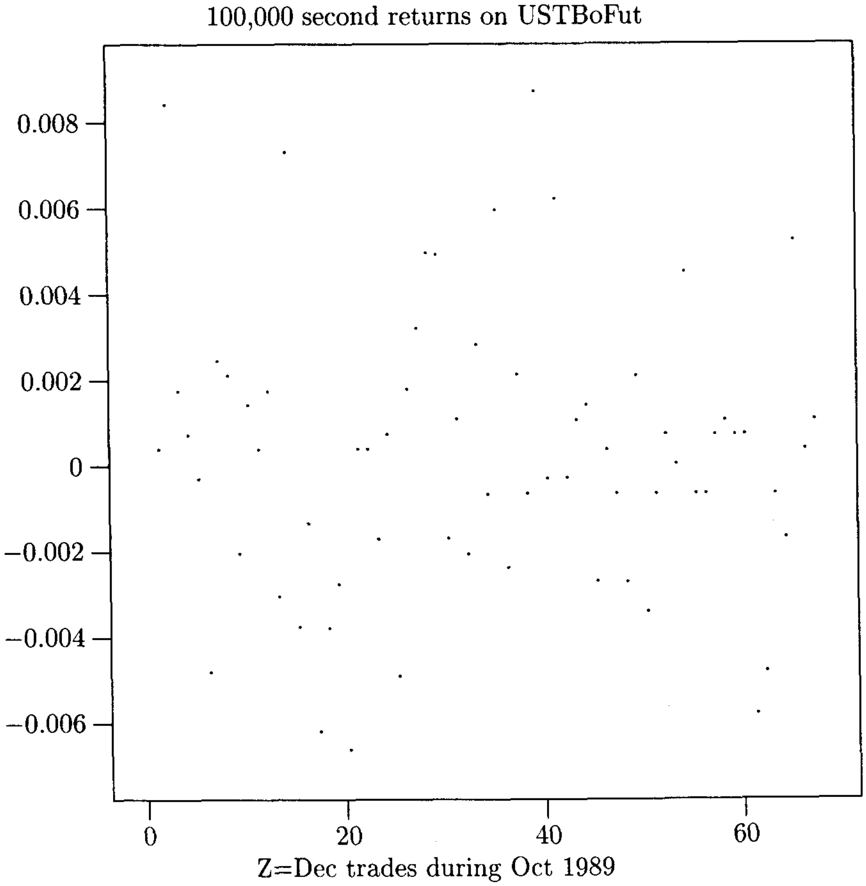
Figure 9. 100,000 Seconds Returns



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